Effects in a pi

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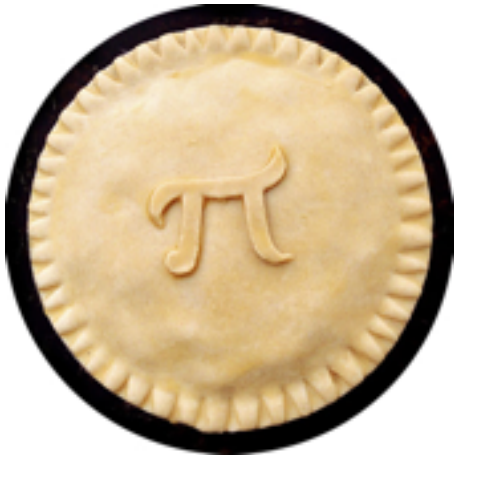


λ-calculus + state

+ simple types

embedding

π-calculus + sessions

+ session types

λ-calculus + state

+ simple types

+ effect systems

embedding

this work*

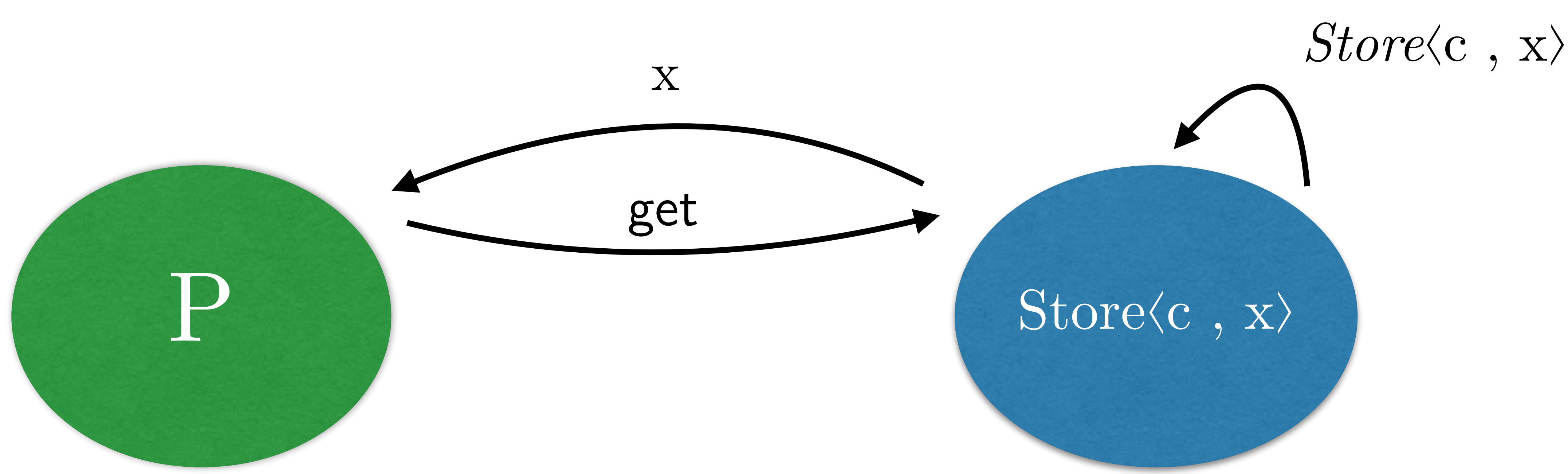
π-calculus + sessions

+ session types

- Impure π-calculus with effect system for free
- Define concurrent state semantics via π-calculus
- Compile into π-calculus

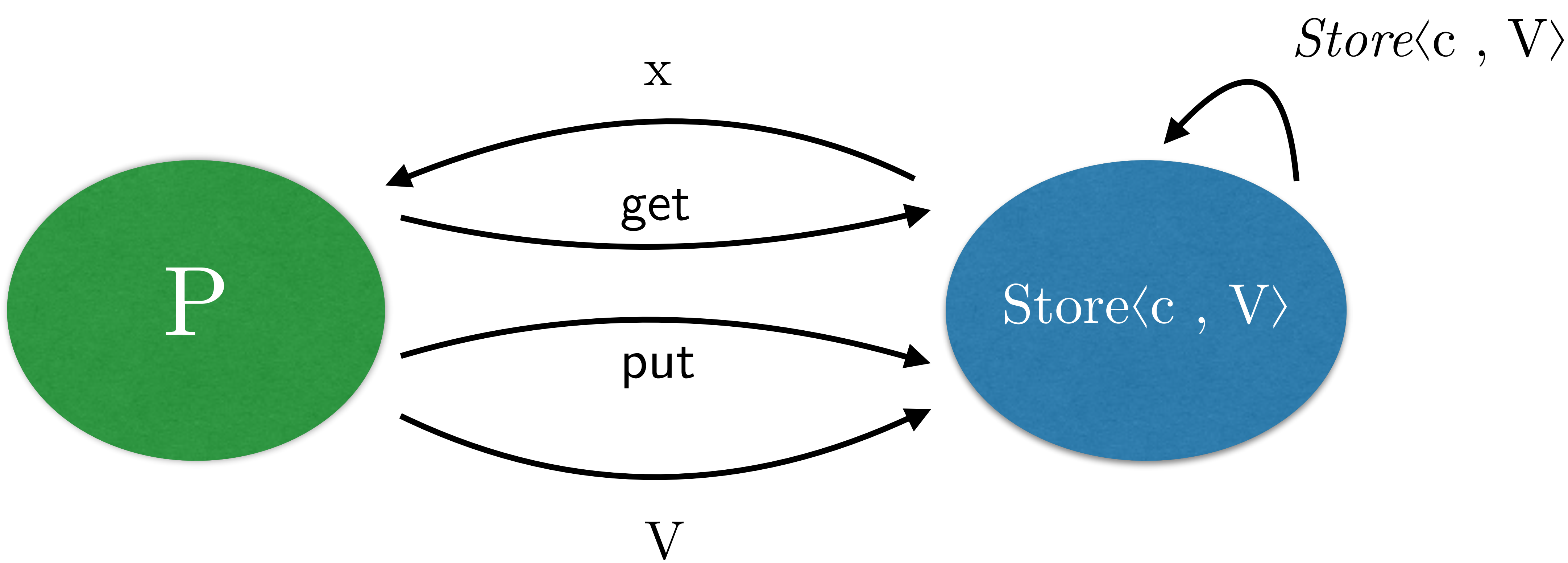
* ***Using session types as an effect system***, Orchard, Yoshida, PLACES'15 pre-proceedings

Variable agent



Variable agent

$$\begin{aligned}\mathbf{def}\ \mathit{Store}(\mathrm{c}, \mathrm{x}) = \mathrm{c} \rhd \{&\mathsf{get} : \mathrm{c}!\langle \mathrm{x}\rangle.\mathit{Store}\langle \mathrm{c}, \mathrm{x}\rangle,\\ &\mathsf{put} : \mathrm{c}?(\mathrm{y}).\mathit{Store}\langle \mathrm{c}, \mathrm{y}\rangle,\\ &\mathsf{stop} : \mathbf{0}\}\\ \mathbf{in}\ \mathit{Store}\langle \mathrm{c}\ , \mathrm{i}\rangle&\end{aligned}$$



Variable agent

$$\begin{aligned}
&\mathbf{def}\ \mathit{Store}(\mathrm{c}, \mathrm{x}) = \mathrm{c} \triangleright \{\mathsf{get} : \mathrm{c}!\langle \mathrm{x}\rangle.\mathit{Store}\langle \mathrm{c}, \mathrm{x}\rangle,\\
&\qquad\qquad\qquad\qquad\quad \mathsf{put} : \mathrm{c}?(\mathrm{y}).\mathit{Store}\langle \mathrm{c}, \mathrm{y}\rangle,\\
&\qquad\qquad\qquad\qquad\quad \mathsf{stop} : \mathbf{0}\}\\
&\mathbf{in}\ \mathit{Store}\langle \mathrm{c}\ ,\ \mathrm{i}\rangle
\end{aligned}$$

Server

$$\begin{aligned}
\mathrm{get}\ (\mathrm{c})(\mathrm{x}).\mathrm{P} &= (\mathrm{c} \triangleleft \mathsf{get}).\mathrm{c}?(\mathrm{x}).\mathrm{P}\\
\mathrm{put}\ (\mathrm{c})\langle \mathrm{V}\rangle.\mathrm{P} &= (\mathrm{c} \triangleleft \mathsf{put}).\mathrm{c}!\langle \mathrm{V}\rangle.\mathrm{P}\\
\mathrm{stop} &= (\mathrm{c} \triangleleft \mathsf{stop}).\mathbf{0}
\end{aligned}$$

Client

e.g. increment store

$$\mathbf{def}\ \mathit{Store}(\mathrm{c}, \mathrm{x}) = \ldots\ \mathbf{in}\ \ (\mathrm{get}(\mathrm{c})(\mathrm{x}).\mathrm{put}(\mathrm{c})\langle \mathrm{x}+1\rangle.\mathrm{stop}\ \ |\ \mathit{Store}\langle \mathrm{c}\ ,\ \mathrm{i}\rangle)$$

$\Gamma \vdash \mathrm{M} : \tau,\ \mathsf{F}$

Effect calculus

abs $$\frac{\Gamma, x : \sigma \vdash M : \tau,\ \mathsf{F}}{\Gamma \vdash \lambda x.M : \sigma \xrightarrow{\mathsf{F}} \tau,\ \varnothing}$$

$$\frac{\Gamma \vdash M : \sigma,\ \mathsf{F} \qquad \Gamma, x : \sigma \vdash N : \tau,\ \mathsf{G}}{\Gamma \vdash \mathbf{let}\ x = M\ \mathbf{in}\ N : \tau,\ \mathsf{F} \bullet \mathsf{G}}$$

app $$\frac{\Gamma \vdash M : \sigma \xrightarrow{\mathsf{H}} \tau,\ \mathsf{F} \qquad \Gamma \vdash N : \sigma,\ \mathsf{G}}{\Gamma \vdash M\ N : \tau,\ \mathsf{F} \bullet \mathsf{G} \bullet \mathsf{H}}$$

var $$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma,\ \varnothing}$$

monoid $(\mathsf{F}, \bullet, \varnothing)$

Effect calculus for state

$$\frac{\Gamma \vdash V : \tau, [\,]}{\Gamma \vdash \mathsf{put}\ V : (), [\mathrm{put}\ \tau]} \qquad \frac{}{\Gamma \vdash \mathsf{get} : \tau, [\mathrm{get}\ \tau]}$$

$$(\mathrm{List}\ \{\mathrm{put}\ \boldsymbol{t}, \mathrm{get}\ \boldsymbol{t} \mid \boldsymbol{t} \in \tau\}, ++, [\,])$$

e.g. increment store

$$\Gamma \vdash \mathbf{let}\ x = \mathsf{get}\ \mathbf{in}\ \mathsf{put}\ (x+1) : \mathrm{int}, [\mathrm{get\ int}, \mathrm{put\ int}]$$

$\Gamma \,;\, \Delta \vdash \mathrm{P}$

π-calculus with sessions

$$(\text{recv})\ \frac{\Gamma, x:\tau; \Delta, c:S \vdash P}{\Gamma; \Delta, c:?[\tau].S \vdash c?(x).P} \qquad (\text{send})\ \frac{\Gamma; \emptyset \vdash e:\tau \quad \Gamma; \Delta, c:S \vdash P}{\Gamma; \Delta, c:![\tau].S \vdash c!\langle e \rangle.P}$$

$$(\text{branch})\ \frac{\Gamma; \Delta, c:S_i \vdash P_i}{\Gamma; \Delta, c:\&[\tilde{l}:\tilde{S}] \vdash c \rhd \{\tilde{l}:\tilde{P}\}} \qquad (\text{select})\ \frac{\Gamma; \Delta, c:S \ \vdash P}{\Gamma; \Delta, c:\oplus[l:S] \vdash c \lhd l.P}$$

e.g. increment store

$$\mathrm{get}\ (\mathrm{c})(\mathrm{x}).\mathrm{P} = \mathrm{c} \lhd \mathrm{get}\ .\ \mathrm{c}?(\mathrm{x}).\mathrm{P}$$

$$\mathrm{put}\ (\mathrm{c})\langle \mathrm{V} \rangle.\mathrm{P} = \mathrm{c} \lhd \mathrm{put}\ .\ \mathrm{c}!\langle \mathrm{V} \rangle.\mathrm{P}$$

$$\Gamma, \mathrm{c} : \oplus[\mathrm{get} : ?[\mathrm{int}].\ \oplus[\mathrm{put} : ![\mathrm{int}].\mathbf{end}\ ]] \ \vdash \mathrm{get}(\mathrm{c})(\mathrm{x}).\mathrm{put}(\mathrm{c})(\mathrm{x}+1).\mathbf{0}$$

cf. effects $[\mathrm{get\ int},\ \mathrm{put\ int}]$

Encoding effect annotations as sessions

$$[\![\ [] \ ]\!] = \mathbf{end}$$

$$[\![\ (\mathrm{get}\ \tau) : \mathsf{F}\ ]\!] = \oplus[\mathsf{get} : ?[\![\tau]\!].\ [\![\ \mathsf{F}\ ]\!]\ ]$$

$$[\![\ (\mathrm{get}\ \tau) : \mathsf{F}\ ]\!] = \oplus[\mathsf{put} : ![\![\tau]\!].\ [\![\ \mathsf{F}\ ]\!]\ ]$$

For list-based effect annotations over elements E

given $\quad \mathrm{embed} : \mathrm{E} \rightarrow (S \rightarrow S)$

then $\quad [\![\ \mathsf{F}\ ]\!] \ : [\mathrm{E}] \rightarrow S$

$$= \mathrm{fold\ embed}\ (\lambda.\ \mathbf{end})\ \mathsf{F}$$

Embedding

$$[\![\Gamma \vdash \mathrm{M} : \tau, \mathsf{F}]\!] \xrightarrow{\text{embedding}} [\![\Gamma]\!];\ (r : ![\![\tau]\!].\mathbf{end},\ \mathit{eff} : [\![\mathsf{F}]\!]) \vdash \nu ei, eo\ .\ ((\!| \Gamma \vdash \mathrm{M} : \tau, \mathsf{F} |\!)_r^{ei,eo} \mid \underline{ei}!\langle \mathit{eff} \rangle.eo(c).\mathbf{0})$$

$$(\!| \Gamma \vdash \mathrm{M} : \tau, \mathsf{F} |\!)_r^{ei,eo} = \forall \mathsf{g}\ .$$

$$[\![\Gamma]\!];\ (r : ![\![\tau]\!].\mathbf{end},\ ei : ?[\![\mathsf{F} \bullet \mathsf{g}]\!],\ \underline{eo} : ![\![\mathsf{g}]\!]) \vdash (\!| \mathrm{M} |\!)_r^{ei,eo}$$

receive channel with effect capabilities

send channel with effect capabilities

Embedding (zeroth-order)

$$\llparenthesis\ \Gamma \vdash x : \tau,\ \varnothing\ \rrparenthesis_{r}^{ei,eo} \quad = \quad ei?(c).r!\langle x\rangle.\underline{eo}!\langle c\rangle$$

where $\forall \mathsf{g}\,.\ \ \llbracket\ \Gamma\ \rrbracket;\ \ r : !\llbracket \tau \rrbracket,\ ei : ?\llbracket\ \mathsf{g}\ \rrbracket,\ \underline{eo} : !\llbracket\ \mathsf{g}\ \rrbracket \vdash ei?(c).r!\langle x\rangle.\underline{eo}!\langle c\rangle$

$$\llparenthesis\ \Gamma \vdash \mathbf{let}\ x = M\ \mathbf{in}\ N : \tau,\ \mathsf{F} \bullet \mathsf{G}\ \rrparenthesis_{r}^{ei,eo} \quad =$$

$$\nu\, q,\ a\,.\ (\llparenthesis\ M\ \rrparenthesis_{q}^{ei,\,a}\ \mid \underline{q}?(x).\llparenthesis\ N\ \rrparenthesis_{r}^{a,\,eo}\ )$$

where $\forall \mathsf{h}\,.\quad q : !\llbracket \sigma \rrbracket,\ ei : ?\llbracket\ \mathsf{F} \bullet \mathsf{G} \bullet \mathsf{h}\ \rrbracket,\ \underline{a} : !\llbracket\ \mathsf{G} \bullet \mathsf{h}\ \rrbracket \vdash \llparenthesis\ M\ \rrparenthesis_{q}^{ei,\,a}$

$r : !\llbracket \tau \rrbracket,\ a : ?\llbracket\ \mathsf{G} \bullet \mathsf{h}\ \rrbracket,\ \ \underline{eo} : !\llbracket\ \mathsf{h}\ \rrbracket \vdash \llparenthesis\ N\ \rrparenthesis_{r}^{a,\,eo}$

Embedding (higher-order)

Must embed *latent effects* $\sigma \xrightarrow{F} \tau$

$$[\![\ \sigma \rightarrow \tau \ ]\!] = \ ![\![\ \sigma \ ]\!] \ . \ ![\ ! \ [\![\tau]\!] \ ] \ . \ \mathbf{end}$$

$$[\![\ \sigma \xrightarrow{F} \tau \ ]\!] = \ ![\![\ \sigma \ ]\!] \ . \ ![?[\![\ F \bullet G \ ]\!]] \ . \ ![![\![\ G \ ]\!]] \ . \ ![\ ! \ [\![\tau]\!] \ ] \ . \ \mathbf{end}$$

send channel which
can receive channel
with **latent** effect capabilities

send channel which
can send channel with effect
capabilities for **continuation**

Embedding (higher-order)

$$\llparenthesis\, \Gamma \vdash \lambda x\,.\, M : \sigma \xrightarrow{\mathsf{F}} \tau, \varnothing \,\rrparenthesis^{ei,eo}_{r} \quad =$$

$$\nu\, y\,.\, (ei?(c).\underline{eo}!\langle c\rangle.r\,!\langle y\rangle.y?(x,\ a,\ b,\ q).\llparenthesis\, M \,\rrparenthesis^{a,b}_{q}\ )$$

$$\forall \mathsf{g}, \mathsf{h}\,.\qquad r : ![\ ![\![\ \sigma\ ]\!]\ .\ ![?[\![\ \mathsf{F} \bullet \mathsf{h}\ ]\!]]\ .\ ![![\![\ \mathsf{h}\ ]\!]]\ .\ ![\ !\ [\![\tau]\!]\ ]\ ],$$

$$ei : ?[\![\ \mathsf{g}\ ]\!],\ \underline{eo} : ![\![\ \mathsf{g}\ ]\!]\ \vdash \nu\ y\ .\ (ei?\ ....)$$

$$\llparenthesis\, \Gamma \vdash M\ N : \tau, \mathsf{F} \bullet \mathsf{G} \bullet \mathsf{H} \,\rrparenthesis^{ei,eo}_{r} \quad =$$

$$\nu\ q,\ s,\ a,\ b\,.\ (\llparenthesis\, M \,\rrparenthesis^{ei,\,a}_{q}\ \mid \llparenthesis\, N \,\rrparenthesis^{a,\,b}_{s}\ \mid \underline{q}?(y).\underline{s}?(arg).y!\langle arg,\ b,\ eo,\ r\rangle)$$

Soundness

$$\Gamma \vdash M \equiv N : \tau, F$$

$$\Longrightarrow$$

$$[\![\Gamma]\!]; (r :! [\![\tau]\!].\mathbf{end}, e : [\![F]\!]) \vdash [\![M]\!]^e_r \approx [\![N]\!]^e_r$$

Future work

- Completeness and observational correspondence
- Subeffecting (via session subtyping)
- Different effect systems (what about sets?)
- Actually handling different kinds of concurrent effects

Conclusion

- Sessions and session types expressive enough to encode effects with an effect system
- Use this to give semantics of *concurrent effects*
  - *e.g., non-interference, atomicity via sessions*
- Effect-informed optimisations, e.g. implicit parallelism

if $\Gamma \vdash M : \sigma, \emptyset$

then $(\!|\, \mathbf{let}\, x \leftarrow M \,\mathbf{in}\, (\mathbf{let}\, y \leftarrow N \,\mathbf{in}\, P) \,|\!)_r^{\mathsf{ei},\mathsf{eo}}$

$$= \nu\, q, s, \mathsf{eb}.\, (\llbracket M \rrbracket_q \mid (\!|\, N \,|\!)_s^{\mathsf{ei},\mathsf{eb}} \mid \overline{q}?(x).\overline{s}?(y).(\!|\, P \,|\!)_r^{\mathsf{eb},\mathsf{eo}})$$

More details in our PLACES'15 *paper; see* dorchard.co.uk