

Query-Based Entailment and Inseparability for \mathcal{ALC} Ontologies

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Query Inseparability for Ontologies

By an **ontology** \mathcal{O} we mean

- a **knowledge base** $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, or
- a **TBox** \mathcal{T} .

Query answering over ontologies is an important reasoning task.

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Ontologies \mathcal{O}_1 and \mathcal{O}_2 are **query-inseparable** when we **cannot distinguish** between them by means of queries.

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Applications

- extracting **modules**
- comparing **versions** of an ontology
- **forgetting** some symbols from an ontology
- **exchanging** knowledge

Query Inseparability for Knowledge Bases

Consider a **class of queries** $\mathcal{Q} \in \{\text{CQ}, \text{UCQ}\}$, and
a **signature** Σ of concept and role names.

KBs $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$ and $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ are **Σ - \mathcal{Q} inseparable**, $\mathcal{K}_1 \equiv_{\Sigma}^{\mathcal{Q}} \mathcal{K}_2$, if

$$\mathcal{K}_1 \models \mathbf{q}(\mathbf{a}) \quad \iff \quad \mathcal{K}_2 \models \mathbf{q}(\mathbf{a})$$

for all Σ -queries $\mathbf{q} \in \mathcal{Q}$ and all individuals \mathbf{a} in \mathcal{K}_1 and \mathcal{K}_2 .

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$$\Sigma = \{ \text{spots} \}$$

Examples

Query inseparability is different from logical equivalence:

$$\mathcal{K}_1 = (\{A \sqsubseteq B\}, \{A(a)\}) \quad \text{and} \quad \mathcal{K}_2 = (\emptyset, \{A(a), B(a)\})$$

$$\mathcal{K}_1 \not\equiv \mathcal{K}_2 \quad \text{but} \quad \mathcal{K}_1 \equiv_{\substack{(U)CQ \\ \{A,B\}}} \mathcal{K}_2$$

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UCQ-inseparability and CQ-inseparability are distinct:

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Signature makes a difference:

$$\mathcal{K}_1 \equiv_{\{A\}}^{UCQ} \mathcal{K}_2$$

Query Inseparability for TBoxes

Consider signatures: Σ_1 for **ABoxes** and Σ_2 for **queries**.

TBoxes \mathcal{T}_1 and \mathcal{T}_2 are **(Σ_1, Σ_2) - $(U)CQ$ inseparable**, $\mathcal{T}_1 \equiv_{(\Sigma_1, \Sigma_2)}^{(U)CQ} \mathcal{T}_2$, if

$$(\mathcal{T}_1, \mathcal{A}) \equiv_{\Sigma_2}^{(U)CQ} (\mathcal{T}_2, \mathcal{A})$$

for all Σ_1 -ABoxes \mathcal{A} .

Main Results

KBs:

- (rooted) CQ-inseparability **undecidable** for \mathcal{ALC} .
- (rooted) UCQ-inseparability **2ExpTime-complete**.

TBoxes:

- (rooted) CQ-inseparability **undecidable** for \mathcal{ALC} .
- CQ/UCQ-inseparability **2ExpTime-complete** for $\text{Horn-}\mathcal{ALC}$.
- rooted CQ/UCQ-inseparability **ExpTime-complete** for $\text{Horn-}\mathcal{ALC}$.

See you
at the poster!

