Circumscribing *DL-Lite*

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Description Logic $DL\text{-}Lite^{H}_{bool}$

Description Logics (DLs) are decidable fragments of First-Order Logic, used as Knowledge Representation formalisms.

$DL\text{-}Lite^{\mathcal{H}}_{bool}$ is a light-weight DL that asserts

- Boolean combinations of *atomic concepts* A, *the domain* $\exists P$ and *the range* $\exists P^-$ of atomic roles P,
- Hierarchy of *atomic roles* P and their *inverses* P^-, and
- ground facts $A(a)$, $P(a, b)$.

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Satisfiability check over a $DL\text{-}Lite^{\mathcal{H}}_{bool}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ can be done in NP in combined complexity and in AC^0 in data complexity.

$DL\text{-}Lite^{H}_{bool}$ Knowledge Base

Encoding of the ‘Tweety’ example in $DL\text{-}Lite^{\mathcal{H}}_{bool}$:

$$
\begin{array}{lrcl}
\text{TBox } \mathcal{T}: & \mathit{Bird} \sqcap \neg \mathit{Abnormal} & \sqsubseteq & \mathit{Flier} \\
 & \mathit{Penguin} & \sqsubseteq & \mathit{Bird} \\
 & \mathit{Penguin} & \sqsubseteq & \mathit{Abnormal} \\
\text{ABox } \mathcal{A}: & \mathit{Bird}(\text{tweety}) & &
\end{array}
$$

Circumscription

Circumscription is a non-monotonic formalism introduced by John McCarthy.

Intuitively, circumscription of a predicate X says that

> *the only objects that satisfy X are those*
> *that can be proven to satisfy it.*

$$
\begin{aligned}
\mathrm{Circ}(X(a); X) &= \forall x \left(X(x) \equiv x = a \right) \\
\mathrm{Circ}(\neg X(a); X) &= \forall x\, \neg X(x) \\
\mathrm{Circ}(\forall x (\Phi(x) \rightarrow X(x)); X) &= \forall x \left(\Phi(x) \equiv X(x) \right) \\
\mathrm{Circ}(\forall x (X(x) \rightarrow \Phi(x)); X) &= \forall x\, \neg X(x)
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\mathrm{Circ}(\forall x \left(X(x) \rightarrow \Phi(x) \right); X) &= \forall x\ \neg X(x)
\end{aligned}$$

predicate completion

The Tweety Example

Recall

$$\begin{array}{lrcl} \text{TBox } \mathcal{T}: & Bird \sqcap \neg Abnormal & \sqsubseteq & Flier \\ & Penguin & \sqsubseteq & Bird \\ & Penguin & \sqsubseteq & Abnormal \\ \text{ABox } \mathcal{A}: & Bird(\text{tweety}) & & \end{array}$$

We have that

$$\mathrm{Circ}(\langle \mathcal{T}, \mathcal{A} \rangle; Abnormal) \models Flier(\text{tweety})$$

Now, let $\mathcal{A}' = \mathcal{A} \cup \{Penguin(\text{tweety})\}$. Then

$$\mathrm{Circ}(\langle \mathcal{T}, \mathcal{A}' \rangle; Abnormal) \not\models Flier(\text{tweety})$$

Note, that

$$\langle \mathcal{T}, \mathcal{A} \rangle \not\models Flier(\text{tweety})$$

Circumscription: Semantics

The models of $\mathsf{Circ}(\mathcal{K}; X)$ are the models of $\mathcal{K}$ such that the extension of X cannot be made smaller without losing the property $\mathcal{K}$.

Formally, let $\mathcal{I}$ and $\mathcal{J}$ be two classical interpretations of $\mathcal{K}$. Then we write $\mathcal{I} \leq^{X} \mathcal{J}$ if

- $\Delta^{\mathcal{I}} = \Delta^{\mathcal{J}}$,
- $Y^{\mathcal{I}} = Y^{\mathcal{J}}$ for every $Y \neq X$.
- $X^{\mathcal{I}} \subseteq X^{\mathcal{J}}$.

An interpretation $\mathcal{I}$ is a *model* of $\mathsf{Circ}(\mathcal{K}; X)$ if

- it is a *model* of $\mathcal{K}$ and
- it is *minimal* relative to $\leq^{X}$.

Circumscribing $DL\text{-}Lite^{H}_{bool}$

- In this paper we show how to compute circumscription of a single predicate (a concept or a role) in a $DL\text{-}Lite^{\mathcal{H}}_{bool}$ KB.

- To simplify presentation, in this talk I show how to circumscribe $DL\text{-}Lite^{\mathcal{H}}_{core}$ KBs.

  Given a $DL\text{-}Lite^{\mathcal{H}}_{core}$ TBox $\mathcal{T}$ and a predicate X, we compute

  $$\mathrm{Circ}(\mathcal{T}; X)$$

  Then we show how an ABox can be added to the theory.

- $DL\text{-}Lite^{\mathcal{H}}_{core}$ is a sub-logic of $DL\text{-}Lite^{\mathcal{H}}_{bool}$ with inclusions of the form

$$B_1 \sqsubseteq B_2 \qquad B_2 \sqsubseteq \neg B_2$$
$$R_1 \sqsubseteq R_2 \qquad R_2 \sqsubseteq \neg R_2$$

(B_i denote A, $\exists P$, or $\exists P^-$, R_i denote P or P^-).

Circumscribing a Concept

In $DL\text{-}Lite^{\mathcal{H}}_{core}$, minimizing an atomic concept A corresponds to *predicate completion*.

Let $\mathcal{T}$ be a $DL\text{-}Lite^{\mathcal{H}}_{core}$ TBox and $\mathsf{Pos}_{\mathcal{T}}(A) = \{B_i \sqsubseteq A\}^{1 \leq i \leq n}$ the set of all inclusions in $\mathcal{T}$ where A appears positively (i.e., without negation on the right-hand side of an ISA inclusion).

Then

$$\mathsf{Circ}(\mathcal{T}; A) = \mathcal{T} \cup \{B_1 \sqcup \cdots \sqcup B_n \equiv A\}$$

Note that when computing circumscription of A we can forget about negative occurrences of A, i.e., axioms of the form $A \sqsubseteq B$ or $B \sqsubseteq \neg A$.

Circumscribing a Role

In $DL\text{-}Lite^{\mathcal{H}}_{core}$, a role P can occur positively in the following inclusions:

$$\begin{array}{rcll} R & \sqsubseteq & P & \text{for a role } R \\ B_1 & \sqsubseteq & \exists P & \text{for a concept } B_1 \\ B_2 & \sqsubseteq & \exists P^- & \text{for a concept } B_2 \end{array}$$

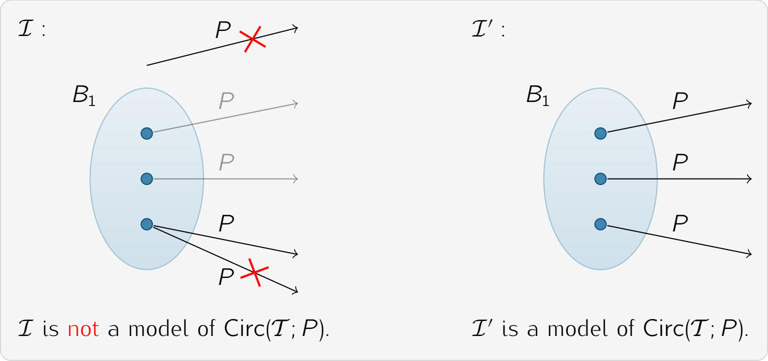
For a $DL\text{-}Lite^{\mathcal{H}}_{core}$ TBox $\mathcal{T}$, if $\mathsf{Pos}_{\mathcal{T}}(P) = \{R_i \sqsubseteq P\}^{1 \leq i \leq n}$ s.t. $R_i \neq P^-$, then this corresponds to the case of *predicate completion* and

$$\mathsf{Circ}(\mathcal{T}; P) = \mathcal{T} \cup \{R_1 \sqcup \cdots \sqcup R_n \equiv P\}.$$

It remains to consider the other cases and their combinations.

Circumscribing a Role: $B_1 \sqsubseteq \exists P$

Assume $\mathcal{T} = \{B_1 \sqsubseteq \exists P\}$.



One can show that

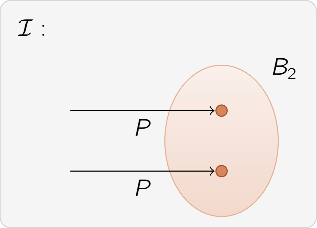
$$\mathsf{Circ}(\mathcal{T}; P) = \{B_1 \equiv \exists P, \mathsf{Funct}(P)\}.$$

Circumscribing a Role: $B_2 \sqsubseteq \exists P^-$

For $\mathcal{T} = \{B_2 \sqsubseteq \exists P^-\}$, symmetrically to the previous case,

$$\mathsf{Circ}(\mathcal{T}; P) = \{B_2 \equiv \exists P^-, \mathsf{Funct}(P^-)\},$$

and models have the following form:

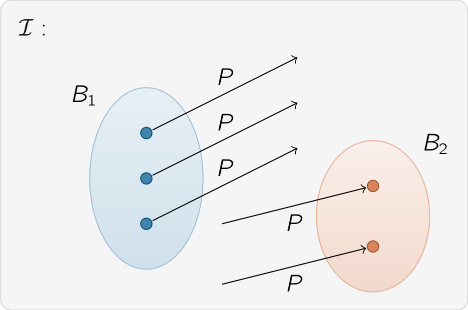


Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$

However, if $\mathcal{T} = \{B_1 \sqsubseteq \exists P, B_2 \sqsubseteq \exists P^-\}$,

$$\mathsf{Circ}(\mathcal{T}; P) \not\models B_1 \equiv \exists P$$
$$\mathsf{Circ}(\mathcal{T}; P) \not\models B_2 \equiv \exists P^-$$

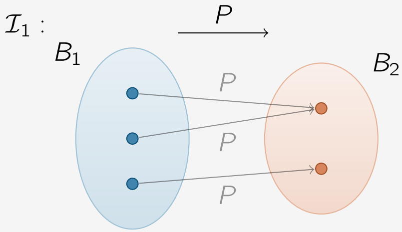
because $\mathcal{I}$ is a model of $\mathsf{Circ}(\mathcal{T}; P)$:



From now on, we assume $\mathcal{T} = \{B_1 \sqsubseteq \exists P, B_2 \sqsubseteq \exists P^-\}$ s.t. $P \notin \Sigma(B_1, B_2)$.

Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$ – 1

First, we restrict the domain and the range of P:



Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$ – 1

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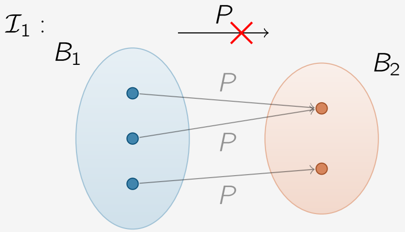


To prohibit such interpretations:

$$\forall x, y \Big(\big(P(x, y) \wedge \neg B_2(y) \wedge \neg B_1(x) \rightarrow \bot \Big)$$

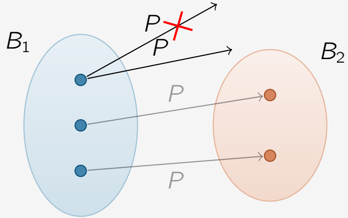
or in the DL syntax ($\mathcal{ALC}$ required)

$$\exists P . \neg B_2 \sqcap \neg B_1 \sqsubseteq \bot$$

Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$ - 2

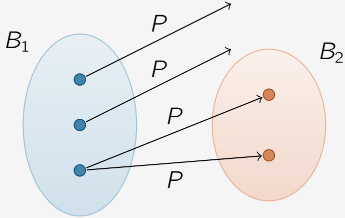
Second, does $\mathrm{Circ}(\mathcal{T}; P)$ entail $\mathrm{Funct}(P)$, $\mathrm{Funct}(P^-)$?

$\mathcal{I}_2$:



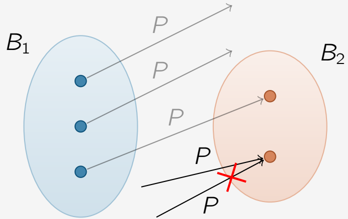
$\mathcal{I}_2$ is not a model of $\mathrm{Circ}(\mathcal{T}; P)$.

$\mathcal{I}_2'$:



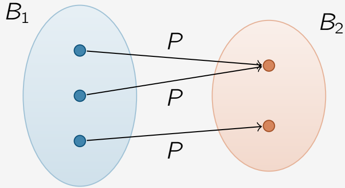
But $\mathcal{I}_2'$ is a model of $\mathrm{Circ}(\mathcal{T}; P)$.

$\mathcal{I}_3$:



$\mathcal{I}_3$ is not a model of $\mathrm{Circ}(\mathcal{T}; P)$.

$\mathcal{I}_3'$:



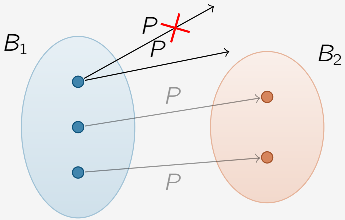
But $\mathcal{I}_3'$ is a model of $\mathrm{Circ}(\mathcal{T}; P)$.

Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$ - 2 contd

How to enforce 'local' functionality of P?

We use qualified number restrictions ($\mathcal{ALCIQ}$ is required) :

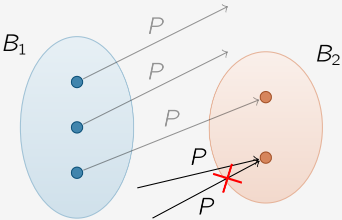
$\mathcal{I}_2$:



To prohibit such interpretations

$$\geq 2\, P.\neg B_2 \sqsubseteq \bot$$

$\mathcal{I}_3$:



To prohibit such interpretations

$$\geq 2\, P^-.\neg B_1 \sqsubseteq \bot$$

More Restrictions

Which interpretations should be still sorted out?

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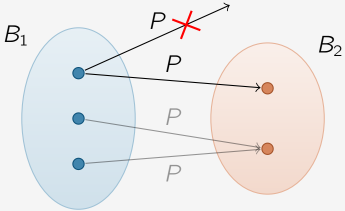
$\mathcal{I}_4$:



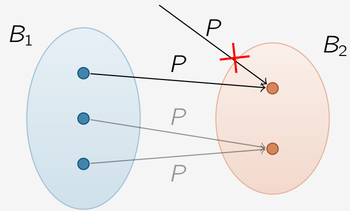
To prohibit such interpretations:

$$\exists P.B_2 \sqcap \exists P.\neg B_2 \sqsubseteq \bot$$

More Restrictions

Which interpretations should be still sorted out?

$\mathcal{I}_5$:



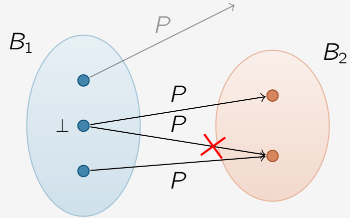
To prohibit such interpretations:

$$\exists P^{-}.B_1 \sqcap \exists P^{-}.\neg B_1 \sqsubseteq \bot$$

More Restrictions

Which interpretations should be still sorted out?

$\mathcal{I}_6$:



To prohibit such interpretations:

$$\geq 2\, P \sqcap \exists P.(\geq 2\, P^{-}) \sqsubseteq \perp$$

Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$ Summary

$\mathrm{Circ}(\{B_1 \sqsubseteq \exists P, B_2 \sqsubseteq \exists P^-\}; P)$ is the following $\mathcal{ALCIQ}$ TBox:

$$B_1 \sqsubseteq \exists P$$
$$B_2 \sqsubseteq \exists P^-$$

1 $\exists P.\neg B_2 \sqsubseteq B_1$

2 $\geq 2\, P.\neg B_2 \sqsubseteq \bot$

3 $\geq 2\, P^-.\neg B_1 \sqsubseteq \bot$

4 $\exists P.B_2 \sqcap \exists P.\neg B_2 \sqsubseteq \bot$

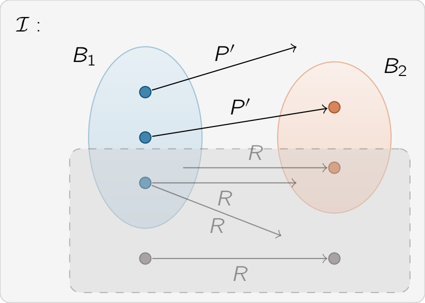
5 $\exists P^-.B_1 \sqcap \exists P^-.\neg B_1 \sqsubseteq \bot$

6 $\geq 2\, P \sqcap \exists P.(\geq 2\, P^-) \sqsubseteq \bot$

Circumscribing a Role: $B_1 \sqsubseteq \exists P$, $B_2 \sqsubseteq \exists P^-$, $R \sqsubseteq P$

Assume that $P \notin \Sigma(R)$, then $\mathrm{Circ}(\{B_1 \sqsubseteq \exists P, B_2 \sqsubseteq \exists P^-, R \sqsubseteq P\}; P)$ is the following $\mathcal{ALCHIQ}$ + union of roles TBox:

$$\mathrm{Circ}(\{B_1 \sqcap \neg\exists R \sqsubseteq \exists P', B_2 \sqcap \neg\exists R^- \sqsubseteq \exists P'^-\}; P')$$
$$P \equiv P' \sqcup R$$



These results can be generalized to arbitrary $DL\text{-}Lite^{\mathcal{H}}_{core}$ TBoxes, including cyclic inclusions on P of the form $\exists P^- \sqsubseteq \exists P$, $P^- \sqsubseteq P$.

Circumscribing $DL\text{-}Lite^{H}_{core}$ Knowledge Bases

It remains to address ABoxes.

Circumscription of an ABox requires nominals and number restrictions:

$$\mathrm{Circ}(\{A(a)\}; A) = A \equiv \{a\}$$

$$\mathrm{Circ}(\{P(a,b)\}; P) = \exists P \sqsubseteq \{a\},\ \exists P^{-} \sqsubseteq \{b\},\ \{a\} \sqsubseteq\ \leq 1\, P,\ \{b\} \sqsubseteq\ \leq 1\, P^{-}$$

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$$\mathrm{Circ}(\{P(a,b)\}; P) \quad = \quad \begin{array}{l} \exists P \sqsubseteq \{a\}, \\ \exists P^{-} \sqsubseteq \{b\}, \\ \{a\} \sqsubseteq \leq 1\, P, \\ \{b\} \sqsubseteq \leq 1\, P^{-} \end{array}$$

Finally, given a $DL\text{-}Lite^{\mathcal{H}}_{core}$ KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ and a predicate X,

$$\mathrm{Circ}(\mathcal{K}; X) = \mathrm{Circ}(\mathcal{T}'; X) \cup \mathrm{Circ}(\mathcal{A}'; X'),$$

where X' is a fresh predicate, $\mathcal{A}' = \mathcal{A}[X/X']$ and $\mathcal{T}' = \mathcal{T} \cup \{X' \sqsubseteq X\}$.

Conclusions and Future Work

1. We computed circumscription of a single predicate in a $DL\text{-}Lite^{\mathcal{H}}_{bool}$ KB.
   - it is first-order expressible and requires the language of $\mathcal{ALCHIOQ}$ augmented with union of roles.
2. To fully address the problem of circumscribing $DL\text{-}Lite^{\mathcal{H}}_{bool}$, we need to consider the following parameters:
   - multiple minimized predicates,
   - varying predicates.
3. There has been work on circumscribed DL KBs by [Bonatti *et al.*, 2009] and [Bonatti *et al.*, 2011].
   - they are mostly interested in checking entailment, in expressive and in tractable DLs.

   Using our characterization we can also check entailment.

Thank you
for your attention!

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