Tutorial 8: Radiosity

Solutions

1. *Form factors*:

Patch	Points		
i	(10, 12, 8)	(10, 13, 8)	(10, 11, 9)
j	(5, 6, 12)	(5, 6, 13)	(8, 6, 12)

The form factor is given by $F_{ij} = \dfrac{\cos\phi_i \cos\phi_j |A_j|}{\pi r^2}$.

The centroids are: (10, 12, 8.33) and (6, 6, 12.33).

The vector **r** joining the patches is (4, 6, -4) and $r^2 = 4^2 + 6^2 + (-4)^2 = 68$, i.e. $r = |\mathbf{r}| = \sqrt{68}$.

Normal vectors can be found from the cross product of the edge vectors:

Patch	Edge vectors	Cross product	Unit normal
i	(0, 1, 0) (0, -1, 1)	(1, 0 , 0)	(1, 0, 0)
j	(0, 0, 1) (3, 0, 0)	(0, 3, 0)	(0, 1, 0)

Thus

$$\cos\phi_i = \frac{\mathbf{n}_i \cdot \mathbf{r}}{|\mathbf{r}|} = \frac{4}{r} \text{ and } \cos\phi_j = \frac{\mathbf{n}_j \cdot \mathbf{r}}{|\mathbf{r}|} = \frac{6}{r}$$

The area of a parallelogram spanned by two vectors is given by the magnitude of the cross product vector. The triangle spanned by the vectors is half the parallelogram, so the patch areas are:

$$|A_i| = \frac{1}{2}\times 1 = \frac{1}{2} \text{ and } |A_j| = \frac{1}{2}\times 3 = \frac{3}{2}$$

So

$$F_{ij} = \frac{\left(\frac{4}{r}\right)\left(\frac{6}{r}\right)\frac{1}{2}}{\pi r^2} = \frac{12}{\pi r^4} = \frac{12}{4624\pi} \approx 0.0008 \text{ and } F_{ji} = \frac{36}{4624\pi} \approx 0.0025$$

2. The hemicube:

Consider one face, say $x = -1$. The unit normal vector is $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. The vector from a point on the face to the origin is $\begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix}$. Making this into a unit vector **p** gives

$$\mathbf{p} = \frac{1}{\sqrt{1 + y^2 + z^2}} \begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix} = \frac{1}{r} \begin{pmatrix} 1 \\ -y \\ -z \end{pmatrix} \quad \text{and} \quad \cos\phi_i = \mathbf{n} \cdot \mathbf{p} = \frac{1}{r}$$

At the origin the unit normal vector is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

The unit vector from the origin towards the point is $-\mathbf{p}$ so $\cos\phi_j = \frac{z}{r}$. Therefore the form factor is

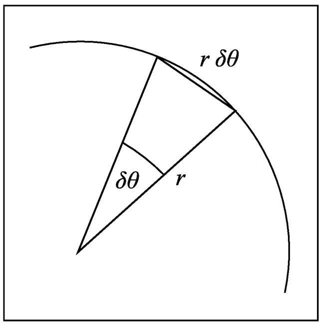
$$\frac{\Delta A \cos\phi_i \cos\phi_j}{\pi\, r^2} = \frac{\Delta A\, z}{\pi\, r^4}$$

The form factors for the other side faces are all the same by symmetry.

3. The hemisphere:

The situation is simpler than for the hemicube because $r = 1, \cos\phi_i = 1$, and $\cos\phi_j = z$.
Thus the delta form factor is just $\frac{\Delta A\, z}{\pi}$

Now we need to estimate ΔA . Each ray passes through a patch bounded by four small arcs that can be viewed as approximating a square. The radius equals 1 so the length of each arc subtended by a small angle of 1 degree is $\frac{\pi}{180}$ (see right).



Assuming that each arc is a side of the approximated square patch gives an area estimate of $\left(\frac{\pi}{180}\right)^2$

Thus the form factor for each patch is $\left(\frac{\pi}{180}\right)^2 \frac{z}{\pi} = \frac{\pi\, z}{180^2}$.

4. r-refinement:

The direction of movement can be normalised to the maximum radiosity. One scheme could be to divide the direction vector by the sum of the total radiosity change

$$|B_1 - B| + |B_2 - B| + |B_3 - B| + |B_4 - B| = \sum_{i=1}^{4} |B_i - B|$$

A problem with this would be the case where *all* the radiosity change was to one neighbour. This would move the point all the way to that neighbour. This suggests that it would be better to relax the change by at least a further half.

For the numeric example:

$$|B_1 - B|\,(\mathbf{P}_1 - \mathbf{P}) = 20 \times \begin{pmatrix} -10 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ 80 \\ 0 \end{pmatrix} \qquad |B_2 - B|\,(\mathbf{P}_2 - \mathbf{P}) = 10 \times \begin{pmatrix} -10 \\ 24 \\ 0 \end{pmatrix} = \begin{pmatrix} -100 \\ 240 \\ 0 \end{pmatrix}$$

$$|B_3 - B|\,(\mathbf{P}_3 - \mathbf{P}) = 0 \times \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad |B_4 - B|\,(\mathbf{P}_4 - \mathbf{P}) = 20 \times \begin{pmatrix} -10 \\ -6 \\ 0 \end{pmatrix} = \begin{pmatrix} -200 \\ -120 \\ 0 \end{pmatrix}$$

These values give the direction $\begin{pmatrix} -500 \\ 200 \\ 0 \end{pmatrix}$. Normalising by the sum of the radiosity changes gives a direction of:

$$\frac{1}{50} \begin{pmatrix} -500 \\ 200 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 4 \\ 0 \end{pmatrix}$$

Relaxing by a factor of 2 gives

$$\begin{pmatrix} -5 \\ 2 \\ 0 \end{pmatrix}$$

So **P** moves from (20, 6, 0) to (15, 8, 0) . This is perhaps rather too large a change for an iterative process. A better scheme might involve using the distances between the points.