Model Checking Temporal-Epistemic Logic using Alternating Tree Automata

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  [KVW00]: model checking CTL via alternating tree automata
- Nonetheless, automata-theoretic techniques for temporal epistemic MAS logics have been investigated only partially.

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Main result:

- in selected cases explicit MC returns negative results fast.
  - $\Rightarrow$ No need to explore the whole state space.

Models: Interpreted Systems

Interpreted systems: typical formalism for reasoning about knowledge in MAS [FHMV95].

- each agent $A_i \in Ag$ is in some local state $l_i \in L_i$
- $\mathcal{S} \subseteq L_1 \times \ldots \times L_m$ is the set of global states

Definition (Interpreted System)

An *interpreted system* is a tuple $\mathcal{P} = \langle R, s_0, \pi \rangle$ such that

(i) R is a non-empty set of *runs* $\rho : \mathbb{N} \rightarrow \mathcal{S}$

(ii) $s_0 \in \mathcal{S}$ is the initial state

(iii) $\pi : \mathcal{S} \rightarrow 2^{AP}$ is a truth-assignment for atomic proposition in AP

Epistemic indistinguishability:

- for every agent $A_i \in Ag$, $(\rho, n) \sim_i (\rho', n')$ iff $\rho_i(n) = \rho'_i(n')$.

IS are temporal epistemic structures, on which we can interpret CTLK.

Specification Language: the temporal epistemic logic CTLK

Let $Ag = \{A_1, \ldots, A_m\}$ be a set of agents.

Definition (CTLK)

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \rightarrow \phi \mid AX\phi \mid A\phi U\phi \mid E\phi U\phi \mid K_i\phi$$

- CTLK combines the temporal modalities in CTL with an epistemic operator K_i for each agent $A_i \in Ag$.

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- $\overline{K}_i\ EF\ recover$ – agent i can't rule out that the system will eventually recover ...
  $EF\ K_i\ recover$ – ... but only when this happens she will be sure of this fact.

Trees

Let Ag_t be the set $Ag \cup \{t\}$.

Definition (Tree)

An Ag_t-tree is a set $T \subseteq (\mathbb{N} \times Ag_t)^*$ s.t. if $x \cdot (c, j) \in T$ and $(c, j) \in \mathbb{N} \times Ag_t$ then

- $x \in T$
- for all $0 \leq c' < c$, also $x \cdot (c', j) \in T$

A Σ-labelled tree is a pair $\langle T, V \rangle$ where T is a tree and $V : T \to \Sigma$.

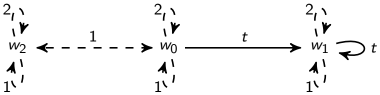
Lemma

Given an IS $\mathcal{P}$, the $\mathcal{S}$-labelled tree $\langle T_{\mathcal{P}}, V_{\mathcal{P}} \rangle$ obtained by unwinding $\mathcal{P}$ is s.t.

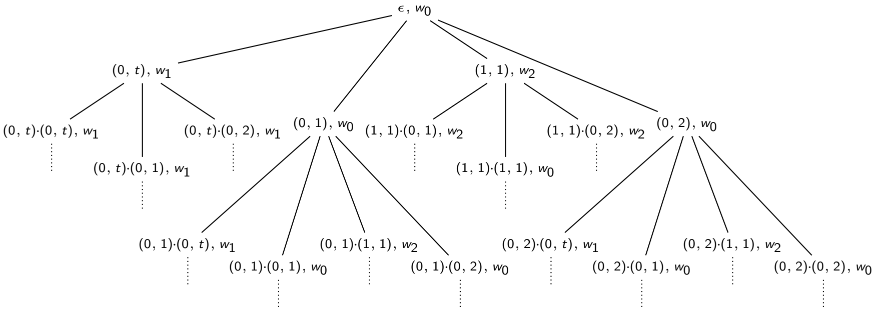
$$T_{\mathcal{P}} \models \phi \quad \textit{iff} \quad \mathcal{P} \models \phi$$

Example 1 – Unwinding an Interpreted System

Consider the IS $\mathcal{P}$ with $Ag = \{1, 2\}$.



The $\mathcal{S}$-labelled tree $\langle T_{\mathcal{P}}, V_{\mathcal{P}} \rangle$ unwinding $\mathcal{P}$ can be given as



Weak Epistemic Alternating Tree Automata

Extension of Alternating Tree Automata [MSS86], i.e., non-deterministic tree automata endowed with a weakness partition.

Definition (Alternating Tree Automaton)

An *alternating tree automaton* is a tuple $\mathcal{A} = \langle \Sigma, \mathcal{D}, Q, \delta, q_0, Ag_t, F \rangle$ such that

(i) Σ and Ag_t are defined as above

(ii) $\mathcal{D} \subset \mathbb{N}$ is a finite set of *degrees* (i.e., branching factors)

(iii) Q is a set of *states* endowed with a *weakness partition*

(iv) $q_0 \in Q$ is the *initial state*

(v) $F \subseteq Q$ is the set of *accepting states*

(vi) $\delta : Q \times \Sigma \times \mathcal{D}^{|Ag_t|} \longrightarrow \mathcal{B}^+(\mathbb{N} \times Ag_t \times Q)$ is the *transition function*

Acceptance is defined w.r.t. a Büchi acceptance condition.

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3. the language $\mathcal{L}(\mathcal{A}_{\mathcal{P},\psi})$ is non-empty iff the tree $\langle T_{\mathcal{P}}, V_{\mathcal{P}} \rangle$ is accepted by $\mathcal{A}_{\mathcal{D},\psi}$, i.e., iff ψ is true in $\mathcal{P}$.

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By extending the results in [KVW00] all these steps can be performed in linear time.

$\Rightarrow$ Compare the situation with alternating tree automata.

Example 2 – from CTLK to WEAA

- Consider the CTLK formula $\varphi' = AGK_1K_2p$.
- Put φ' into NNF with all abbreviations expanded: $\varphi = A\left(\mathbf{false}\overline{U}K_1K_2p\right)$.
- The closure of φ is $cl(\varphi) = \{\varphi, K_1K_2p, K_2p, p\}$.
- The accepting states are $F = \{\varphi, K_1K_2p, K_2p\}$.
- We define $\mathcal{A}_{\mathcal{D},\varphi} = \langle 2^{\{p\}}, \mathcal{D}, cl(\varphi), \delta, \varphi, F\rangle$ where the transition relation δ is defined as

q	$\delta(q, p, k)$	$\delta(q, \emptyset, k)$
φ	$\bigwedge_{c=0}^{k_t-1}(c, t, \varphi) \wedge \bigwedge_{c=0}^{k_i-1}(c, i, p) \wedge \bigwedge_{c=0}^{k_i-1}(c, i, K_ip)$	
K_1K_2p	$\bigwedge_{c=0}^{k_1-1}(c, 1, K_2p) \wedge \bigwedge_{c=0}^{k_1-1}(c, 1, K_1K_2p)$	
K_2p	$\bigwedge_{c=0}^{k_2-1}(c, 2, p) \wedge \bigwedge_{c=0}^{k_2-1}(c, 2, K_2p)$	
p	**true**	**false**

Implementation

- ETAV – Epistemic Tree Automata Verifier: explicit-state model checker (written in C++).
  - Open source release available from `http://bitbucket.org/etav/etav/`
- Approach taken: depth-first construction of product automaton $\mathcal{A}_{\mathcal{P},\psi}$, interleaved with the non-emptiness check.
- If we can decide whether the run is accepting without building the full product automata, ETAV will return this result early and save on computation.
- Optimisations:
  1. information on the satisfaction of a formula at a node is reused.
  2. the sibling for a node is constructed iff the current node is not sufficient to decide path acceptance.
  3. the transition relation is constructed only when required.

The Gossip Protocol

$GP_1 \quad EF\left(\bigwedge_{i \in Ag} complete_i\right)$

$GP_2 \quad K_{G1} EF\left(\bigwedge_{i \in Ag} complete_i\right)$

$GP_3 \quad AG\left(complete_{G1} \rightarrow K_{G1} AF\left(\bigwedge_{i \in Ag} complete_i\right)\right)$

$\|A\|$	Formula	Memory (KiB)	Time (s)	Nodes
3	GP_1	3336	0.002	35
	GP_2	3336	0.002	66
	GP_3	3336	0.002	131
4	GP_1	3576	0.030	69
	GP_2	3576	0.031	531
	GP_3	3576	0.029	46
5	GP_1	452444	84.646	95
	GP_2	452308	84.573	41596
	GP_3	452100	84.649	232

- the high execution times for $|A| = 5$ arises from parsing the large, explicitly-declared state space.

The Faulty Train Gate Controller

TGC_1 $AG\,(train1\_in\_tunnel \rightarrow EF\neg train1\_in\_tunnel)$
TGC_2 $AG\,(\neg train1\_in\_tunnel \vee \neg train2\_in\_tunnel)$
TGC_3 $AG(train1\_in\_tunnel \rightarrow K_{Train1}\neg train2\_in\_tunnel)$
TGC_4 $AG(K_{Train1}\,(\neg train1\_in\_tunnel \vee \neg train2\_in\_tunnel))$

Depth	Formula	Memory (KiB)	Time (s)	Nodes
1	TGC_1	12020	1.383	308
	TGC_2	12024	1.381	199
	TGC_3	12020	1.384	114
	TGC_4	30600	1.973	298284
6	TGC_1	7932	0.695	1751
	TGC_2	7932	0.697	1118
	TGC_3	7932	0.695	55
	TGC_4	12936	0.838	82098
w	TGC_1	8910	0.638	27822
	TGC_2	8914	0.625	27140
	TGC_3	9037	0.626	29401
	TGC_4	26414	1.106	307169

- unsatisfiable formulas (depth $= 1$ or 6) lead to smaller state-spaces.

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In future work we aim at:

- comparing (fairly) the automata-theoretic approach with existing symbolic techniques.
- implementing a *real* "on-the-fly" model checking procedure.
- exploring the deployment of partial order reduction techniques, which often rely on an automata-theoretic approach.

Questions?

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