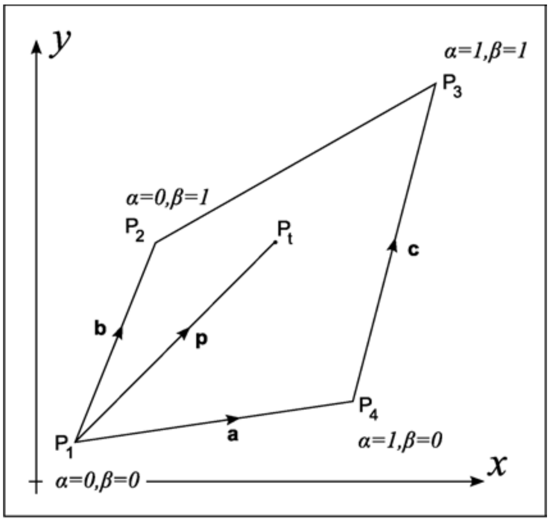
Tutorial 5: Texture Mapping

Solutions

Q1. Using the notation in the notes and writing vectors as columns, we have the following edge vectors for three of the quadrilateral's edges:

$$\mathbf{a} = \mathbf{P}_4 - \mathbf{P}_1 = \begin{pmatrix} 35 \\ 5 \end{pmatrix} \quad \mathbf{b} = \mathbf{P}_2 - \mathbf{P}_1 = \begin{pmatrix} 10 \\ 25 \end{pmatrix} \quad \mathbf{c} = \mathbf{P}_3 - \mathbf{P}_4 = \begin{pmatrix} 10 \\ 40 \end{pmatrix} \quad \mathbf{p} = \mathbf{P}_t - \mathbf{P}_1 = \begin{pmatrix} 25 \\ 25 \end{pmatrix}$$

The vector equation $\mathbf{p} = \alpha\beta(\mathbf{c} - \mathbf{b}) + \alpha\,\mathbf{a} + \beta\,\mathbf{b}$ can therefore be written:



$$\begin{pmatrix} 25 \\ 25 \end{pmatrix} = \alpha\beta \begin{pmatrix} 10-10 \\ 40-25 \end{pmatrix} + \alpha \begin{pmatrix} 35 \\ 5 \end{pmatrix} + \beta \begin{pmatrix} 10 \\ 25 \end{pmatrix}$$

Simplifying and separating the components, we obtain two equations

$$\begin{aligned} 25 &= 0\alpha\beta + 35\alpha + 10\beta \\ 25 &= 15\alpha\beta + 5\alpha + 25\beta \end{aligned}$$

Which can be simplified to

$$\begin{aligned} 7\alpha + 2\beta &= 5 \\ 3\alpha\beta + \alpha + 5\beta &= 5 \end{aligned}$$

The first equation gives $\beta = \dfrac{5 - 7\alpha}{2}$ which can be substituted into the second to obtain

$$3\alpha\left(\frac{5-7\alpha}{2}\right) + \alpha + 5\left(\frac{5-7\alpha}{2}\right) = 5$$

$$\Rightarrow 21\alpha^2 + 18\alpha - 15 = 0$$

$$\Rightarrow 7\alpha^2 + 6\alpha - 5 = 0$$

$$\Rightarrow \alpha \approx 0.52 \text{ or } -1.38$$

Ignoring the negative value of α, we have $\alpha \approx 0.52$ and $\beta \approx 0.68 = 0.5 \times (5 - 7 \times 0.52)$.

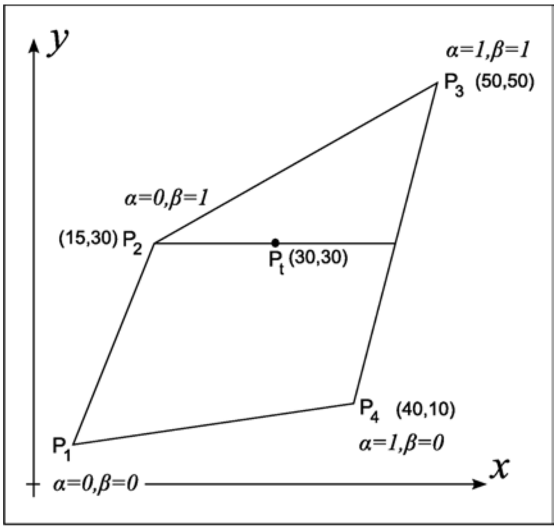
The intensity value to be given to the pixel should therefore be

$$I \approx \frac{(0.52 + 0.68)}{2} I_{\max} = 0.6 I_{\max}$$

Q2. The differentials of α and β along the for lines are as follows:

Start	End	x-change	y-change	No. Of pixels	$\Delta\alpha$	$\Delta\beta$
$\mathbf{P}_1$	$\mathbf{P}_2$	10	25	26	0	$1/25$
$\mathbf{P}_2$	$\mathbf{P}_3$	35	20	36	$1/35$	0
$\mathbf{P}_1$	$\mathbf{P}_4$	35	5	36	$1/35$	0
$\mathbf{P}_4$	$\mathbf{P}_3$	10	40	41	0	$1/40$

Q3. The horizontal line through $\mathbf{P}_t$ intersects the quadrilateral at two locations. One intersection is at the point (15, 30) which is the vertex $\mathbf{P}_2$, and has $\alpha = 0$, $\beta = 1$. The other intersection is half way along the line from $\mathbf{P}_4$ to $\mathbf{P}_3$ and has $\alpha = 1$, $\beta = 0.5$.



Since the pixel coordinate of the second intersection is (45, 30), the horizontal line has 31 pixels.

The differentials of α and β along the horizontal line are therefore:

$$\Delta\alpha = \frac{1-0}{30} = 1/30 \qquad \Delta\beta = \frac{0.5-1}{30} = -1/60$$

Q4. The pixel at $\mathbf{P}_t$ is the 16^{th} along the horizontal line. Its α and β values are therefore given by:

$$\alpha = 0 + 15\Delta\alpha = 0.5 \qquad \beta = 1 + 15\Delta\beta = 0.75$$

The intensity given to the pixel based on these texture coordinates is therefore

$$I \approx \frac{(0.5+0.75)}{2} I_{\max} = 0.625\, I_{\max}$$

Q5. To compute the values of α and β along the sides of the quadrilateral the differential method uses linear interpolation, and since in every case one or other of the texture coordinates is a constant along each edge, the bi-linear interpolation equation reduces to a linear equation. Hence these values on the edges given by the differential method and the bi-linear interpolation method are the same.

However, along the horizontal line, both α and β vary and y is fixed. Thus the $\alpha\beta$ term in the interpolation equation remains, giving a 2nd order relationship of the form:

$$x = K_1\alpha^2 + K_2\alpha + K_3 \quad \text{and} \quad x = L_1\beta^2 + L_2\beta + L_3$$

Hence, linear interpolation will not give the same result along the line. In practice, the differences will be small if the edge vectors **b** and **c** are similar.