Tutorial 5: Texture Mapping

This tutorial concerns texture mapping onto polygons. We will consider the case of a terminal in which individual pixels can be set to any intensity in the range $[0, I_{max}]$.

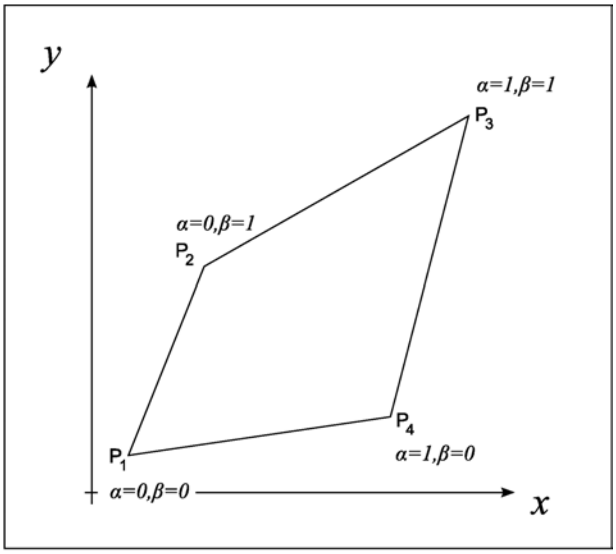
Suppose that the following function is used to transform texture coordinates (α, β) into an intensity value:

$$I = \frac{(\alpha + \beta)}{2} I_{\max}$$

I is therefore the intensity given to a pixel which corresponds to position (α, β) in texture space.

A quadrilateral is projected onto the screen. The screen coordinates and texture coordinates of its vertices are as follows:

Vertex	Screen coordinates (x, y)	Texture coordinates (α, β)
$\mathbf{P}_1$	(5,5)	(0, 0)
$\mathbf{P}_2$	(15,30)	(0, 1)
$\mathbf{P}_3$	(50,50)	(1, 1)
$\mathbf{P}_4$	(40,10)	(1, 0)



1. Let $\mathbf{P}_t$ be the point with screen coordinates (30, 30). Calculate the texture coordinates (α, β) corresponding to $\mathbf{P}_t$ and find the intensity to be applied to $\mathbf{P}_t$.

N.B. Use bi-linear interpolation to find the texture coordinates, i.e. solve for (α, β) the equation:

$$\mathbf{p} = \alpha\beta(\mathbf{c} - \mathbf{b}) + \alpha\,\mathbf{a} + \beta\,\mathbf{b}$$

Where

$\mathbf{p} = \mathbf{P}_t - \mathbf{P}_1$ $\quad$ $\mathbf{a} = \mathbf{P}_4 - \mathbf{P}_1$ $\quad$ $\mathbf{b} = \mathbf{P}_2 - \mathbf{P}_1$ $\quad$ $\mathbf{c} = \mathbf{P}_3 - \mathbf{P}_4$

Computing texture coordinates for every pixel by using bi-linear interpolation is very time consuming, since the solution of a quadratic is required. By doing the calculations differentially, much computation time can be saved.

For example, consider the line from $\mathbf{P}_1$ to $\mathbf{P}_2$: At $\mathbf{P}_1$ $\beta = 0$ and at $\mathbf{P}_2$ $\beta = 1$ and there are 26 pixels on the line (the biggest change in the screen coordinates is along the y-component, from 5 to 30, which gives 26 pixels including the end-points).

Moving along the line from $\mathbf{P}_1$ to $\mathbf{P}_2$, the differential change in β from one pixel to the next is $\frac{1}{25}$ because 26 pixels (including end-points) have 25 spaces between them.

Thus, we can estimate the values of β at each pixel along the line simply by adding the differential as we move from one pixel to its neighbour.

2. What are the differentials in α and β for the four lines bounding the quadrilateral?

3. Now consider the horizontal line through the point (30, 30). Using the differential method above, find the values of α and β at the points where it intersects the sides of the quadrilateral, and hence find the differentials in α and β along the line.

4. Use these differential values to compute α and β at the point (30, 30). Find also the corresponding intensity to be given to the pixel. Compare your results with the values obtained in part 1.

5. Can you suggest why the methods in parts 1 and 3 do not give exactly the same result?

Notice that with the differential method, calculating α and β for most of the pixels in the quadrilateral requires only two additions, and hence is much faster.