Tutorial 7: Ray Tracing

Answers

Q1.

Tests to exclude simple cases where we can be sure a ray starting at (x_{pix}, y_{pix}) does not intersect the object:

Cylinder :

Radius : r
Centre 1 : (x_1, y_1, z_1)
Centre 2 : (x_2, y_2, z_2)
Bounding rectangle in xy-plane:

$x_{min} = \min(x_1, x_2) - r$ $\quad$ $x_{max} = \max(x_1, x_2) + r$
$y_{min} = \min(y_1, y_2) - r$ $\quad$ $y_{max} = \max(y_1, y_2) + r$

Test :

No intersection if

$(x_{pix} \le x_{\min})$ OR $(x_{\max} \le x_{pix})$ OR $(y_{pix} \le y_{\min})$ OR $(y_{\max} \le y_{pix})$

Sphere:

Radius : r
Centre : (c_x, c_y, c_z)
Bounding rectangle in xy-plane:

$x_{min} = c_x - r$ $\quad$ $x_{max} = c_x + r$
$y_{min} = c_y - r$ $\quad$ $y_{max} = c_y + r$

Test :

No intersection if

$(x_{pix} \le x_{\min})$ OR $(x_{\max} \le x_{pix})$ OR $(y_{pix} \le y_{\min})$ OR $(y_{\max} \le y_{pix})$

Box:

Apex
A : (a_x, a_y, a_z),
Edges:
$\mathbf{e_1}$: (x_1, y_1, z_1)
$\mathbf{e_2}$: (x_2, y_2, z_2)
$\mathbf{e_3}$: (x_3, y_3, z_3)

We can represent each vertex of the box as a vector sum starting at the apex using different combinations of the edges

$$
\begin{array}{lllll}
\mathbf{v}_0 & = & \mathbf{A} + 0\times\mathbf{e}_1 + 0\times\mathbf{e}_2 + 0\times\mathbf{e}_3 & = & \mathbf{A} \\
\mathbf{v}_1 & = & \mathbf{A} + 1\times\mathbf{e}_1 + 0\times\mathbf{e}_2 + 0\times\mathbf{e}_3 & = & \mathbf{A} + \mathbf{e}_1 \\
\mathbf{v}_2 & = & \mathbf{A} + 1\times\mathbf{e}_1 + 1\times\mathbf{e}_2 + 0\times\mathbf{e}_3 & = & \mathbf{A} + \mathbf{e}_1 + \mathbf{e}_2 \\
 & \vdots & & \vdots & \\
\mathbf{v}_7 & = & \mathbf{A} + 1\times\mathbf{e}_1 + 1\times\mathbf{e}_2 + 1\times\mathbf{e}_3 & = & \mathbf{A} + \mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3
\end{array}
$$

Writing the x-component of vertex $\mathbf{v}_i$ as $\mathbf{v}_{i,x}$ and the y-component as $\mathbf{v}_{i,y}$, we can write the limits of the bounding rectangle in the xy-plane as:

$$x_{\min} = \min_i(\mathbf{v}_{i,x}) \ , \ x_{\max} = \max_i(\mathbf{v}_{i,x})$$
$$y_{\min} = \min_i(\mathbf{v}_{i,y}) \ , \ y_{\max} = \max_i(\mathbf{v}_{i,y})$$

In the special case where the edges of the box are aligned with the x, y and z axes, the bounding rectangle in the xy-plane can be written more simply using the x- and y-components of the apex and the edge vectors $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$

$$x_{\min} = a_x + \min(x_1, x_2, x_3) \quad , \ x_{\max} = a_x + \max(x_1, x_2, x_3)$$
$$y_{\min} = a_y + \min(y_1, y_2, y_3) \ , \ y_{\max} = a_y + \max(y_1, y_2, y_3)$$

In both cases, the test is once again:

No intersection if
$(x_{pix} \le x_{\min})$ OR $(x_{\max} \le x_{pix})$ OR $(y_{pix} \le y_{\min})$ OR $(y_{\max} \le y_{pix})$

Q2.

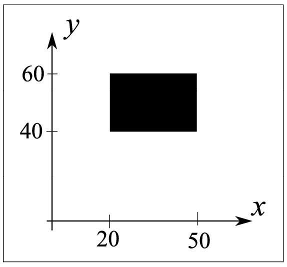
Using the tests of the previous question, can the rays starting at (32,52) and (32,58) be ruled out from intersecting with the given shapes:

	C_1	**C_2**	r
Cylinder 1	(20, 50, 50)	(50, 50, 50)	10

x_{min} = 20 - 10 = 10 $\quad$ x_{max} = 50 + 10 = 60
y_{min} = 50 - 10 = 40 $\quad$ y_{max} = 50 + 10 = 60

The simple test fails for both Ray1 and Ray2, i.e. each ray could still intersect with cylinder 1 and a further test is needed to decide for sure.

For this particular cylinder, the central axis runs parallel to the x-axis, this means that the cylinder projects onto a simple rectangle in the xy-plane.



It is then easy to see that both rays actually do intersect the cylinder.

	$\mathbf{C_1}$	$\mathbf{C_2}$	r
Cylinder 2	(35, 55, 40)	(35, 55, 60)	5

$x_{min} = 35 - 5 = 30$ $\quad$ $x_{max} = 35 + 5 = 40$
$y_{min} = 55 - 5 = 50$ $\quad$ $y_{max} = 55 + 5 = 60$

The simple test fails for both Ray1 and Ray2, i.e. each ray could still intersect with cylinder 2 and a further test is needed to decide for sure.

For this particular cylinder, the central axis runs parallel to the z-axis, this means that the cylinder projects onto a circle with radius 5 in the xy-plane. A second test can be carried out by calculating the distance between each ray's pixel and the centre of the circle and testing if it is less than the radius.

	C	r
Sphere 1	(20, 50, 50)	10

$x_{min} = 20\text{-}10 = 10$ $\quad$ $x_{max} = 20 + 10 = 30$
$y_{min} = 50 - 10 = 40$ $\quad$ $y_{max} = 50 + 10 = 60$

Both Ray1 and Ray2 are outside the bounding rectangle, i.e. we can be sure they miss the sphere.

Both boxes in Q1 have edges aligned with the coordinate axes. This means we can use the simpler definition for their bounding rectangles in the xy-plane.

	A	$\mathbf{e_1}$	$\mathbf{e_2}$	$\mathbf{e_3}$
Box 1	(35, 45, 40)	(15, 0, 0)	(0, 15, 0)	(0, 0, 20)

$x_{min} = 35+0 = 35$ $\quad$ $x_{max} = 35+15 = 50$
$y_{min} = 45+0 = 45$ $\quad$ $y_{max} = 45 + 15 = 60$

Both rays are outside.

	A	$\mathbf{e_1}$	$\mathbf{e_2}$	$\mathbf{e_3}$
Box 2	(30, 55, 40)	(5, 0, 0)	(0, -5, 0)	(0, 0, 20)

$x_{min} = 30$ $\quad$ $x_{max} = 35$
$y_{min} =50$ $\quad$ $y_{max} = 55$

The test excludes Ray2, it is definitely outside the bounding rectangle so it will not intersect the box.

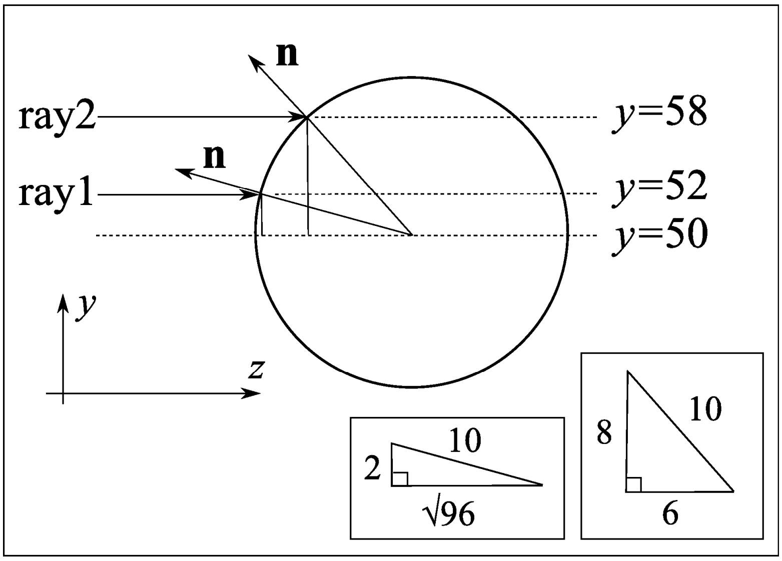
The test fails for Ray1, so it may intersect the box. Note however that the edges of Box 2 are all parallel with the coordinate axes which means that Ray1 does in fact intersect the box.

Q3.

Cylinder 1:

As noted in the previous question, this cylinder's main axis runs parallel to the *x*-axis. This means that the normals at the points where a ray intersects are in the *yz*-plane, i.e. they have no x component.

Consider a section through the cylinder parallel to the *yz*-plane:



Right angled triangles can be obtained using the point where a ray intersects the cylinder, the point on the diameter below the intersection and the centre of the cylinder's cross-section. These triangles can be used to obtain the components of the normals at the ray's intersection point.

Ray	Components	Normalised
1	$(0, 2, -\sqrt{96})$	$\frac{1}{5}(0, 1, -2\sqrt{6})$
2	$(0, 8, -6)$	$(0, 0.8, -0.6)$

Cylinder 2:
Both rays miss.

Box 1:
Both rays miss

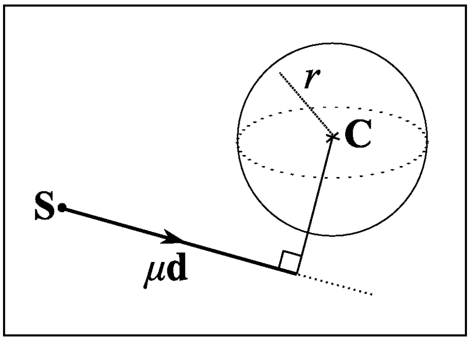
Box 2:
Ray 1 hits end on, normal vector (0, 0, -1). Ray 2 misses

Q4.

For perspective projection, placing bounding spheres around the objects is the best bet. The test is to calculate the distance of the centre of the sphere from the ray. If the ray is given by $\mathbf{S} + \mu\mathbf{d}$, and the centre of the bounding sphere is $\mathbf{C}$, the closest point that the ray gets to the centre of the sphere is given by the value of μ for which:

$$(\mathbf{C} - \mathbf{S} - \mu\mathbf{d})\cdot\mathbf{d} = 0$$

i.e. we are finding a perpendicular from the point **C** to the ray.



If we use a unit vector for **d**, we have:

$$\mu = \frac{(\mathbf{C}\cdot\mathbf{d} - \mathbf{S}\cdot\mathbf{d})}{\mathbf{d}\cdot\mathbf{d}} = \mathbf{C}\cdot\mathbf{d} - \mathbf{S}\cdot\mathbf{d}$$

Using the calculated value of μ, we can test if

$$\left|\mathbf{C} - \mathbf{S} - \mu\,\mathbf{d}\right| < r$$

for a possible intersection. We can solve for the value of r where $(\mathbf{C} - \mathbf{S} - \mu\mathbf{d}) \cdot (\mathbf{C} - \mathbf{S} - \mu\mathbf{d}) - r^2 = 0$. In practice a simple to compute condition can be obtained.