

# Representability in $DL-Lite_{\mathcal{R}}$ Knowledge Base Exchange

Marcelo Arenas<sup>1</sup>   Elena Botoeva<sup>2</sup>   Diego Calvanese<sup>2</sup>  
Vladislav Ryzhikov<sup>2</sup>   Evgeny Sherkhonov<sup>3</sup>

<sup>1</sup> Dept. of Computer Science, PUC Chile  
[marenas@ing.puc.cl](mailto:marenas@ing.puc.cl)

<sup>2</sup> KRDB Research Centre, Free Univ. of Bozen-Bolzano, Italy  
[lastname@inf.unibz.it](mailto:lastname@inf.unibz.it)

ISLA, University of Amsterdam, Netherlands  
[e.sherkhonov@uva.nl](mailto:e.sherkhonov@uva.nl)

Description Logics Workshop  
7 June 2012, Rome



# Outline

- 1 Knowledge Base Exchange
- 2 Representability in  $DL-Lite_{\mathcal{R}}$
- 3 Conclusions



# Outline

- 1 Knowledge Base Exchange
- 2 Representability in  $DL-Lite_{\mathcal{R}}$
- 3 Conclusions



# Knowledge Base Exchange

## Problem

given a mapping  $\mathcal{M}$  and a source knowledge base (KB)  $\mathcal{K}_1$ ,  
compute a target KB  $\mathcal{K}_2$  that is a *solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$ .



source signature



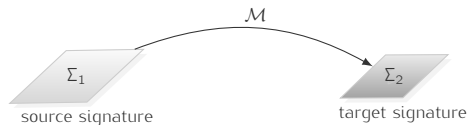
target signature



# Knowledge Base Exchange

## Problem

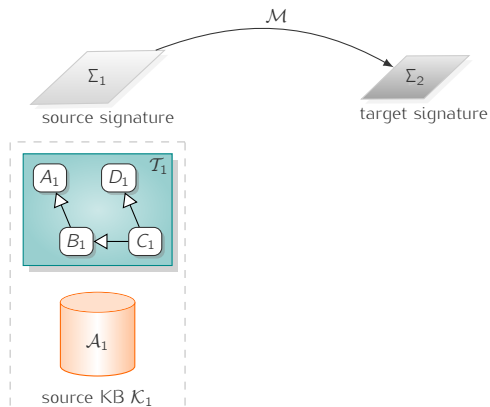
given a mapping  $\mathcal{M}$  and a source knowledge base (KB)  $\mathcal{K}_1$ , compute a target KB  $\mathcal{K}_2$  that is a *solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$ .



# Knowledge Base Exchange

## Problem

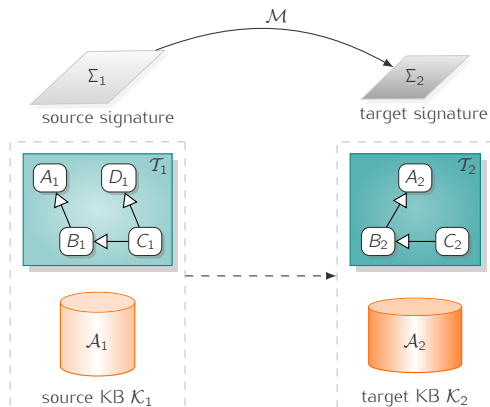
given a mapping  $\mathcal{M}$  and a source knowledge base (KB)  $\mathcal{K}_1$ , compute a target KB  $\mathcal{K}_2$  that is a *solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$ .



# Knowledge Base Exchange

## Problem

given a mapping  $\mathcal{M}$  and a source knowledge base (KB)  $\mathcal{K}_1$ , compute a target KB  $\mathcal{K}_2$  that is a *solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$ .

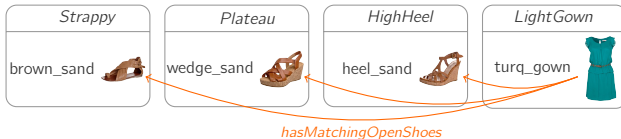


# Knowledge Base Exchange: Example

$\mathcal{T}_1$ :

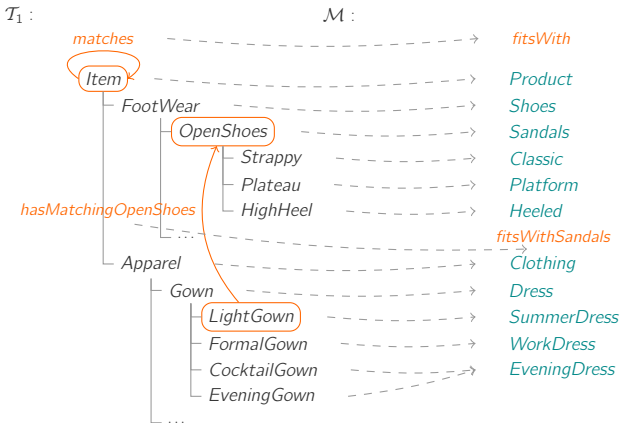


$\mathcal{A}_1$ :





# Knowledge Base Exchange: Example



$\mathcal{A}_1$  :



*hasMatchingOpenShoes*

# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .

$\mathcal{T}_1 \cup \mathcal{M}$ :



$\mathcal{A}_1$ :



$\mathcal{A}_2^{univ}$ :

# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .

$\mathcal{T}_1 \cup \mathcal{M}$ :



$\mathcal{A}_1$ :



$\mathcal{A}_2^{univ}$ :



# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .

$\mathcal{T}_1 \cup \mathcal{M}$ :



$\mathcal{A}_1$ :



$\mathcal{A}_2^{univ}$ :



# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .

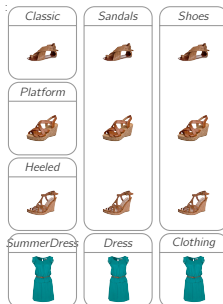
$\mathcal{T}_1 \cup \mathcal{M}$ :



$\mathcal{A}_1$ :



$\mathcal{A}_2^{\text{univ}}$ :



# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .

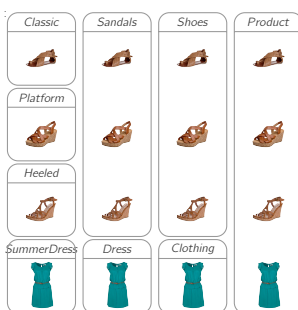
$\mathcal{T}_1 \cup \mathcal{M}$ :



$\mathcal{A}_1$ :



$\mathcal{A}_2^{univ}$ :



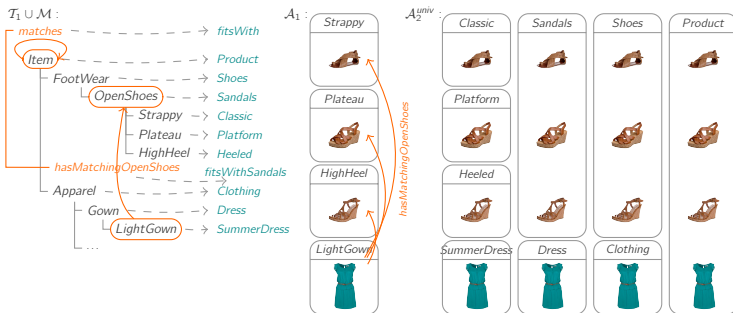
# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



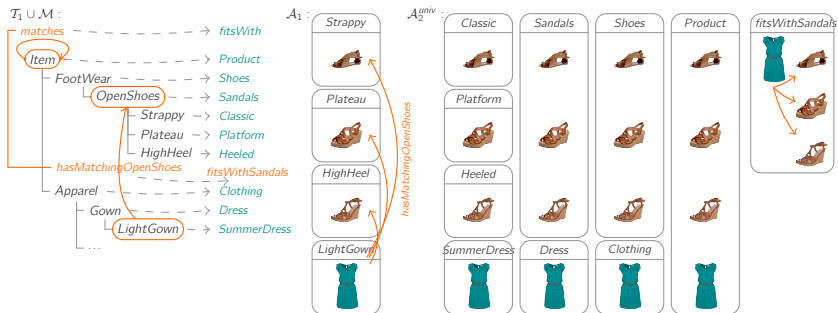
# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .





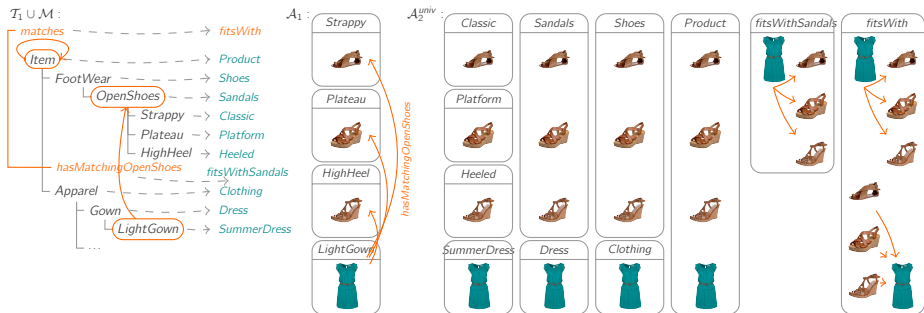
# Example: Universal Solution

## Definition

$\mathcal{K}_2$  is a *universal solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if  $\text{Mod}(\mathcal{K}_2) = \text{Mod}(\mathcal{K}_1 \cup \mathcal{M})|_{\Sigma_2}$ .

## Theorem

If  $\text{chase}_{\mathcal{T}_1}(\mathcal{A}_1)$  is finite, then  $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is a universal solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Example: Universal UCQ-solution

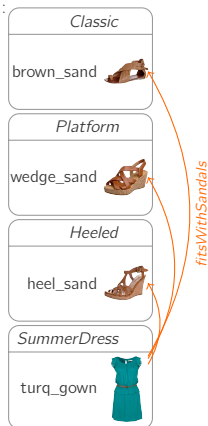
## Definition

$\mathcal{K}_2$  is a *universal UCQ-solution* for  $\mathcal{K}_1$  under  $\mathcal{M}$  if for each UCQ  $q$  over  $\Sigma_2$ ,  
 $\text{cert}(q, \langle \mathcal{T}_1 \cup \mathcal{M}, \mathcal{A}_1 \rangle) = \text{cert}(q, \mathcal{K}_2)$ .

$\mathcal{T}_2$  :



$\mathcal{A}_2$  :



# Outline

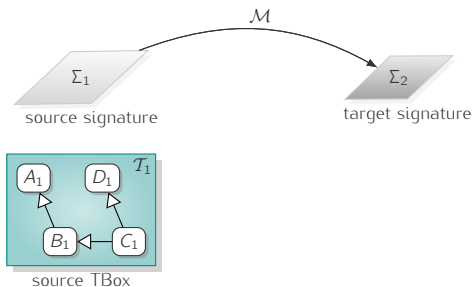
- 1 Knowledge Base Exchange
- 2 Representability in  $DL-Lite_{\mathcal{R}}$
- 3 Conclusions



# Representability Problem

## Problem

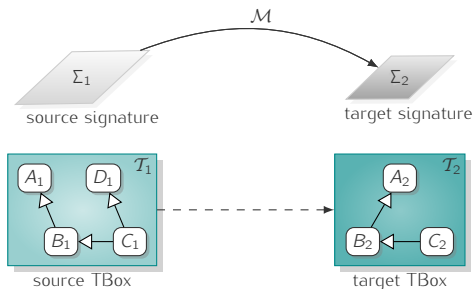
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a target TBox  $\mathcal{T}_2$ , such that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,  $\langle \mathcal{T}_2, \text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Representability Problem

## Problem

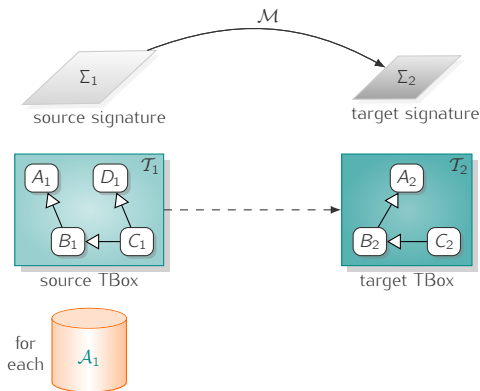
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a target TBox  $\mathcal{T}_2$ , such that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,  $\langle \mathcal{T}_2, \text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Representability Problem

## Problem

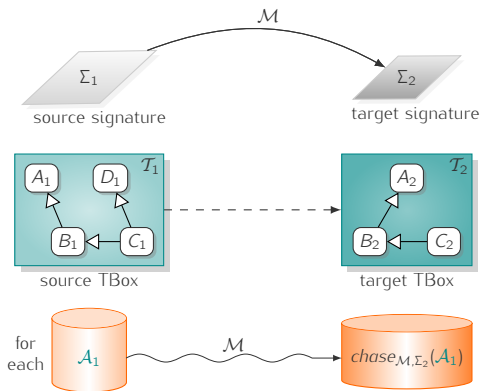
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a target TBox  $\mathcal{T}_2$ , such that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,  $\langle \mathcal{T}_2, \text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Representability Problem

## Problem

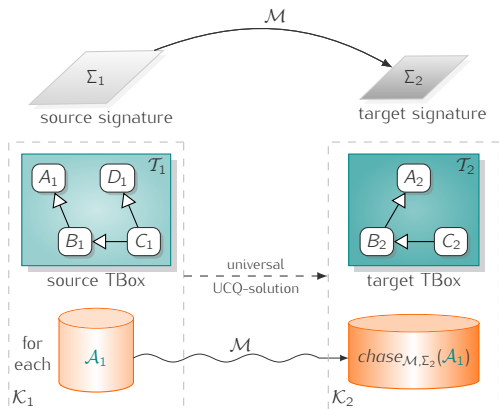
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a target TBox  $\mathcal{T}_2$ , such that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,  $\langle \mathcal{T}_2, \text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Representability Problem

## Problem

given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a target TBox  $\mathcal{T}_2$ , such that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,  $\langle \mathcal{T}_2, \text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .





# Representability: Example

 $\mathcal{T}_1 :$  $\mathcal{M} :$ *FootWear*└ *OpenShoes*

-----&gt;

*Shoes*

-----&gt;

*Sandals*

# Representability: Example



# Representability: Example



It is easy to see that  $\mathcal{T}_2$  is a UCQ-representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ :

for each ABox  $\mathcal{A}_1$  of the form  $\{OpenShoes(s)\}$  or  $\{FootWear(s)\}$ ,

$$chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1) = chase_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1),$$

hence  $\langle \mathcal{T}_2, chase_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rangle$  is a universal UCQ-solution for  $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$  under  $\mathcal{M}$ .



# Representability: Progress

- In DL11, we solved the representability problem for  $DL-Lite_{RDFS}$ , i.e., TBox inclusions of the form

$$\begin{array}{lcl} A_1 & \sqsubseteq & A_2 \\ \exists R_1 & \sqsubseteq & A_2 \\ R_1 & \sqsubseteq & R_2 \end{array}$$



# Representability: Progress

- In DL11, we solved the representability problem for  $DL\text{-Lite}_{RDFS}$ , i.e., TBox inclusions of the form

$$\begin{array}{lcl} A_1 & \sqsubseteq & A_2 \\ \exists R_1 & \sqsubseteq & A_2 \\ R_1 & \sqsubseteq & R_2 \end{array}$$

- In this work we address inclusions of the form

$$A_1 \sqsubseteq \exists R_2$$



# Representability: Progress

- In DL11, we solved the representability problem for  $DL\text{-Lite}_{RDFS}$ , i.e., TBox inclusions of the form

$$\begin{array}{l} A_1 \sqsubseteq A_2 \\ \exists R_1 \sqsubseteq A_2 \\ R_1 \sqsubseteq R_2 \end{array}$$

- In this work we address inclusions of the form

$$A_1 \sqsubseteq \exists R_2$$

Thus, we solve the representability problem for  $DL\text{-Lite}_{\mathcal{R}}^{pos}$ , the fragment of  $DL\text{-Lite}_{\mathcal{R}}$  without disjointness assertions.



# Checking Representability

- We first show how to check whether a given TBox  $\mathcal{T}_2$  is a representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ .



# Checking Representability

- We first show how to check whether a given TBox  $\mathcal{T}_2$  is a representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ .
  - ▶ We provide polynomial time conditions to verify that.





# Checking Representability

- We first show how to check whether a given TBox  $\mathcal{T}_2$  is a representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ .
  - ▶ We provide polynomial time conditions to verify that.
  - ▶ These conditions ensure that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,

$chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is homomorphically equivalent to  $chase_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ .



# Checking Representability

- We first show how to check whether a given TBox  $\mathcal{T}_2$  is a representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ .
  - ▶ We provide polynomial time conditions to verify that.
  - ▶ These conditions ensure that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,

$chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is homomorphically equivalent to  $chase_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ .

- ▶ We also show that these conditions are necessary.



# Checking Representability

- We first show how to check whether a given TBox  $\mathcal{T}_2$  is a representation of  $\mathcal{T}_1$  under  $\mathcal{M}$ .
  - ▶ We provide polynomial time conditions to verify that.
  - ▶ These conditions ensure that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$ ,

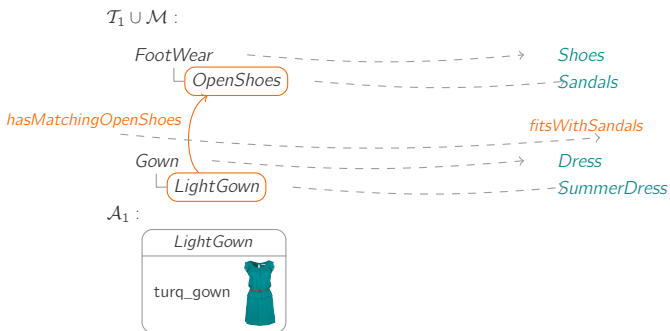
$chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$  is homomorphically equivalent to  $chase_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ .

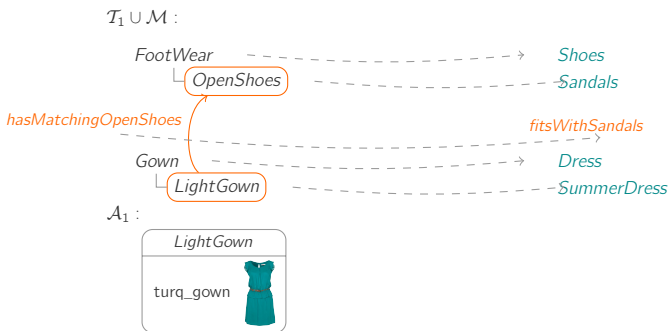
- ▶ We also show that these conditions are necessary.

In what follows, I will show how to ensure that for each ABox  $\mathcal{A}_1$  over  $\Sigma_1$

$$chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1) \rightarrow chase_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1).$$

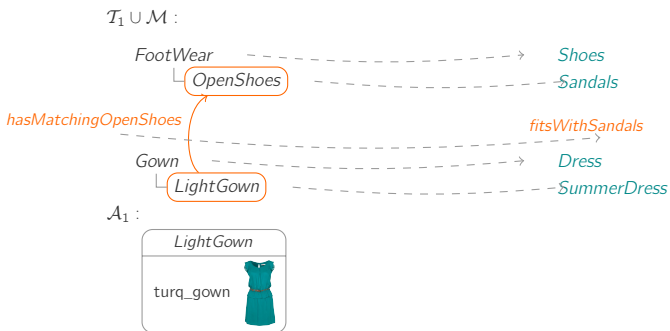


Inclusion  $A_1 \sqsubseteq \exists R_2$ 

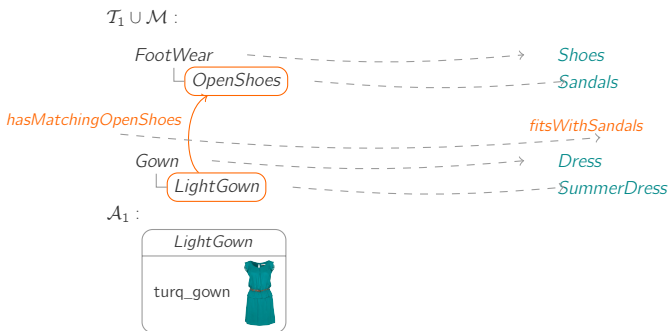
Inclusion  $A_1 \sqsubseteq \exists R_2$ 

$\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

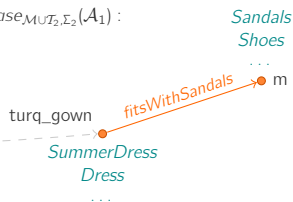


Inclusion  $A_1 \sqsubseteq \exists R_2$ 
 $chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ 

 $chase_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ 

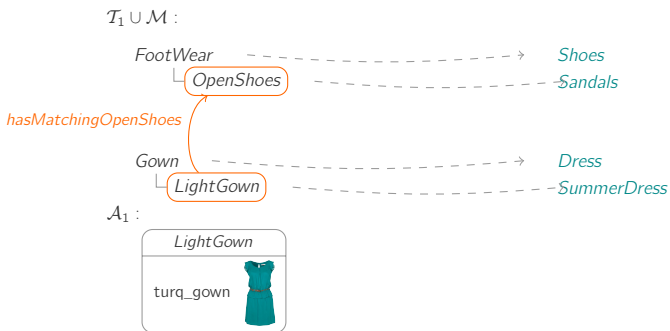

Inclusion  $A_1 \sqsubseteq \exists R_2$ 
 $chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ 

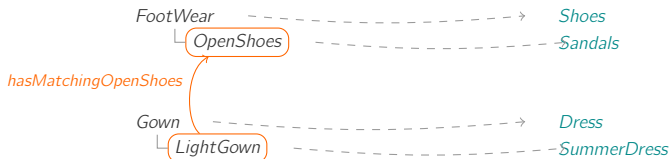
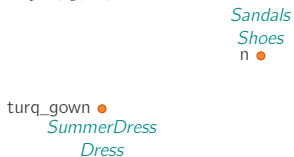
 $chase_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ 

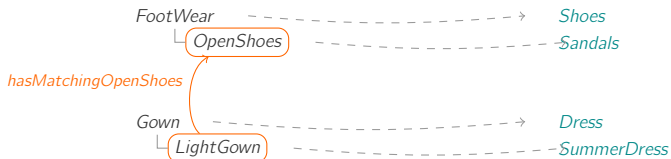

Inclusion  $A_1 \sqsubseteq \exists R_2$  $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ : $\text{chase}_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ :





Inclusion  $A_1 \sqsubseteq \exists R_2$  cont.

Inclusion  $A_1 \sqsubseteq \exists R_2$  cont. $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

Inclusion  $A_1 \sqsubseteq \exists R_2$  cont. $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $chase_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

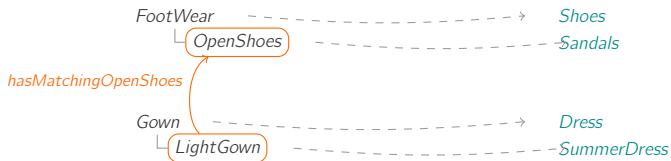
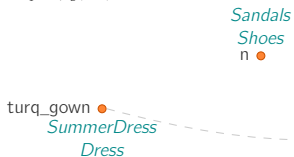
turq\_gown ●  
*SummerDress*  
*Dress*

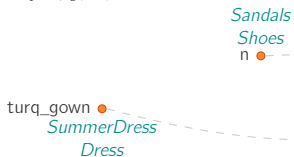
*Sandals*  
*Shoes*  
 n ●

 $chase_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

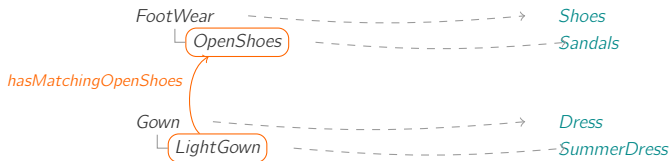
turq\_gown ●  
*SummerDress*



Inclusion  $A_1 \sqsubseteq \exists R_2$  cont. $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ : $\text{chase}_{\mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

Inclusion  $A_1 \sqsubseteq \exists R_2$  cont. $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ : $\text{chase}_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ :



Inclusion  $A_1 \sqsubseteq \exists R_2$  cont. $\mathcal{T}_1 \cup \mathcal{M}$ : $\mathcal{A}_1$ : $\text{chase}_{\mathcal{T}_1 \cup \mathcal{M}, \Sigma_2}(\mathcal{A}_1)$ :

turq\_gown  $\bullet$   
 SummerDress  
 Dress

Sandals  
 Shoes  
 n  $\bullet$

 $\text{chase}_{\mathcal{M} \cup \mathcal{T}_2, \Sigma_2}(\mathcal{A}_1)$ :

turq\_gown  $\bullet$   
 SummerDress  
 Dress  
 ...

Sandals  
 Shoes

$R_2^1$   $\bullet$   $m_1$   
 $\dots$   
 $R_2^k$   $\bullet$   $m_k$   
 $m_{k-1}$







# Deciding Representability in $DL\text{-Lite}_{\mathcal{R}}^{\text{pos}}$

## Theorem

Let  $\mathcal{M}$  be a  $DL\text{-Lite}_{\mathcal{R}}^{\text{pos}}$  mapping and  $\mathcal{T}_1$  a  $DL\text{-Lite}_{\mathcal{R}}^{\text{pos}}$  TBox over  $\Sigma_1$ . Then we can check whether  $\mathcal{T}_1$  is representable under  $\mathcal{M}$  in polynomial time.

This algorithm is similar to the one for  $DL\text{-Lite}_{\text{RDFS}}$ :

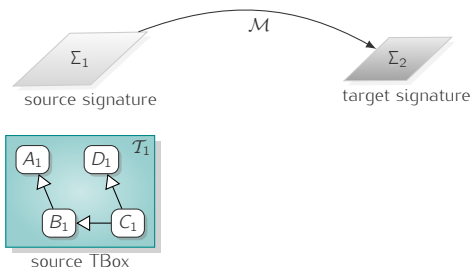
- we construct the maximal possible TBox over  $\Sigma_2$  and check whether it is a representation.
- If the check fails, then  $\mathcal{T}_1$  is not representable under  $\mathcal{M}$ . Otherwise, this TBox is a representation.



# Weak Representability

## Problem

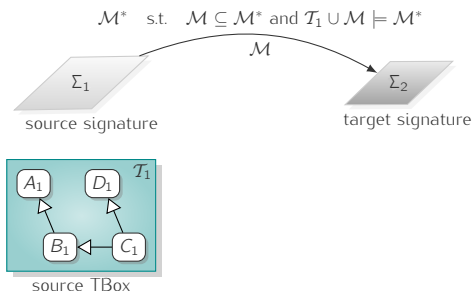
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .



# Weak Representability

## Problem

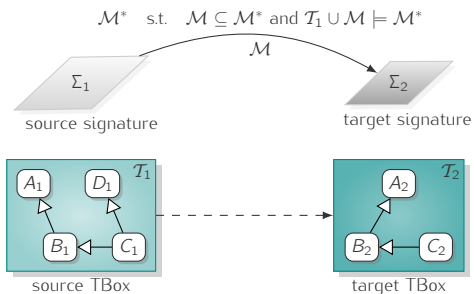
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .



# Weak Representability

## Problem

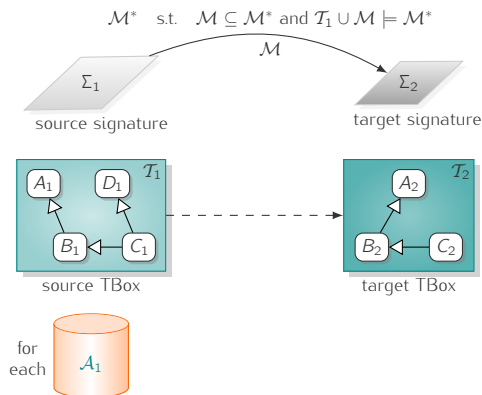
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .



# Weak Representability

## Problem

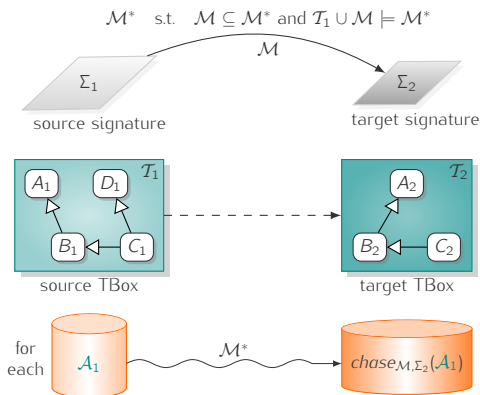
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .



# Weak Representability

## Problem

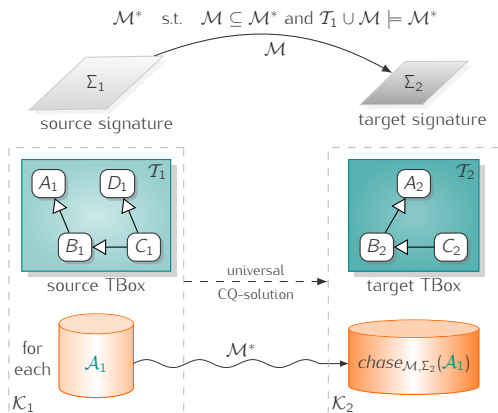
given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .



# Weak Representability

## Problem

given a mapping  $\mathcal{M}$  and a source TBox  $\mathcal{T}_1$ , decide whether there exists a mapping  $\mathcal{M}^*$  such that  $\mathcal{M} \subseteq \mathcal{M}^*$ ,  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ , and  $\mathcal{T}_1$  is UCQ-representable under  $\mathcal{M}^*$ .





## Deciding Weak Representability

Recall that every  $DL\text{-Lite}_{RDFS}$  TBox is weakly representable under every  $DL\text{-Lite}_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

However, there exist a  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .

# Deciding Weak Representability

Recall that every  $DL-Lite_{RDFS}$  TBox is weakly representable under every  $DL-Lite_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

However, there exist a  $DL-Lite_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .



# Deciding Weak Representability

Recall that every  $DL\text{-Lite}_{RDFS}$  TBox is weakly representable under every  $DL\text{-Lite}_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

However, there exist a  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .

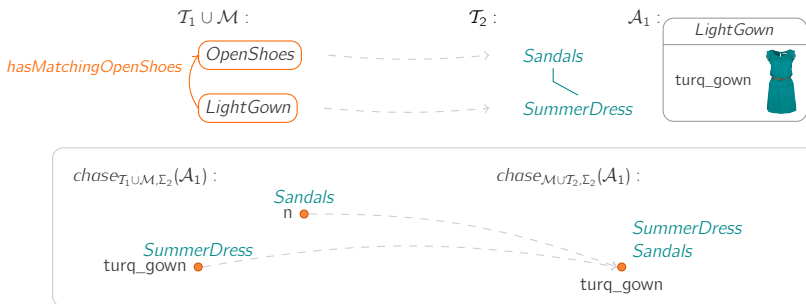




# Deciding Weak Representability

Recall that every  $DL\text{-Lite}_{RDFS}$  TBox is weakly representable under every  $DL\text{-Lite}_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

However, there exist a  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .



# Deciding Weak Representability

Recall that every  $DL\text{-Lite}_{RDFS}$  TBox is weakly representable under every  $DL\text{-Lite}_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

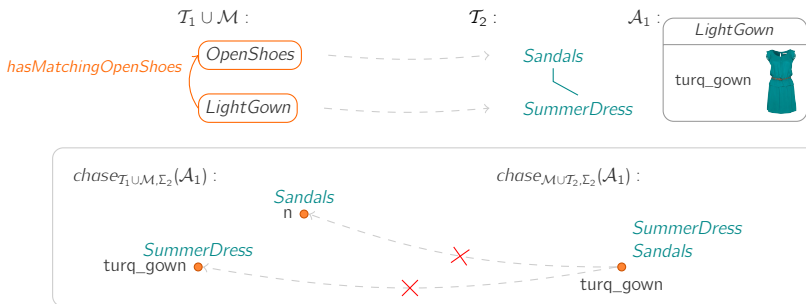
However, there exist a  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .



# Deciding Weak Representability

Recall that every  $DL\text{-Lite}_{RDFS}$  TBox is weakly representable under every  $DL\text{-Lite}_{RDFS}$  mapping ( $\mathcal{M}^*$  is the maximal mapping s.t.  $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$ ).

However, there exist a  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mapping  $\mathcal{M}$  and TBox  $\mathcal{T}_1$  s.t.  $\mathcal{T}_1$  is not weakly representable under  $\mathcal{M}$ .



## Theorem

Weak representability can be solved in polynomial time for  $DL\text{-Lite}_{\mathcal{R}}^{pos}$  mappings and TBoxes.

# Outline

- 1 Knowledge Base Exchange
- 2 Representability in  $DL-Lite_{\mathcal{R}}$
- 3 Conclusions





## Conclusions and Future Work

- We continued our work on KB exchange in  $DL-Lite$ .
- We addressed the problem of UCQ-representability in  $DL-Lite_{\mathcal{R}}^{pos}$ , the fragment of  $DL-Lite_{\mathcal{R}}$  without disjointness assertions.
- We showed that the problem of representability and weak representability of a  $DL-Lite_{\mathcal{R}}^{pos}$  TBox under a  $DL-Lite_{\mathcal{R}}^{pos}$  mapping is decidable in polynomial time.
- To solve representability in full  $DL-Lite_{\mathcal{R}}$  it remains to add disjointness constraints.
- There are also other open problems in the context of KB exchange, e.g., computing universal solutions, checking solution etc.



Thank you  
for your attention!

