Computing Solutions in OWL 2 QL Knowledge Base Exchange

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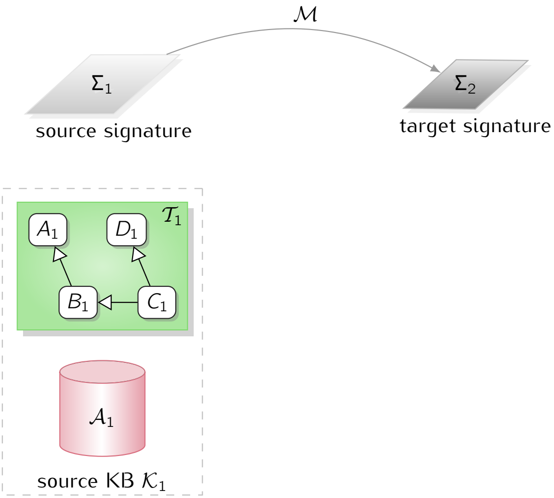
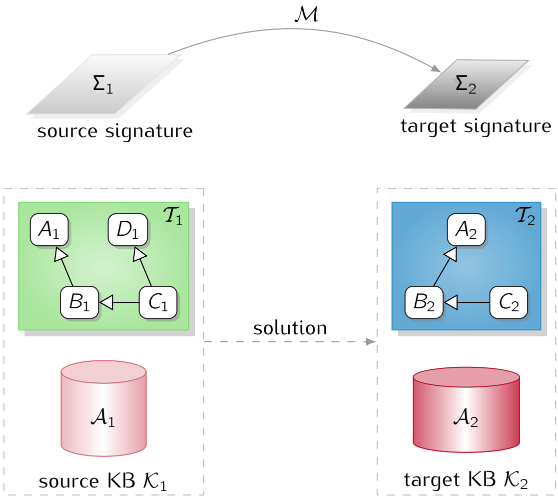
Description Logics
July 2013, Ulm

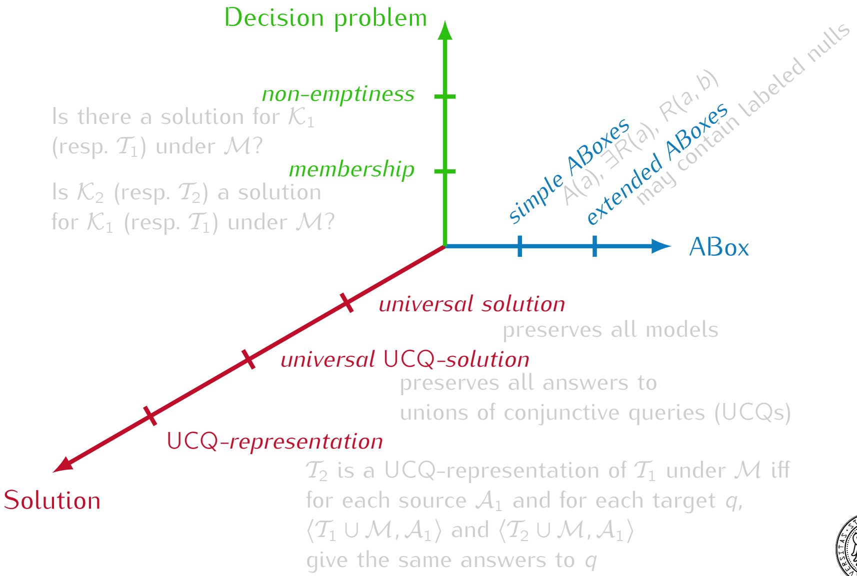
Knowledge Base Exchange Framework



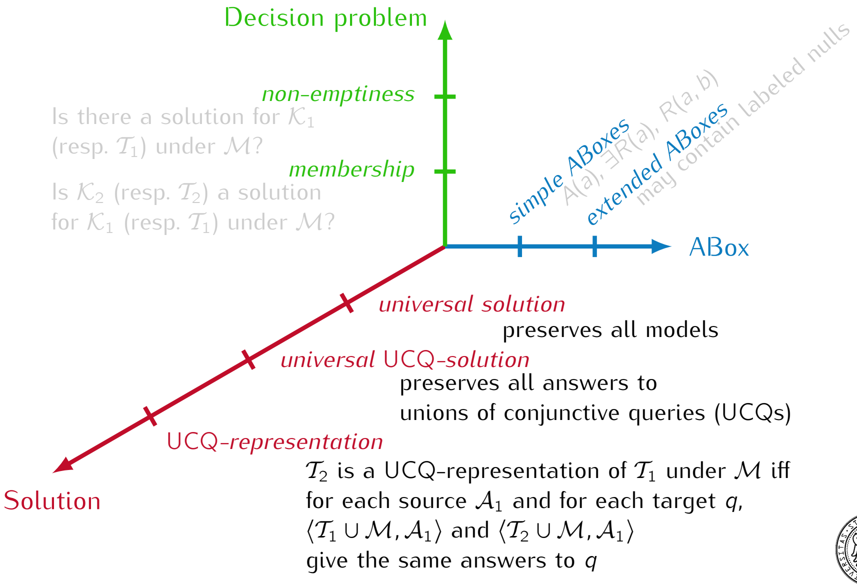
Knowledge Base Exchange Framework



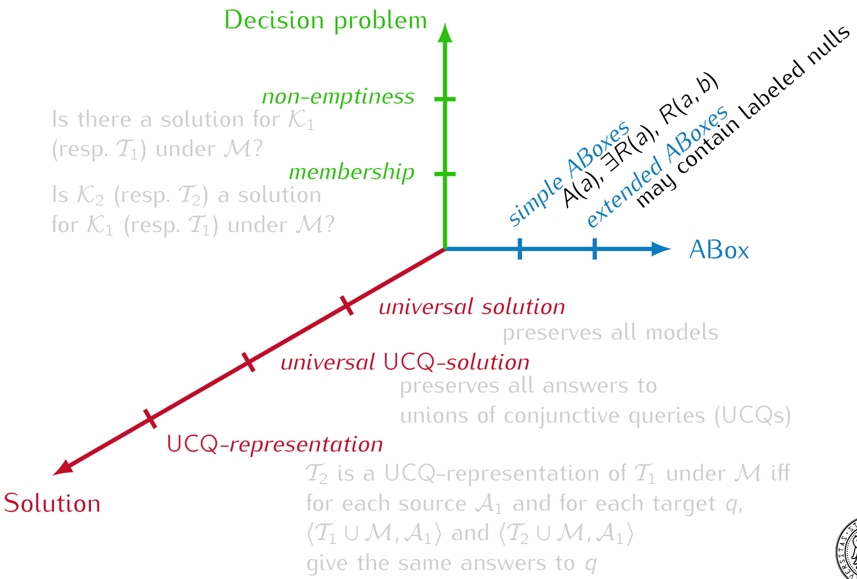
Reasoning Problems



Reasoning Problems



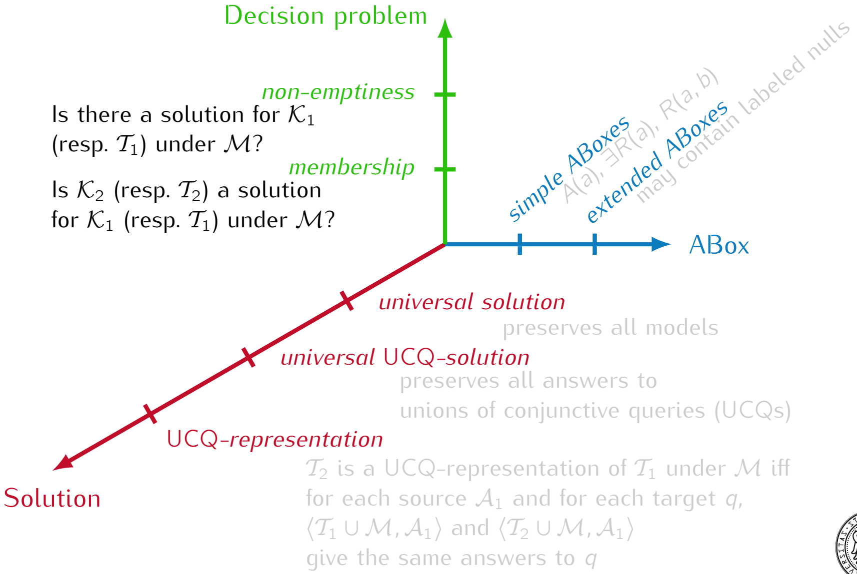
Reasoning Problems



Reasoning Problems

Is there a solution for $\mathcal{K}_1$ (resp. $\mathcal{T}_1$) under $\mathcal{M}$?

Is $\mathcal{K}_2$ (resp. $\mathcal{T}_2$) a solution for $\mathcal{K}_1$ (resp. $\mathcal{T}_1$) under $\mathcal{M}$?



Knowledge Base Exchange: Universal Solution

$$\mathcal{M}: \quad \begin{array}{lcl} \mathit{AuthorOf}^- & \sqsubseteq & \mathit{WrittenBy} \\ \mathit{TaxNumber} & \sqsubseteq & \mathit{SSN} \\ \exists \mathit{AuthorOf}^- & \sqsubseteq & \exists \mathit{BookGenre} \end{array}$$

$$\mathcal{T}_1: \quad \begin{array}{l} \exists \mathit{AuthorOf} \sqsubseteq \mathit{Author} \\ \mathit{Author} \sqsubseteq \exists \mathit{TaxNumber} \end{array}$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
tolkien	lotr

Knowledge Base Exchange: Universal Solution

$\mathcal{M}$:

$$AuthorOf^- \sqsubseteq WrittenBy$$
$$TaxNumber \sqsubseteq SSN$$
$$\exists AuthorOf^- \sqsubseteq \exists BookGenre$$

$\mathcal{T}_1$:

$$\exists AuthorOf \sqsubseteq Author$$
$$Author \sqsubseteq \exists TaxNumber$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
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$\mathcal{A}_2$:

WrittenBy	
lolita	nabokov
lotr	tolkien

SSN	
nabokov	m_1
tolkien	m_2

BookGenre	
lolita	m_3
lotr	m_4

$\mathcal{A}_2$ is a universal solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ (with extended ABoxes).

Knowledge Base Exchange: Universal UCQ-Solution

$$\mathcal{M}: \begin{array}{rcl} AuthorOf^- & \sqsubseteq & WrittenBy \\ TaxNumber & \sqsubseteq & SSN \\ \exists AuthorOf^- & \sqsubseteq & \exists BookGenre \end{array}$$

$$\mathcal{T}_1: \begin{array}{l} \exists AuthorOf \sqsubseteq Author \\ Author \sqsubseteq \exists TaxNumber \end{array}$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
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Knowledge Base Exchange: Universal UCQ-Solution

$$\mathcal{M}: \quad \begin{array}{rcl} AuthorOf^- & \sqsubseteq & WrittenBy \\ TaxNumber & \sqsubseteq & SSN \\ \exists AuthorOf^- & \sqsubseteq & \exists BookGenre \end{array}$$

$$\mathcal{T}_1: \quad \begin{array}{l} \exists AuthorOf \sqsubseteq Author \\ Author \sqsubseteq \exists TaxNumber \end{array}$$

$\mathcal{A}_1$:

AuthorOf	
nabokov	lolita
tolkien	lotr

$$\mathcal{T}_2: \quad \begin{array}{l} \exists WrittenBy^- \sqsubseteq \exists SSN \\ \exists WrittenBy \sqsubseteq \exists BookGenre \end{array}$$

$\mathcal{A}_2$:

WrittenBy	
lolita	nabokov
lotr	tolkien

$q(b) \leftarrow \exists g.BookGenre(b, g)$: $\quad cert(q, \langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle) = \{lolita, lotr\} = cert(q, \langle \mathcal{T}_2, \mathcal{A}_2 \rangle)$

$q(b, g) \leftarrow \exists b, g.BookGenre(b, g)$: $\quad cert(q, \langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle) = \{\} = cert(q, \langle \mathcal{T}_2, \mathcal{A}_2 \rangle)$

$\langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal-UCQ solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ (with simple ABoxes).

Knowledge Base Exchange: UCQ-Representation

$$\mathcal{M}: \begin{array}{rcl} AuthorOf^- & \sqsubseteq & WrittenBy \\ TaxNumber & \sqsubseteq & SSN \\ \exists AuthorOf^- & \sqsubseteq & \exists BookGenre \end{array}$$

$$\mathcal{T}_1: \quad \exists AuthorOf \sqsubseteq \exists TaxNumber$$

Knowledge Base Exchange: UCQ-Representation

$$\mathcal{M}: \begin{array}{rcl} \mathit{AuthorOf}^- & \sqsubseteq & \mathit{WrittenBy} \\ \mathit{TaxNumber} & \sqsubseteq & \mathit{SSN} \\ \exists \mathit{AuthorOf}^- & \sqsubseteq & \exists \mathit{BookGenre} \end{array}$$

$$\mathcal{T}_1: \quad \exists \mathit{AuthorOf} \sqsubseteq \exists \mathit{TaxNumber}$$

$$\mathcal{T}_2: \quad \begin{array}{l} \exists \mathit{WrittenBy}^- \sqsubseteq \exists \mathit{SSN} \\ \exists \mathit{WrittenBy} \sqsubseteq \exists \mathit{BookGenre} \end{array}$$

$\mathcal{T}_2$ is a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M}$.

Knowledge Base Exchange: UCQ-Representation

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$$\mathcal{T}_1: \quad \exists \mathit{AuthorOf} \sqsubseteq \exists \mathit{TaxNumber}$$

$$\mathcal{T}_2: \quad \begin{array}{l} \exists \mathit{WrittenBy}^- \sqsubseteq \exists \mathit{SSN} \\ \exists \mathit{WrittenBy} \sqsubseteq \exists \mathit{BookGenre} \end{array}$$

$\mathcal{A}_1$:

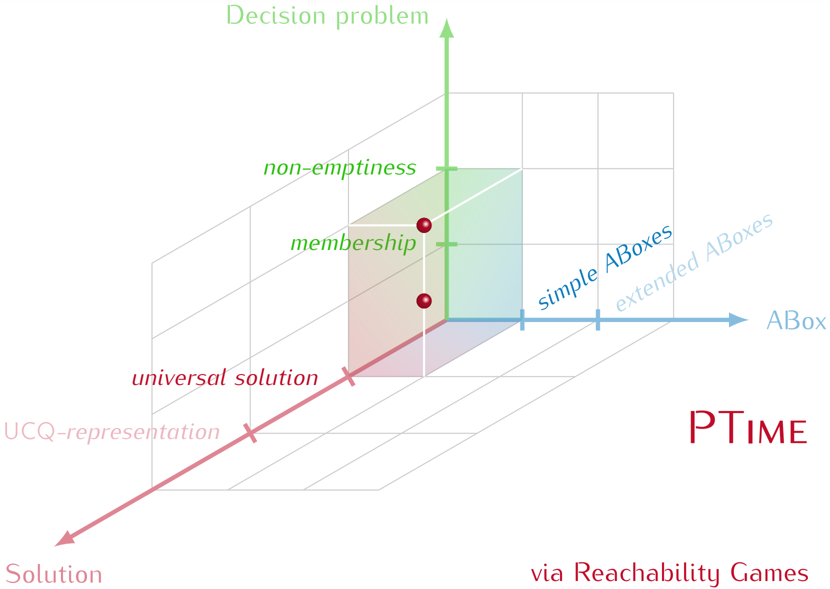
AuthorOf	
nabokov	lolita

TaxNumber	
smith	000

for each source $\mathcal{A}_1$ and for each target q,
$\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle$ and $\langle \mathcal{T}_2 \cup \mathcal{M}, \mathcal{A}_1 \rangle$
give the same answers to q

$\mathcal{T}_2$ is a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M}$.

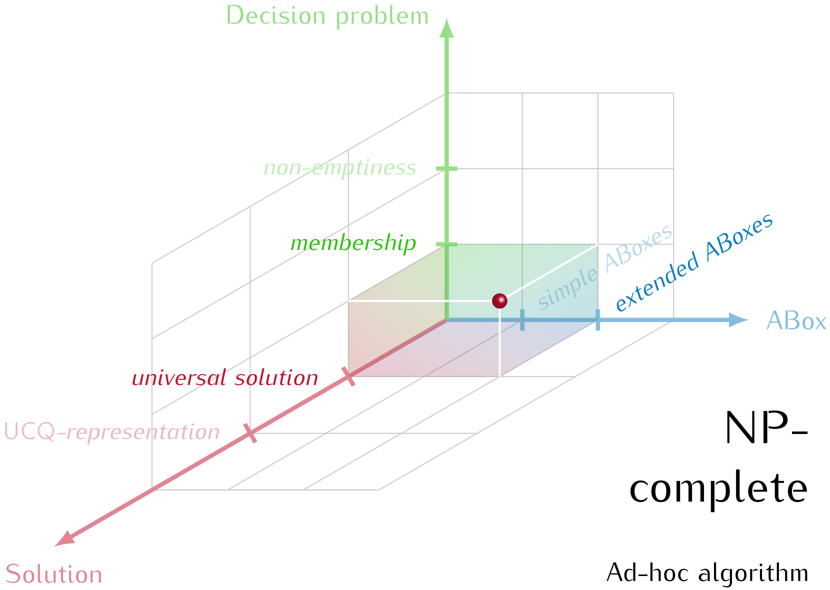
Summary of Results



PTIME

via Reachability Games

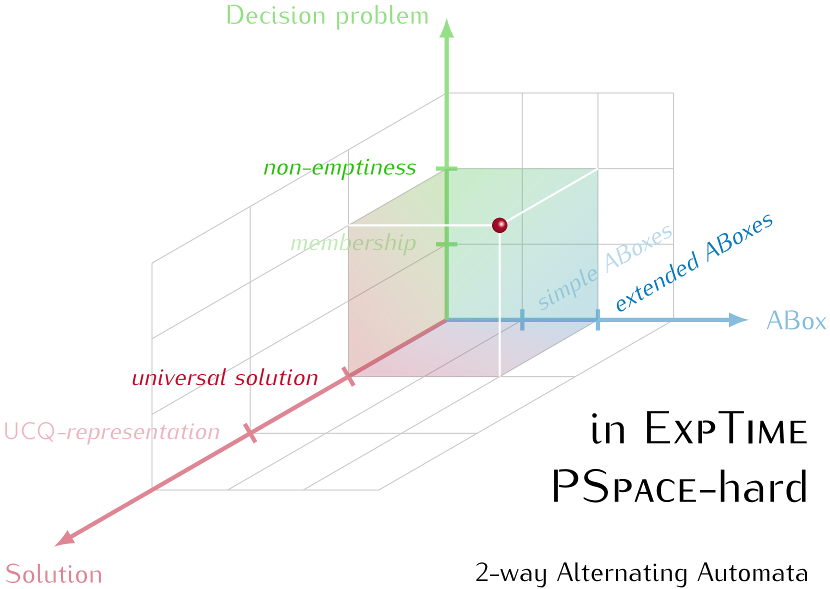
Summary of Results



NP-complete

Ad-hoc algorithm
3-colorability

Summary of Results



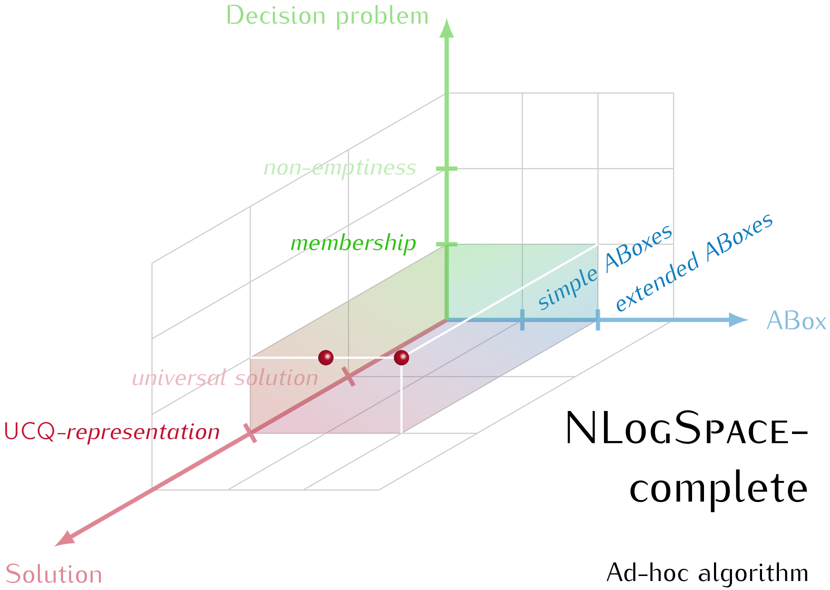
in ExpTime

PSpace-hard

2-way Alternating Automata

QBF validity

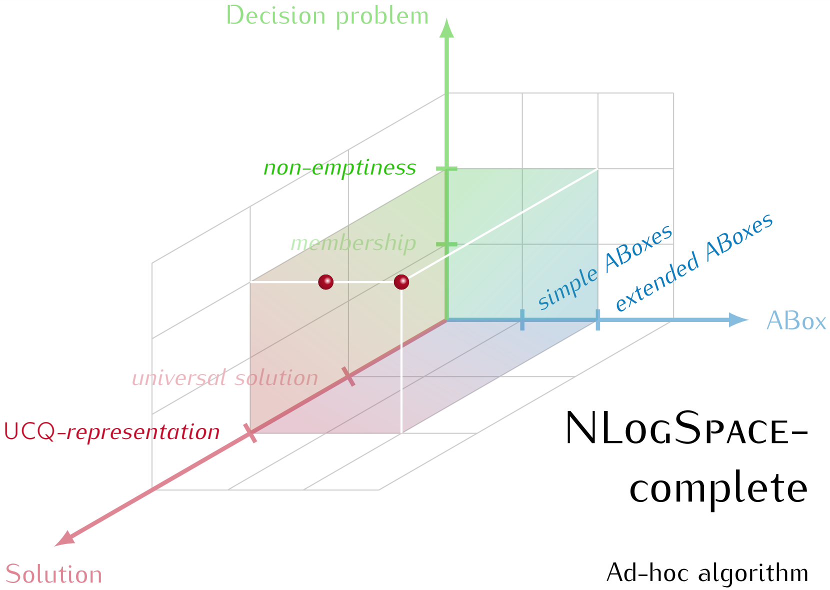
Summary of Results



NLogSpace-complete

Ad-hoc algorithm (improved)

Summary of Results



NLogSpace-complete

Ad-hoc algorithm (improved)

Membership for Simple Universal Solutions is in PTime

$\mathcal{A}_2$ is a universal solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ iff[1] there exist

- a homomorphism from $\mathcal{U}_{\mathcal{A}_2}$ to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$,
- a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$

on the target symbols.

[1] when $\mathcal{T}_1$ and $\mathcal{M}$ are positive, otherwise one more condition has to be added

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EASY

on the target symbols.

[1] when $\mathcal{T}_1$ and $\mathcal{M}$ are positive, otherwise one more condition has to be added

Membership for Simple Universal Solutions is in PTime

$\mathcal{A}_2$ is a universal solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ iff[1] there exist

- a homomorphism from $\mathcal{U}_{\mathcal{A}_2}$ to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$, EASY
- a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$ via Reachability Games on graphs

on the target symbols.

[1] when $\mathcal{T}_1$ and $\mathcal{M}$ are positive, otherwise one more condition has to be added

Membership for Simple Universal Solutions is in PTIME

- a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$ via Reachability Games on graphs

on the target symbols.

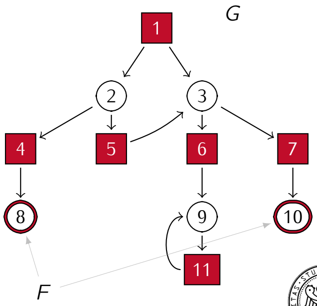
We construct a reachability game $\mathcal{G} = (G, F)$ such that

there exists a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$

iff

Duplicator has a strategy against Spoiler in G to avoid F.

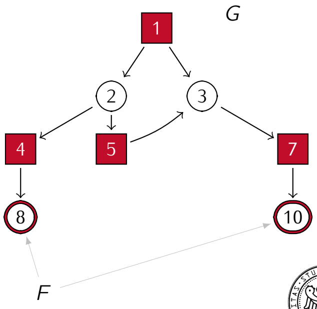
Duplicator wins from 1



Duplicator has a strategy against Spoiler in G to avoid F.

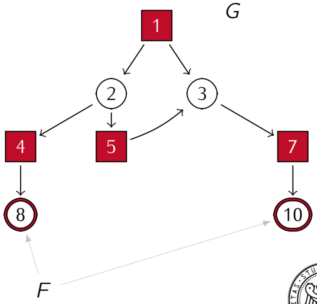
Membership for Simple Universal Solutions is in PTime

Spoiler wins from 1



Duplicator has a strategy against Spoiler in G to avoid F.

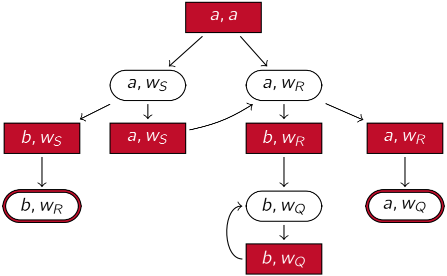
Membership for Simple Universal Solutions is in PTime



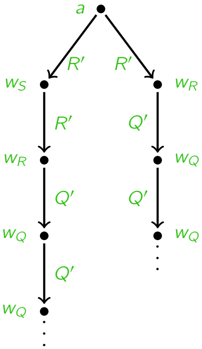
Duplicator has a strategy against Spoiler in G to avoid F. Decidable in PTime.

Games and Homomorphisms

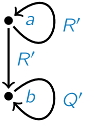
Each play starting from a, a defines a homomorphism (on the target symbols) from a path in $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$.



The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$

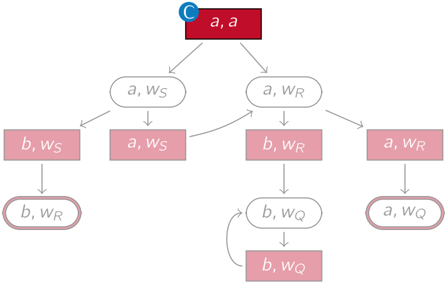


$\mathcal{U}_{\mathcal{A}_2}$

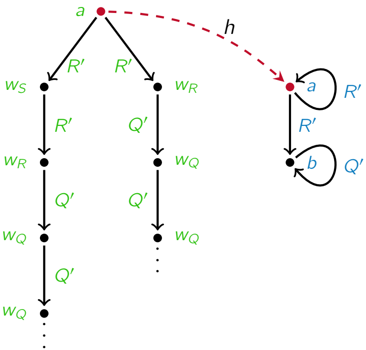
for $\mathcal{T}_1 = \{\exists S^- \sqsubseteq \exists R, \exists R^- \sqsubseteq \exists Q, \exists Q^- \sqsubseteq \exists Q\}$, $\mathcal{A}_1 = \{\exists S(a), \exists R(a)\}$, and $\mathcal{M} = \{S \sqsubseteq R', R \sqsubseteq R', Q \sqsubseteq Q'\}$, and $\mathcal{A}_2 = \{R'(a, a), R'(a, b), Q'(b, b)\}$.

Games and Homomorphisms

Each play starting from (a, a) defines a homomorphism (on the target symbols) from a path in $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$.



The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\mathcal{U}_{\mathcal{A}_2}$

for $\mathcal{T}_1 = \{\exists S^- \sqsubseteq \exists R, \exists R^- \sqsubseteq \exists Q, \exists Q^- \sqsubseteq \exists Q\}$, $\mathcal{A}_1 = \{\exists S(a), \exists R(a)\}$, and $\mathcal{M} = \{S \sqsubseteq R', R \sqsubseteq R', Q \sqsubseteq Q'\}$, and $\mathcal{A}_2 = \{R'(a,a), R'(a,b), Q'(b,b)\}$.

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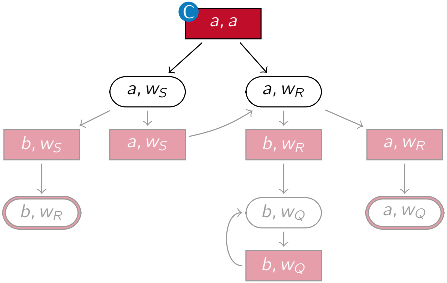
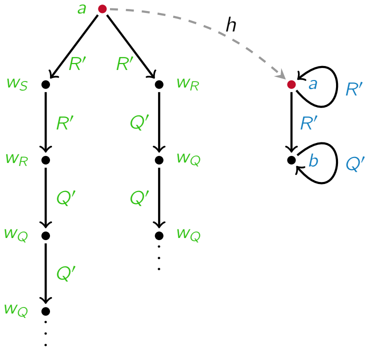


The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\mathcal{U}_{\mathcal{A}_2}$

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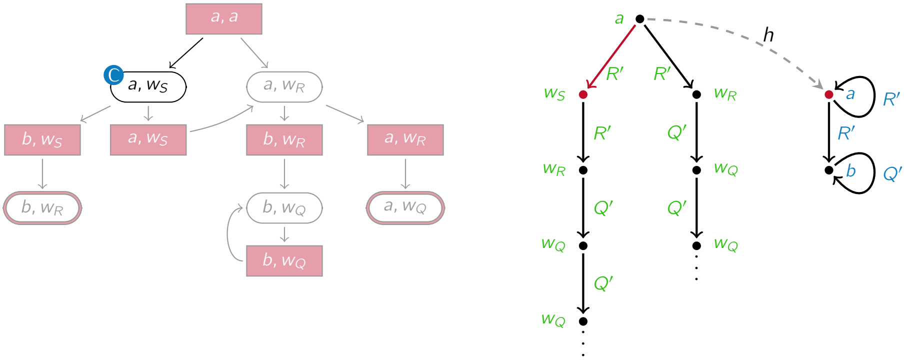
Games and Homomorphisms

Each play starting from **a, a** defines a homomorphism (on the target symbols) from a path in $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$.



The reachability game $\mathcal{G} = (G, F)$

$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$

$\mathcal{U}_{\mathcal{A}_2}$

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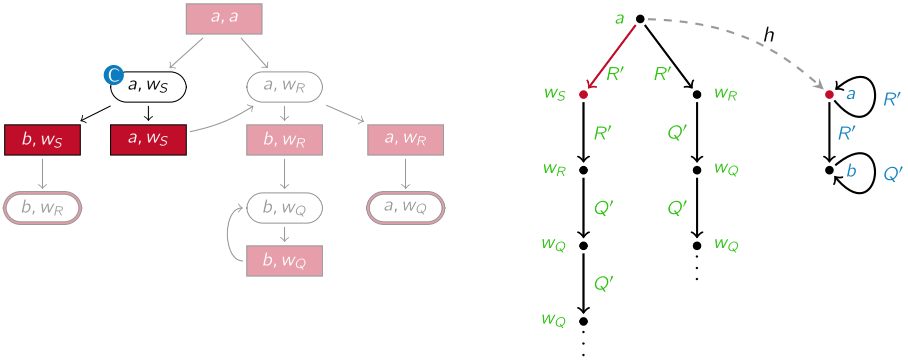

The reachability game $\mathcal{G} = (G, F)$ $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\mathcal{U}_{\mathcal{A}_2}$

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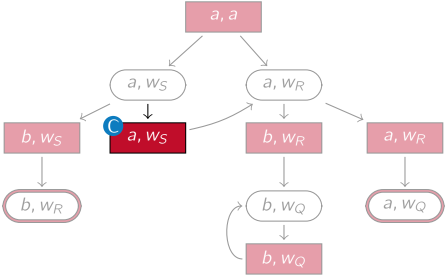
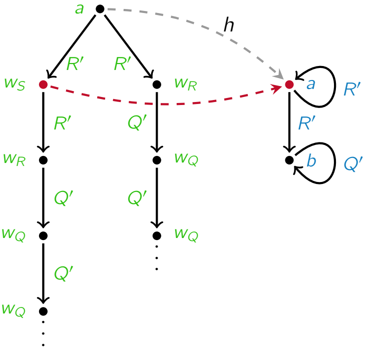


The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\mathcal{U}_{\mathcal{A}_2}$

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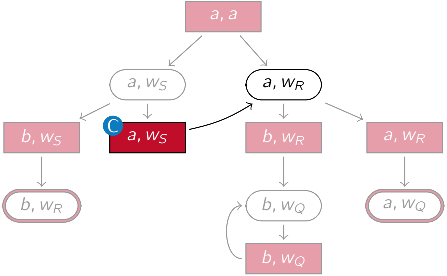
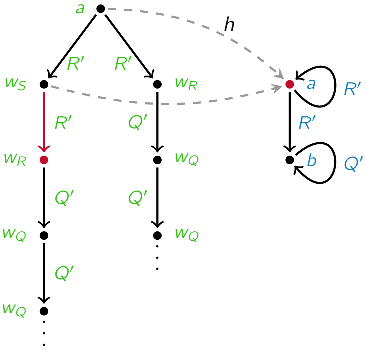


The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\mathcal{U}_{\mathcal{A}_2}$

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Games and Homomorphisms

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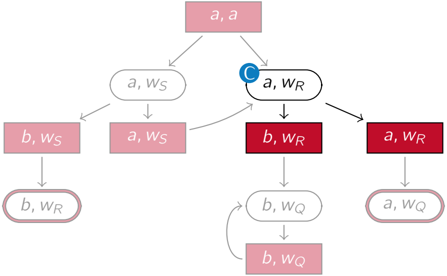
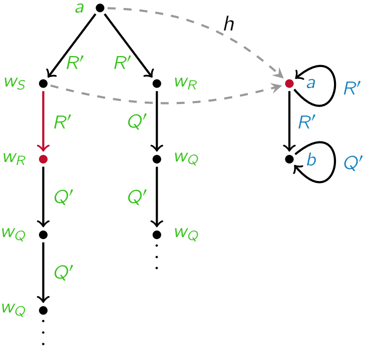


The reachability game $\mathcal{G} = (G, F)$



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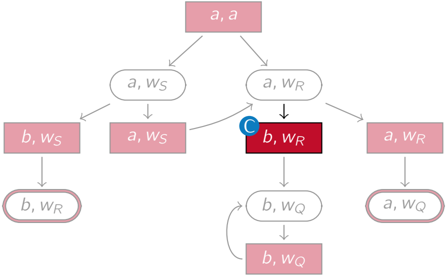
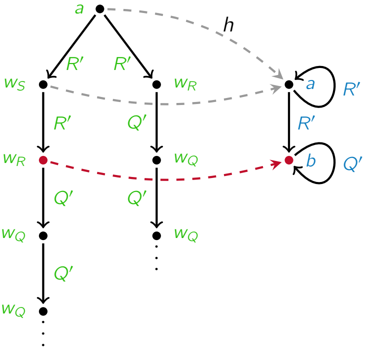


The reachability game $\mathcal{G} = (G, F)$



$\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ $\qquad$ $\mathcal{U}_{\mathcal{A}_2}$

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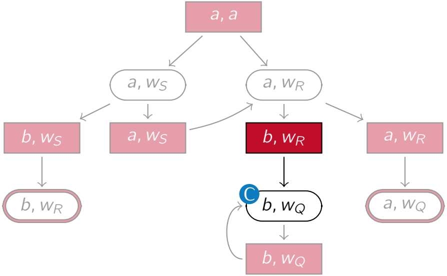

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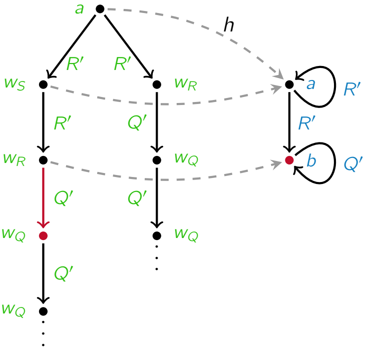

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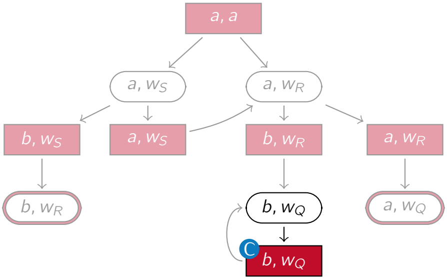

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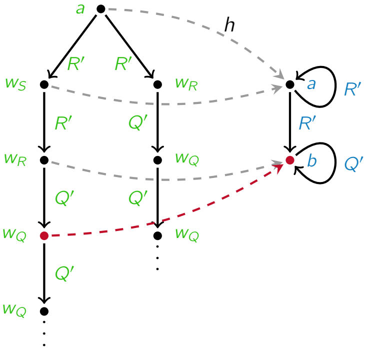

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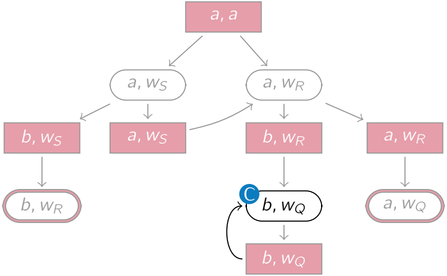

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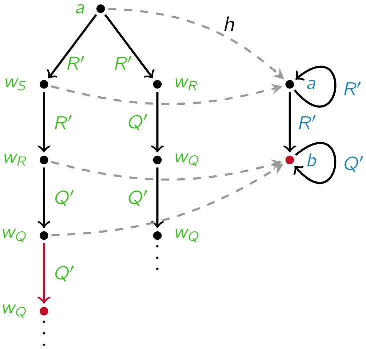

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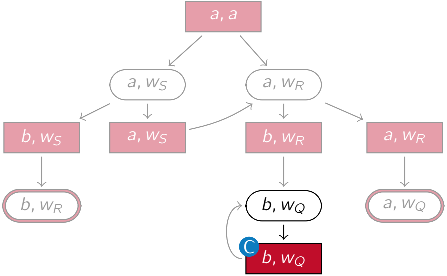

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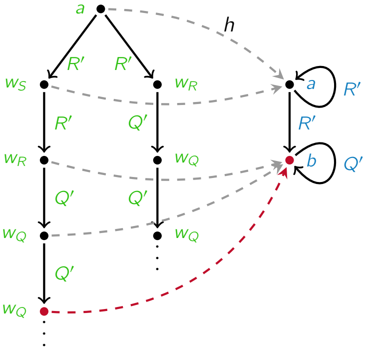

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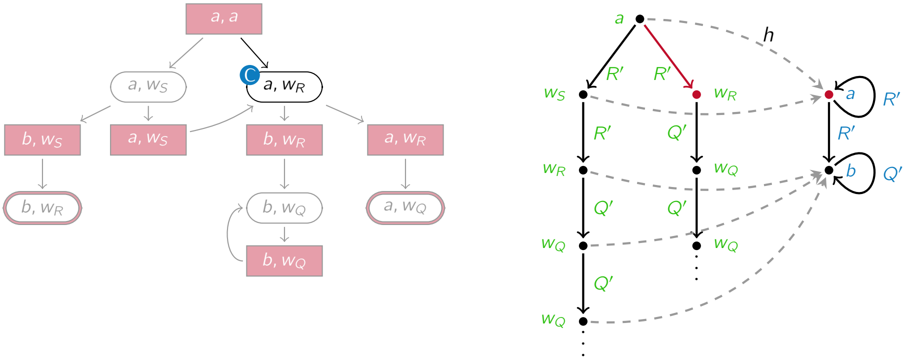

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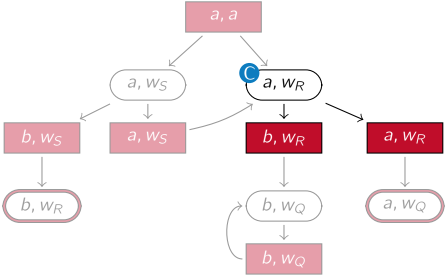

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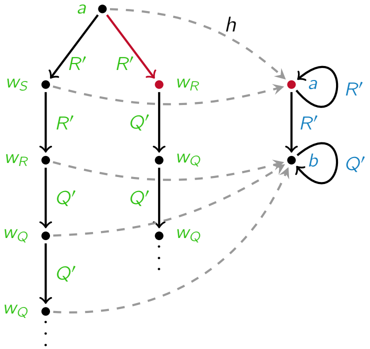

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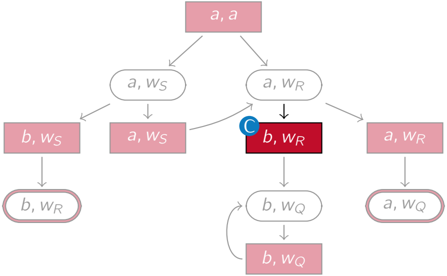

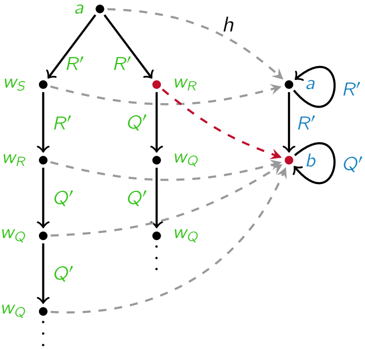

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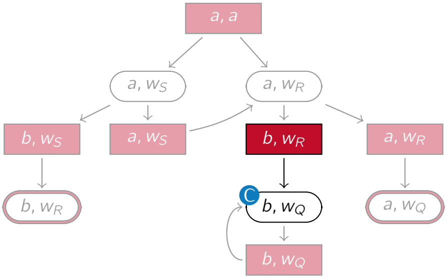

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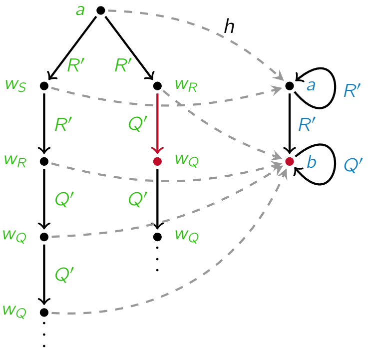

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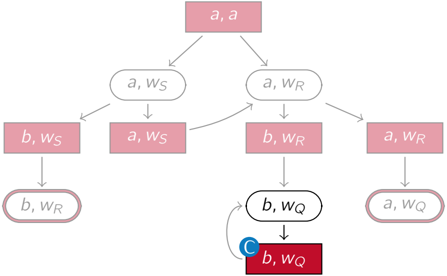

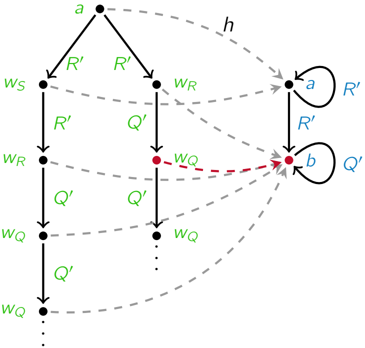

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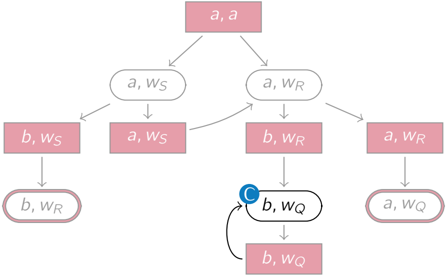

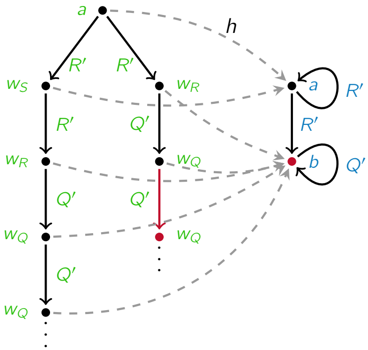

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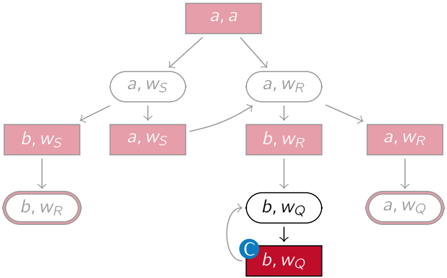

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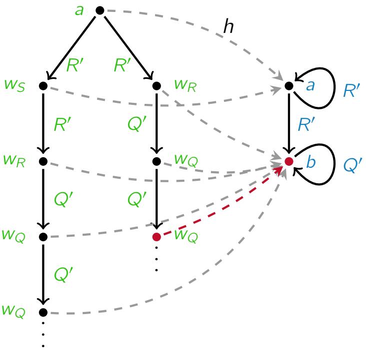

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Non-emptiness for Extended Universal solutions, UB

There exists a universal solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ iff[2] there exists a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle}$ to a finite subset of it, on the target symbols.

[2] when $\mathcal{T}_1$ and $\mathcal{M}$ are positive, otherwise one more condition has to be added

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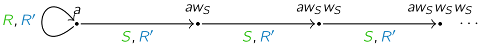

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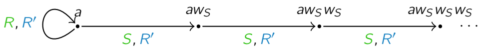
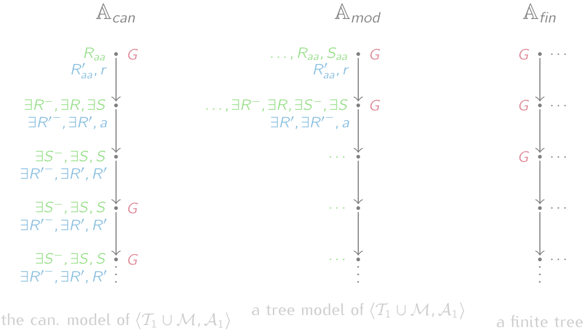


Two-way Alternating Automata. We construct three automata:



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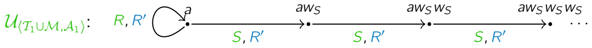

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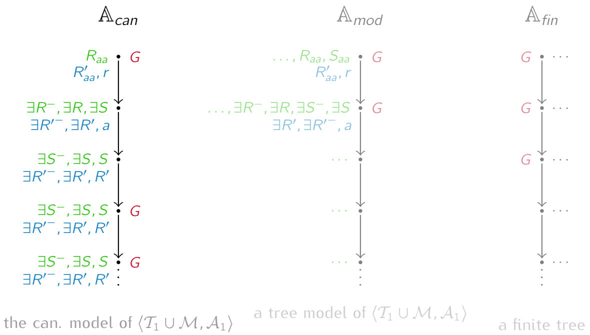

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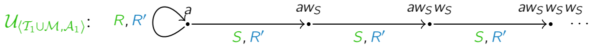

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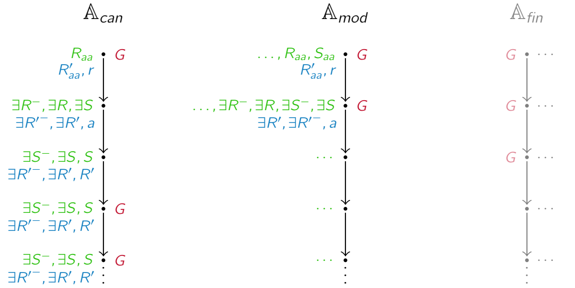

the can. model of $\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle$ a tree model of $\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle$ a finite tree

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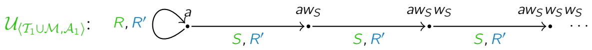

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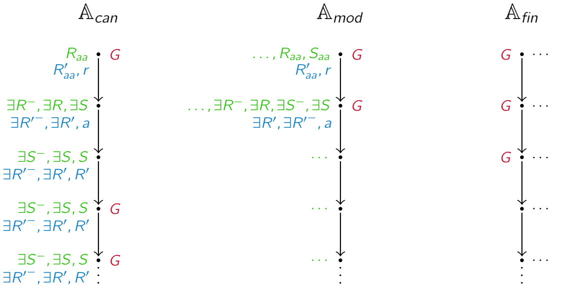

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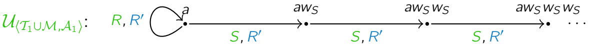

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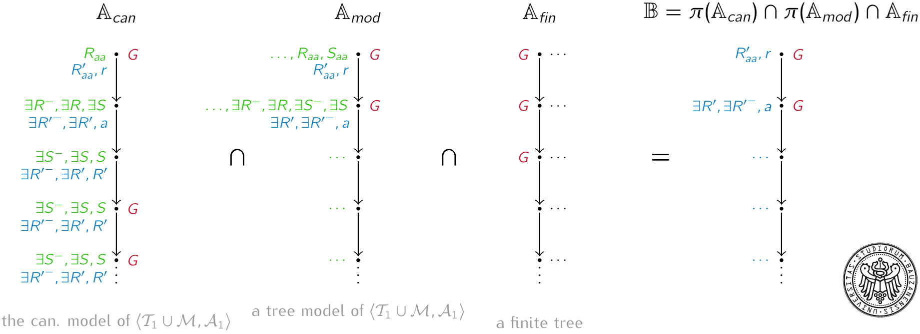

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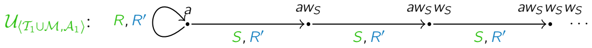
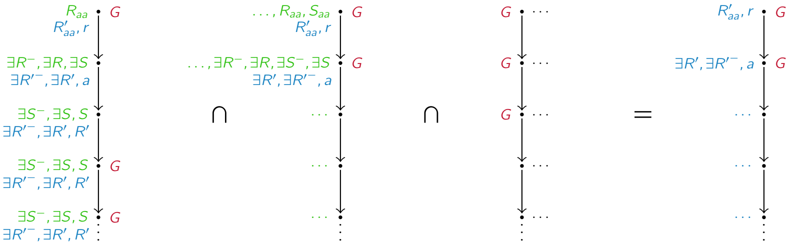


Then $\mathcal{A}_2 = \{R'(a,a)\}$ is a universal solution for $\mathcal{K}_1$ under $\mathcal{M}$.

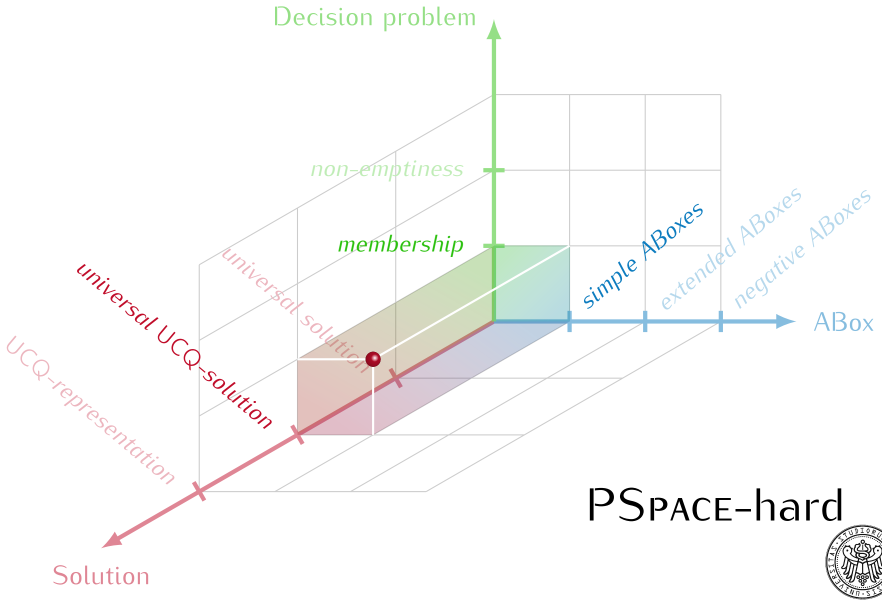
Two-way Alternating Automata. We construct three automata:

$\mathbb{A}_{can}$ $\quad$ $\mathbb{A}_{mod}$ $\quad$ $\mathbb{A}_{fin}$ $\quad$ $\mathbb{B} = \pi(\mathbb{A}_{can}) \cap \pi(\mathbb{A}_{mod}) \cap \mathbb{A}_{fin}$



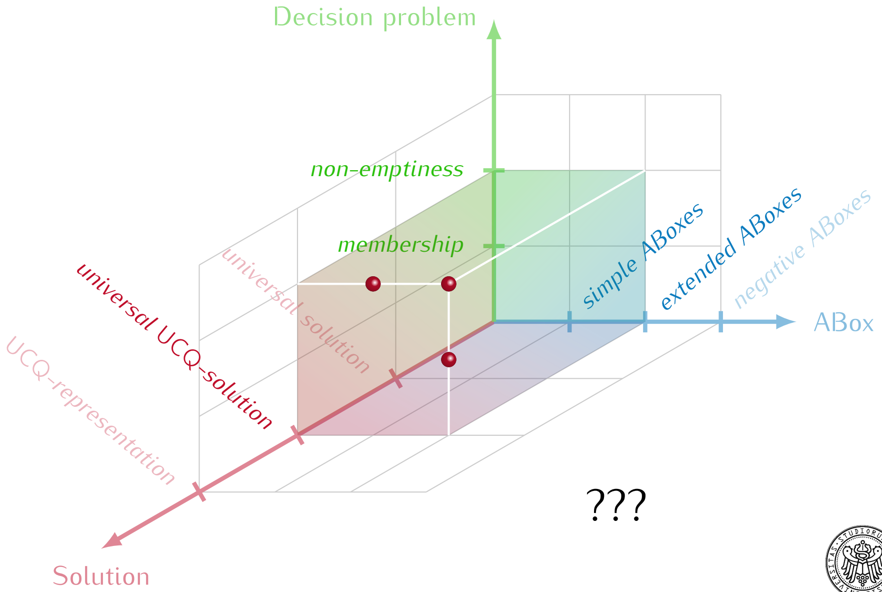
the can. model of $\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle$ $\quad$ a tree model of $\langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle$ $\quad$ a finite tree

Future Work



PSPACE-hard

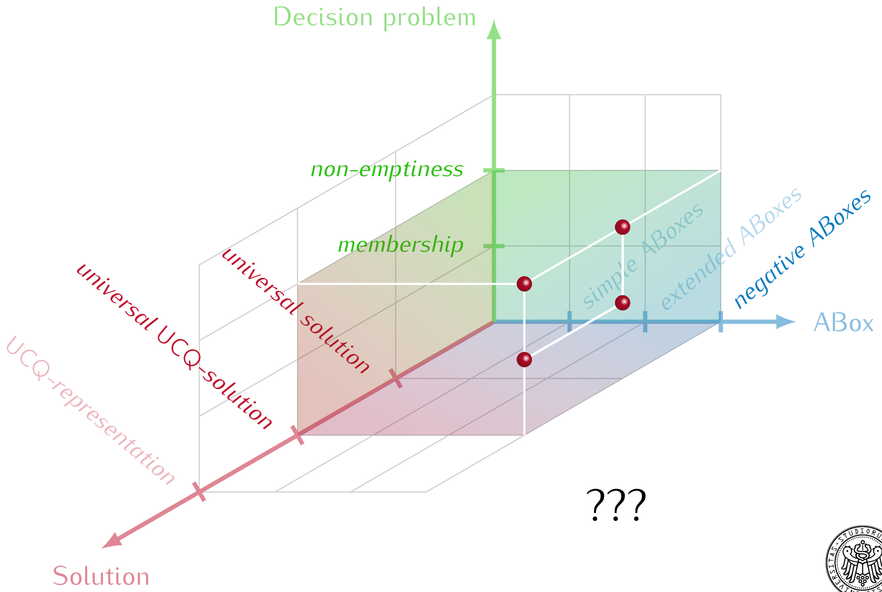
Future Work



Decision problem
non-emptiness
membership
simple ABoxes
extended ABoxes
negative ABoxes
ABox
universal UCQ-solution
universal solution
UCQ-representation
Solution

???

Future Work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Thank you
for your attention!