Fundamenta Informaticae XXI (2001) 1001–1016
IOS Press

First-Order Linear-time Epistemic Logic with Group Knowledge: An Axiomatisation of the Monodic Fragment

Francesco Belardinelli [C]

Alessio Lomuscio
Department of Computing
Imperial College London, UK
{*F.Belardinelli, A.Lomuscio*}*@imperial.ac.uk*

Abstract. We investigate quantified interpreted systems, a computationally grounded semantics for a first-order temporal epistemic logic on linear time. We report a completeness result for the monodic fragment of a language that includes LTL modalities as well as distributed and common knowledge. We exemplify possible uses of the formalism by analysing message passing systems, a typical framework for distributed systems, in a first-order setting.

Keywords: First-order Modal Logic, Temporal Logic, Epistemic Logic, Multi-Agent Systems, Completeness.

1. Introduction

Propositional modal logics to reason about knowledge and time have been thoroughly investigated by researchers in logic and artificial intelligence both as regards their fundamental theoretical properties (completeness, decidability, complexity) [3, 8, 10], as well as their suitability for the specification and verification of multi-agent systems [5, 21, 31].

In one line of research epistemic modalities have been added to represent group knowledge such as distributed and common knowledge [9, 11]. In another one, the temporal fragment has been modified

Address for correspondence: Department of Computing
Imperial College London, UK
[C]Corresponding author

according to different models of time (e.g., linear or branching, discrete or continuous) [17, 19]. In yet another line, temporal epistemic logic has been studied within a first order setting [1, 6, 15].

In this paper we extend a combination of epistemic and temporal logic to the predicate level. We provide this language with a computationally grounded semantics [30] given in terms of *quantified interpreted systems* [1, 2], and we present a sound and complete axiomatisation of the *monodic* fragment of this logic, where at most one free variable appears in the scope of any modal operator. Finally, we apply this formalism to the modeling of message passing systems, a typical framework in distributed systems [18, 5].

Our starting point is a number of results by Hodkinson, Wolter, and Zakharyaschev, among others, regarding the axiomatisability [25, 29], decidability [15, 28], and complexity [13, 14] of first-order modal logics, including both positive [12, 24] and negative results [16, 26, 27]. Specifically, we prove the completeness of our first-order temporal epistemic logic via a *quasimodel* construction. These structures have been introduced in [15] to prove decidability for *monodic* fragments of first-order temporal logic (FOTL) on a variety of flows of time. These investigations were further pursued in [16], where branching flows of time are analysed, and in [12], which deals with the packed fragment of FOTL. In [13, 14] the complexity of the decision problem for a number of monodic fragments of FOTL is considered.

As regards general first-order modal logic, the decidability of monodic fragments has been investigated in [28]. In [27] it is proved that first-order epistemic logic with common knowledge is not axiomatisable. However, in [26] it is shown that its monodic fragment is. Finally, this paper relies on results in [25, 29]. In [29] the authors present a complete axiomatisation for the monodic fragment of FOTL on the natural numbers. In [25] we have a similar result for a variety of first-order epistemic logics with common knowledge. None of these references use interpreted systems [5, 20] as the underlying semantics, as we do here.

Our motivation for this contribution comes from an interest in reactive, autonomous, distributed systems, or multi-agent systems (MAS), whose high-level properties may usefully be modeled by first-order temporal epistemic formalisms [4, 23, 31], and behaviours programmed by languages based on interpreted systems such as ISPL [22]. While temporal epistemic logics are well understood at the propositional level, their usefulness has been demonstrated in a number of applications (security and communication protocols, robotics), and model checking tools have been developed for them [7, 22], still we believe there is a growing need in web-services, security, as well as other areas, to extend these languages to the first order. As a preliminary contribution, in [2] we introduced a "static" version of quantified interpreted systems to model a first-order epistemic formalism. This was then extended to the temporal dimension in [1]. Differently from these previous works, here we explicitly assume linear-time operators and the natural numbers as the flow of time. Both features are crucial for applications, but they also increase the complexity of the formalism.

Scheme of the paper. In Section 2 we introduce the first-order temporal epistemic language $\mathcal{L}_m$ for a set $A = \{1, \ldots, m\}$ of agents, and in Section 3 we provide it with a computationally grounded semantics in terms of quantified interpreted systems, and present its monodic fragment. In Section 4 we explore its expressive power in specifying message passing systems. In Sections 5 and 6 we introduce an axiomatisation for the monodic fragment of $\mathcal{L}_m$ and prove its completeness.

2. Syntax

The first-order temporal epistemic language $\mathcal{L}_m$ contains individual variables $x_1, x_2, \ldots$, individual constants $c_1, c_2, \ldots$, and n-ary predicative letters $P_1^n, P_2^n, \ldots$ for $n \in \mathbb{N}$, the propositional connectives $\neg$ and $\rightarrow$, the universal quantifier $\forall$, the linear-time operators $\bigcirc$ and $\mathcal{U}$, the epistemic operators K_i for $i \in A$, the distributed knowledge operator D, and the common knowledge operator C.

The only terms $t_1, t_2, \ldots$ in $\mathcal{L}_m$ are individual variables and constants.

Definition 2.1. Formulas in $\mathcal{L}_m$ are defined in the Backus-Naur form as follows:

$$\phi \quad ::= \quad P^k(t_1, \ldots, t_k) \mid \neg\psi \mid \psi \rightarrow \psi' \mid \forall x\psi \mid \bigcirc\psi \mid \psi\mathcal{U}\psi' \mid K_i\psi \mid D\psi \mid C\psi$$

The formulas $\bigcirc\phi$ and $\phi\mathcal{U}\phi'$ are read as "*ϕ holds at the next step*" and "*ϕ' will eventually hold and ϕ is the case until that moment*". The formula $K_i\phi$ represents "*agent i knows ϕ*", while formulas $D\phi$ and $C\phi$ respectively mean "*ϕ is distributed knowledge*" and "*ϕ is common knowledge*" in the group A of agents.

We define the symbols $\wedge$, $\vee$, $\leftrightarrow$, $\exists$, G (always in the future), and F (some time in the future) as standard; while $\bar{K}_i\phi$ and $\bar{D}\phi$ are shorthands for $\neg K_i\neg\phi$ and $\neg D\neg\phi$ respectively. Further, $E\phi = \bigwedge_{i \in A} K_i\phi$, and for $\triangle$ equal to E or $\bigcirc$, $\triangle^k\phi$ is defined as follows for every $k \in \mathbb{N}$: $\triangle^0\phi = \phi$ and $\triangle^{k+1}\phi = \triangle\triangle^k\phi$.

Finally, by $\phi[\vec{y}]$ we mean that $\vec{y} = y_1, \ldots, y_n$ are all the free variables in ϕ; while $\phi[\vec{y}/\vec{t}]$ is the formula obtained by substituting simultaneously some, possibly all, free occurrences of $\vec{y}$ in ϕ with $\vec{t} = t_1, \ldots, t_n$, renaming bounded variables if necessary.

3. Quantified Interpreted Systems

In this section we present a dynamic version of the "static" quantified interpreted systems in [2] by assuming the natural numbers $\mathbb{N}$ as the underlying flow of time. Specifically, for each agent $i \in A$ in a multi-agent system we introduce a set L_i of local states $l_i, l'_i, \ldots$, and a set Act_i of actions $\mathrm{a}_i, \mathrm{a}'_i, \ldots$. We consider local states and actions for the environment e as well. The set $\mathcal{S} \subseteq L_e \times L_1 \times \ldots \times L_m$ contains the global states of the MAS, while $Act \subseteq Act_e \times Act_1 \times \ldots \times Act_m$ is the set of joint actions. We also introduce a transition function $\tau : Act \rightarrow (\mathcal{S} \rightarrow \mathcal{S})$. Intuitively, $\tau(\mathrm{a})(s) = s'$ encodes that agents access the global state s' from s by performing the joint action a. We say that the global state s' is *reachable in one step* from s, or $s \sqsubset s'$, iff there is $\mathrm{a} \in Act$ such that $\tau(\mathrm{a})(s) = s'$.

To represent the temporal evolution of the MAS we consider the flow of time $\langle \mathbb{N}, < \rangle$ of natural numbers $\mathbb{N}$ with the strict total order $<$. A run r over $\langle \mathcal{S}, Act, \tau, \mathbb{N} \rangle$ is a function from $\mathbb{N}$ to $\mathcal{S}$ such that $r(n) \sqsubset r(n+1)$. Intuitively, a run represents a possible evolution of the MAS according to the transition function τ and assuming $\mathbb{N}$ as the flow of time. We now define the quantified interpreted systems for the language $\mathcal{L}_m$ as follows:

Definition 3.1. (QIS)
A quantified interpreted system over $\langle \mathcal{S}, Act, \tau, \mathbb{N} \rangle$ is a triple $\mathcal{P} = \langle \mathcal{R}, \mathcal{D}, I \rangle$ such that:

(i) $\mathcal{R}$ is a non-empty set of runs over $\langle \mathcal{S}, Act, \tau, \mathbb{N} \rangle$;

(ii) $\mathcal{D}$ is a non-empty set of individuals;

(iii) I is an interpretation of $\mathcal{L}_m$ such that $I(c) \in \mathcal{D}$, and for $r \in \mathcal{R}$, $n \in \mathbb{N}$, $I(P^k, r, n)$ is a k-ary relation on $\mathcal{D}$.

We denote by $\mathcal{QIS}$ the class of all quantified interpreted systems.

Note that the individual constants in $\mathcal{L}_m$ are interpreted rigidly, that is, their interpretation is the same in every global state. Following standard notation [5] a pair (r, n) is a *point* in $\mathcal{P}$. If $r(n) = \langle l_e, l_1, \ldots, l_m \rangle$ is the global state at the point (r, n), then $r_e(n) = l_e$ and $r_i(n) = l_i$ are the environment's and agent i's local state at (r, n) respectively. Further, a QIS is *synchronous* if for every $i \in A$, $r_i(n) = r'_i(n')$ implies $n = n'$, that is, time is part of the local state of any agent. We denote by $\mathcal{QIS}^{sync}$ the class of all synchronous QIS.

Now we assign a meaning to the formulas of $\mathcal{L}_m$ in quantified interpreted systems. Let σ be an assignment from the variables to the individuals in $\mathcal{D}$, the valuation $I^\sigma(t)$ of a term t is defined as $\sigma(y)$ for $t = y$, and $I^\sigma(t) = I(c)$ for $t = c$. A variant $\sigma\binom{x}{a}$ of an assignment σ assigns $a \in \mathcal{D}$ to x and coincides with σ on all the other variables.

Definition 3.2. The satisfaction relation $\models$ for $\phi \in \mathcal{L}_m$, $(r, n) \in \mathcal{P}$, and an assignment σ is defined as follows:

$(\mathcal{P}^\sigma, r, n) \models P^k(t_1, \ldots, t_k)$	iff	$\langle I^\sigma(t_1), \ldots, I^\sigma(t_k) \rangle \in I(P^k, r, n)$
$(\mathcal{P}^\sigma, r, n) \models \neg\psi$	iff	$(\mathcal{P}^\sigma, r, n) \not\models \psi$
$(\mathcal{P}^\sigma, r, n) \models \psi \rightarrow \psi'$	iff	$(\mathcal{P}^\sigma, r, n) \not\models \psi$ or $(\mathcal{P}^\sigma, r, n) \models \psi'$
$(\mathcal{P}^\sigma, r, n) \models \forall x \psi$	iff	for all $a \in \mathcal{D}$, $(\mathcal{P}^{\sigma\binom{x}{a}}, r, n) \models \psi$
$(\mathcal{P}^\sigma, r, n) \models \bigcirc\psi$	iff	$(\mathcal{P}^\sigma, r, n+1) \models \psi$
$(\mathcal{P}^\sigma, r, n) \models \psi \mathcal{U} \psi'$	iff	there is $n' \geq n$ such that $(\mathcal{P}^\sigma, r, n') \models \psi'$ and for all $n'', n \leq n'' < n'$ implies $(\mathcal{P}^\sigma, r, n'') \models \psi$
$(\mathcal{P}^\sigma, r, n) \models K_i\psi$	iff	for all $(r', n'), r_i(n) = r'_i(n')$ implies $(\mathcal{P}^\sigma, r', n') \models \psi$
$(\mathcal{P}^\sigma, r, n) \models D\psi$	iff	$r_i(n) = r'_i(n')$ for all $i \in A,$ implies $(\mathcal{P}^\sigma, r', n') \models \psi$
$(\mathcal{P}^\sigma, r, n) \models C\psi$	iff	for all $k \in \mathbb{N}, (\mathcal{P}^\sigma, r, n) \models E^k\psi$

The truth conditions for $\wedge$, $\vee$, $\leftrightarrow$, $\exists$, G, and F are defined from those above. A formula $\phi \in \mathcal{L}_m$ is *true at a point* (r, m) iff it is satisfied at (r, m) by every σ; ϕ is *valid on a QIS* $\mathcal{P}$ iff it is true at every point in $\mathcal{P}$; ϕ is *valid on a class* $\mathcal{C}$ *of QIS* iff it is valid on every QIS in $\mathcal{C}$.

3.1. The monodic fragment

In the rest of the paper we focus on the monodic fragment of the language $\mathcal{L}_m$.

Definition 3.3. The monodic fragment $\mathcal{L}^1_m$ is the set of formulas $\phi \in \mathcal{L}_m$ such that any subformula of ϕ of the form $K_i\psi$, $D\psi$, $C\psi$, $\bigcirc\psi$, or $\psi_1 \mathcal{U} \psi_2$ contains at most one free variable.

The monodic fragments of a number of first-order modal logics have been thoroughly investigated [14, 15, 25, 28, 29]. In the case of $\mathcal{L}_m$ this fragment is quite expressive as it contains formulas like the following:

$$\forall y(\textit{Resource}(y) \rightarrow C(\forall z \textit{Available}(y, z) \mathcal{U} \exists x \textit{Request}(x, y)) \quad (1)$$

$$D \bigcirc \forall xyz(\textit{Request}(x, y) \rightarrow \neg\textit{Available}(y, z)) \rightarrow \bigcirc D \forall xyz(\textit{Request}(x, y) \rightarrow \neg\textit{Available}(y, z)) \quad (2)$$

Formula (1) states that it is common knowledge that every resource will eventually be requested, but until that time the resource remains universally available. Formula (2) represents that if it is distributed knowledge that at the next step any resource is not available whenever it is requested, then at the next step it is distributed knowledge that this is the case. However, note that the formula

$$\forall x K_i(\textit{Process}(x) \rightarrow \forall y F\ \textit{Access}(x, y)) \tag{3}$$

which intuitively means that agent i knows that every process will eventually try to access every resource, is not monodic. Still, the monodic fragment of $\mathcal{L}_m$ is quite expressive as it contains all *de dicto* formulas, i.e., formulas where no free variable appears in the scope of modal operators, as in (2).

4. Message Passing Systems

In this section we model message passing systems [5, 18] in the framework of QIS. A message passing system (MPS) is a MAS in which the only actions for the agents are sending and receiving messages. This setting is common to a variety of distributed systems, well beyond the realms of MAS and AI. Indeed, any synchronous or asynchronous networked system can be seen as an MPS.

To define our message passing QIS we introduce a set *Msg* of messages $\mu_1, \mu_2, \ldots$, and define the local state l_i for agent i as a *history* over *Msg*, that is, a sequence of events of the form *send*(i, j, μ) and *rec*(i, j, μ) for $i, j \in A$, $\mu \in$ *Msg*. Intuitively, *send*(i, j, μ) represents the event in which *agent* i *sends message* μ *to* j, while the meaning of *rec*(i, j, μ) is that *agent* i *receives message* μ *from* j. A global state $s \in \mathcal{S}$ is a tuple $\langle l_e, l_1, \ldots, l_n \rangle$ where $l_1, \ldots, l_n$ are local states as above, and l_e contains all the events in $l_1, \ldots, l_n$.

A run r over $\langle \mathcal{S}, \mathbb{N} \rangle$ is a function from the natural numbers $\mathbb{N}$ to $\mathcal{S}$ such that:

MP1 $r_i(0)$ is a sequence of length zero, and $r_i(m+1)$ is either identical to $r_i(m)$ or results from appending an event to $r_i(m)$.

By MP1 the local state of each agent records the messages she has sent or received, and at each step at most a single event occurs to any agent. We define message passing QIS (MPQIS) as the class of quantified interpreted systems $\mathcal{P} = \langle \mathcal{R}, \mathcal{D}, I \rangle$ where $\mathcal{R}$ is a non-empty set of runs satisfying MP1, $\mathcal{D}$ contains the agents in A and the messages in *Msg*, and I is an interpretation for $\mathcal{L}_m$. We use the same notation for objects in the model and syntactic elements, the distinction will be made clear by the context.

For the specification of MPS we introduce a predicative letter *Send* such that $(\mathcal{P}^\sigma, r, n) \models$ *Send*(i, j, μ) iff event *send*(i, j, μ) occurs to agent i at time n in run r, i.e., $r_i(n)$ is the result of appending *send*(i, j, μ) to $r_i(n-1)$. Also, we introduce the predicate *Sent* such that $(\mathcal{P}^\sigma, r, n) \models$ *Sent*(i, j, μ) iff event *send*(i, j, μ) occurs to agent i before time n in run r, i.e., *send*(i, j, μ) appears in $r_i(n)$. The predicates *Rec* and *Rec'ed* are similarly defined for event *rec*(i, j, μ).

Let us now explore the range of specifications that can be expressed in this formalism. A property often required in MPS is *channel reliability*. We express this by stating that every sent message is eventually received. Note that, according to the definition of message passing QIS, it is possible that a message is lost during a run of the system. We can force channel reliability by requiring the following specification to hold on MPQIS:

$$\forall \mu (\exists i j \textit{Send}(i, j, \mu) \rightarrow F \exists i' j' \textit{Rec}(j', i', \mu)) \tag{4}$$

In fact, we can be more specific and require that every message is received *at most* (*at least*) k steps after being sent, or *exactly* k steps after being sent:

$$\forall \mu(\exists ij\mathit{Send}(i,j,\mu) \rightarrow \bigcirc^k \exists i'j'\mathit{Rec'ed}(j',i',\mu)) \tag{5}$$

$$\forall \mu(\exists ij\mathit{Sent}(i,j,\mu) \rightarrow \bigcirc^k \exists i'j'\mathit{Rec}(j',i',\mu)) \tag{6}$$

$$\forall \mu(\exists ij\mathit{Send}(i,j,\mu) \rightarrow \bigcirc^k \exists i'j'\mathit{Rec}(j',i',\mu)) \tag{7}$$

Note that all of (4)-(7) are monodic. In these specifications the identities of the sender and the receiver are left unspecified. So, in cases in which we are not interested in singling out the addresser and the addressee, the monodic fragment suffices.

Another property often required on MPQIS is that there are no "ghost" messages: if agent i receives a message μ, then i knows that μ must actually have been sent by some agent j. This specification is expressible as a monodic formula:

$$\forall \mu(\exists j\mathit{Rec}(i,j,\mu) \rightarrow K_i \exists j'\mathit{Sent}(j',i,\mu)) \tag{8}$$

We compare (8) with a further relevant property of MPQIS, i.e., *authentication*: if agent i receives a message μ from agent j, then i knows that μ has actually been sent by j. This specification can be expressed as the *de re* version of (8):

$$\forall \mu j(\mathit{Rec}(i,j,\mu) \rightarrow K_i\mathit{Sent}(j,i,\mu)) \tag{9}$$

Note that, differently from (8), (9) is not monodic.

Even if we allow an agent i not to know whether a received message μ has actually been sent, that is, we reject (8), it can be checked on arbitrary MPQIS that the following formula holds:

$$\forall \mu(\exists ij(\mathit{Sent}(i,j,\mu) \wedge \mathit{Rec'ed}(j,i,\mu)) \rightarrow D\exists i'j'(\mathit{Sent}(i',j',\mu) \wedge \mathit{Rec'ed}(j',i',\mu)))$$

In other words, it is distributed knowledge that a message μ has been sent and received as soon as it has been received. On the other hand, the corresponding monodic formulas

$$\forall \mu(\exists j(\mathit{Sent}(i,j,\mu) \wedge \mathit{Rec'ed}(j,i,\mu)) \rightarrow K_i\exists j'(\mathit{Sent}(i,j',\mu) \wedge \mathit{Rec'ed}(j',i,\mu)))$$
$$\forall \mu(\exists i(\mathit{Sent}(i,j,\mu) \wedge \mathit{Rec'ed}(j,i,\mu)) \rightarrow K_j\exists i'(\mathit{Sent}(i',j,\mu) \wedge \mathit{Rec'ed}(j,i',\mu)))$$

are not valid for any agent i, j.

Furthermore, in $\mathcal{L}^1_m$ we can express that an agent i cannot aquire the knowledge that message μ has been sent to her, other than by receiving the message:

$$\forall \mu(\exists j\mathit{Sent}(j,i,\mu) \rightarrow (\neg K_i \exists j'\mathit{Sent}(j',i,\mu)\mathcal{U}\exists j''\mathit{Rec}(i,j'',\mu))$$

Finally, we might want to check whether at a certain point in the evolution of the MPQIS it will be common knowledge that a message has been sent or received:

$$\forall \mu(\exists ij\mathit{Sent}(i,j,\mu) \rightarrow FC(\exists i'j'\mathit{Sent}(i',j',\mu))) \tag{10}$$

$$\forall \mu(\exists ij\mathit{Rec'ed}(i,j,\mu) \rightarrow FC(\exists i'j'\mathit{Rec'ed}(i',j',\mu))) \tag{11}$$

From results in [5] regarding the attainability of common knowledge in systems with unreliable communication, we may infer that some assumption on channel reliability in MPQIS is needed in order to validate specifications (10) and (11).

The conclusion we draw from the observations above is that the monodic fragment of the language $\mathcal{L}_m$ allows for rich specifications on MPS, notwithstanding the limitation on free variables in modal contexts.

5. Axiomatisation

In this section we present a sound and complete axiomatisation of the set of monodic validities in the class of quantified interpreted systems. This result shows that, even though language $\mathcal{L}_m^1$ is highly expressive, QIS provide a perfectly adequate semantics for it. This also opens the possibility of developing automated verification methods for this formalism.

The system QKT_m^1 is a first-order multi-modal version of the propositional epistemic system $S5$ combined with the linear temporal logic LTL. Hereafter we list the postulates of QKT_m^1. Note that $\Rightarrow$ is the inference relation between formulas, while $\Box$ is a placeholder for any of the epistemic operators K_i for $i \in A$, D, or C.

Definition 5.1. The system QKT_m^1 contains the following schemes of axioms and inference rules, where ϕ, ψ and χ are formulas in $\mathcal{L}_m^1$:

Taut	instances of classic propositional tautologies
MP	$\phi \to \psi, \phi \Rightarrow \psi$
K	$\bigcirc(\phi \to \psi) \to (\bigcirc\phi \to \bigcirc\psi)$
T1	$\bigcirc\neg\phi \leftrightarrow \neg \bigcirc \phi$
T2	$\phi\mathcal{U}\psi \leftrightarrow \psi \vee (\phi \wedge \bigcirc(\phi\mathcal{U}\psi))$
Nec	$\phi \Rightarrow \bigcirc\phi$
T3	$\chi \to \neg\psi \wedge \bigcirc\chi \Rightarrow \chi \to \neg(\phi\mathcal{U}\psi)$
K	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$
T	$\Box\phi \to \phi$
4	$\Box\phi \to \Box\Box\phi$
5	$\neg\Box\phi \to \Box\neg\Box\phi$
Nec	$\phi \Rightarrow \Box\phi$
D	$K_i\phi \to D\phi$
C1	$C\phi \leftrightarrow (\phi \wedge EC\phi)$
C2	$\phi \to (\psi \wedge E\phi) \Rightarrow \phi \to C\psi$
BF	$\bigcirc\forall x\phi \leftrightarrow \forall x \bigcirc \phi$
BF	$\Box\forall x\phi \leftrightarrow \forall x\Box\phi$
Ex	$\forall x\phi \to \phi[x/t]$
Gen	$\phi \to \psi[x/t] \Rightarrow \phi \to \forall x\psi$ for x not free in ϕ

Table 1. the system QKT_m^1

The operators K_i, D, and C are $S5$ modalities, while the next $\bigcirc$ and until $\mathcal{U}$ operators are axiomatised as linear-time modalities. To this we add the classic postulates Ex and Gen for quantification, which are both sound as we are considering a unique domain $\mathcal{D}$ of individuals for each QIS. We consider the standard definitions of *proof* and *theorem*: $\vdash \phi$ means that $\phi \in \mathcal{L}^1_m$ is a theorem in QKT^1_m.

It is a routine exercise to check that the axioms of QKT^1_m are valid on every QIS and the inference rules preserve validity. As a consequence, we have the following soundness result:

Theorem 5.1. (Soundness)
The system QKT^1_m is sound with respect to the class $\mathcal{QIS}$ of quantified interpreted systems.

The next corollary directly follows from the fact that $\mathcal{QIS}^{sync}$ is a subset of $\mathcal{QIS}$.

Corollary 5.1. (Soundness)
The system QKT^1_m is sound with respect to the class $\mathcal{QIS}^{sync}$ of synchronous QIS.

Now we show that the axioms in QKT^1_m are not only necessary, but also sufficient to prove all monodic validities on $\mathcal{QIS}$ and $\mathcal{QIS}^{sync}$.

5.1. Kripke Models

Although quantified interpreted systems are useful for modeling MAS, to prove the completeness of QKT^1_m we first introduce an appropriate class of Kripke models, and prove completeness for these models. Then we apply a correspondence result between Kripke models and QIS to obtain the desired result.

Definition 5.2. A *Kripke model* for $\mathcal{L}_m$ is a tuple $\mathcal{M} = \langle \langle \mathbb{N}_j, <_j \rangle_{j \in J}, \{\sim_i\}_{i \in A}, \mathcal{D}, I \rangle$ such that:

(i) for $j \in J$, $\mathbb{N}_j$ is a copy of the natural numbers with the strict total order $<_j$;

(ii) for $i \in A$, $\sim_i$ is an equivalence relation on $\bigcup_{j \in J} \mathbb{N}_j$;

(iii) $\mathcal{D}$ is a non-empty set of individuals;

(iv) the interpretation I is such that $I(c) \in \mathcal{D}$, and for $n_j \in \mathbb{N}_j$, $I(P^k, n_j)$ is a k-ary relation on $\mathcal{D}$.

The class of all Kripke models is denoted by $\mathcal{K}$.

A Kripke model is *synchronous* if for every $i \in A$, $n_j \sim_i n'_{j'}$ implies $n = n'$. By $\mathcal{K}^{sync}$ we denote the class of all synchronous Kripke models. The satisfaction relation $\models$ for an assignment σ is inductively defined as follows:

$(\mathcal{M}^\sigma, n_j) \models P^k(t_1, \ldots, t_k)$	iff	$\langle I^\sigma(t_1), \ldots, I^\sigma(t_k) \rangle \in I(P^k, n_j)$
$(\mathcal{M}^\sigma, n_j) \models \neg\psi$	iff	$(\mathcal{M}^\sigma, n_j) \not\models \psi$
$(\mathcal{M}^\sigma, n_j) \models \psi \to \psi'$	iff	$(\mathcal{M}^\sigma, n_j) \not\models \psi$ or $(\mathcal{M}^\sigma, n_j) \models \psi'$
$(\mathcal{M}^\sigma, n_j) \models \forall x \psi$	iff	for all $a \in \mathcal{D}$, $(\mathcal{M}^{\sigma\binom{x}{a}}, n_j) \models \psi$
$(\mathcal{M}^\sigma, n_j) \models \bigcirc\psi$	iff	$(\mathcal{M}^\sigma, n+1_j) \models \psi$
$(\mathcal{M}^\sigma, n_j) \models \psi \mathcal{U} \psi'$	iff	there is $n'_j \geq_j n_j$ such that $(\mathcal{M}^\sigma, n'_j) \models \psi'$ and for all n''_j, $n_j \leq_j n''_j <_j n'_j$ implies $(\mathcal{M}^\sigma, n''_j) \models \psi$
$(\mathcal{M}^\sigma, n_j) \models K_i\psi$	iff	for all $n'_{j'}$, $n_j \sim_i n'_{j'}$ implies $(\mathcal{M}^\sigma, n'_{j'}) \models \psi$
$(\mathcal{M}^\sigma, n_j) \models D\psi$	iff	for all $n'_{j'}$, $(n_j, n'_{j'}) \in (\bigcap_{i \in A} \sim_i)$ implies $(\mathcal{M}^\sigma, n'_{j'}) \models \psi$
$(\mathcal{M}^\sigma, n_j) \models C\psi$	iff	for all $n'_{j'}$, $(n_j, n'_{j'}) \in (\bigcup_{i \in A} \sim_i)^*$ implies $(\mathcal{M}^\sigma, n'_{j'}) \models \psi$

where $(\bigcup_{i\in A} \sim_i)^*$ is the reflexive and transitive closure of the relation $(\bigcup_{i\in A} \sim_i)$.

We compare Kripke models and quantified interpreted systems by means of a map $g : \mathcal{K} \to \mathcal{QIS}$. Let $\mathcal{M} = \langle\langle\mathbb{N}_j, <_j\rangle_{j\in J}, \{\sim_i\}_{i\in A}, \mathcal{D}, I\rangle$ be a Kripke model. For every equivalence relation $\sim_i$, for $n_j \in \mathbb{N}_j$, let the equivalence class $[n_j]_{\sim_i} = \{n'_{j'} \mid n_j \sim_i n'_{j'}\}$ be a local state for agent i, while $\mathbb{N}_j$ is the set of local states for the environment. Then define $g(\mathcal{M})$ as the tuple $\langle\mathcal{R}, \mathcal{D}, I'\rangle$ where $\mathcal{R}$ contains the runs r_j for $j \in J$ such that $r_j(n) = \langle n_j, [n_j]_{\sim_1}, \ldots, [n_j]_{\sim_m}\rangle$, $\mathcal{D}$ is the same as in $\mathcal{M}$, and $I'(P^k, r_j, n) = I(P^k, n_j)$. The structure $g(\mathcal{M})$ is a QIS that satisfies the following result:

Lemma 5.1. For every $\phi \in \mathcal{L}_m$, $n \in \mathbb{N}$,

$$(\mathcal{M}^\sigma, n_j) \models \phi \quad \text{iff} \quad (g(\mathcal{M})^\sigma, r_j, n) \models \phi$$

We omit the proof of this lemma, which can be easily proved by induction on the lenght of ϕ. Note that if $\mathcal{M}$ is synchronous, then also $g(\mathcal{M})$ is synchronous, i.e., g also defines a map from $\mathcal{K}^{sync}$ to $\mathcal{QIS}^{sync}$.

6. Completeness

The completeness of the system QKT^1_m with respect to the classes $\mathcal{QIS}$ and $\mathcal{QIS}^{sync}$ of quantified interpreted systems is proved by means of a quasimodel construction [6]. In particular, the version of quasimodels here considered combines the purely epistemic structures in [25] with the purely temporal structures in [29]. Intuitively, a quasimodel for a monodic formula $\phi \in \mathcal{L}^1_m$ is a relational structure whose points are sets of sets of subformulas of ϕ. Each set of sets of subformulas describes a "possible state of affairs", and contains sets of subformulas defining the individuals in the point. In what follows we provide the exact definitions.

Definition 6.1. Given a formula $\phi \in \mathcal{L}^1_n$ we denote by $sub\phi$ the set of subformulas of ϕ, and define $sub_C\phi = sub\phi \cup \{EC\psi \mid C\psi \in sub\phi\} \cup \{K_iC\psi \mid C\psi \in sub\phi, i \in A\}$. Further, let $sub_{C\bigcirc}\phi = sub_C\phi \cup \{\neg\psi \mid \psi \in sub_C\phi\} \cup \{\bigcirc\psi \mid \psi \in sub_C\phi\} \cup \{\bigcirc\neg\psi \mid \psi \in sub_C\phi\}$.

Let $sub_n\phi$ be the subset of $sub_{C\bigcirc}\phi$ containing formulas with at most n free variables and let x be a variable not occurring in ϕ, we define $sub_x\phi = \{\psi[y/x] \mid \psi[y] \in sub_1\phi\}$. Clearly, x is the only free variable in $sub_x\phi$. By $con\phi$ we denote the set of all constants occurring in ϕ.

Definition 6.2. (Type)
A *type* for ϕ is any subset $\mathfrak{t}$ of $sub_x\phi$ such that for every $\psi, \chi \in sub_x\phi$, (i) $\psi \wedge \chi \in \mathfrak{t}$ iff $\psi \in \mathfrak{t}$ and $\chi \in \mathfrak{t}$; and (ii) $\neg\psi \in \mathfrak{t}$ iff $\psi \notin \mathfrak{t}$.

This definition of type is completely standard [6, 25, 29]. In what follows we do not distinguish between a type $\mathfrak{t}$ and the conjunction $\bigwedge_{\psi\in\mathfrak{t}} \psi$ of its formulas. Note that $sub_0\phi$ is the set of sentences in $sub_x\phi$. Two types $\mathfrak{t}$, $\mathfrak{t}'$ *agree* on $sub_0\phi$ iff $\mathfrak{t} \cap sub_0\phi = \mathfrak{t}' \cap sub_0\phi$, i.e., they share the same set of sentences. Finally, given a type $\mathfrak{t}$ for ϕ and a constant $c \in con\phi$, the pair $\langle\mathfrak{t}, c\rangle$ is called an *indexed type* for ϕ.

The following definition of state candidate is also standard.

Definition 6.3. (State Candidate)
Let T be a set of types for ϕ that agree on $sub_0\phi$, and T^{con} a set containing for each $c \in con\phi$ an indexed type $\langle \mathfrak{t}, c\rangle$ such that $\mathfrak{t} \in T$, then the pair $\mathfrak{C} = \langle T, T^{con}\rangle$ is a *state candidate* for ϕ.

Given a state candidate $\mathfrak{C} = \langle T, T^{con}\rangle$ we define the formula $\alpha_{\mathfrak{C}}$ as follows:

$$\alpha_{\mathfrak{C}} \quad := \quad \bigwedge_{\mathfrak{t}\in T} \exists x \mathfrak{t}[x] \wedge \forall x \bigvee_{\mathfrak{t}\in T} \mathfrak{t}[x] \wedge \bigwedge_{\langle \mathfrak{t}, c\rangle \in T^{con}} \mathfrak{t}[x/c]$$

Note that $\alpha_{\mathfrak{C}}$ is monodic. A state candidate $\mathfrak{C}$ is *consistent* iff the formula $\alpha_{\mathfrak{C}}$ is consistent with QKT^1_m, i.e., $\nvdash \neg\alpha_{\mathfrak{C}}$. Consistent state candidates will be the points of our quasimodel. We now define a relation of *suitability* for types and state candidates that constitute the relational part of our quasimodel.

Definition 6.4. 1. A pair $(\mathfrak{t}_1, \mathfrak{t}_2)$ of types is $\bigcirc$*-suitable* iff the formula $\mathfrak{t}_1 \wedge \bigcirc\mathfrak{t}_2$ is consistent. A pair $(\mathfrak{t}_1, \mathfrak{t}_2)$ is i*-suitable* iff the formula $\mathfrak{t}_1 \wedge \bar{K}_i\mathfrak{t}_2$ is consistent, and it is D*-suitable* iff the formula $\mathfrak{t}_1 \wedge \bar{D}\neg\mathfrak{t}_2$ is consistent.

2. A pair $(\mathfrak{C}_1, \mathfrak{C}_2)$ of state candidates is $\bigcirc$*-suitable* iff the formula $\alpha_{\mathfrak{C}_1} \wedge \bigcirc\alpha_{\mathfrak{C}_2}$ is consistent. A pair $(\mathfrak{C}_1, \mathfrak{C}_2)$ is i*-suitable* iff the formula $\alpha_{\mathfrak{C}_1} \wedge \bar{K}_i\alpha_{\mathfrak{C}_2}$ is consistent, and it is D*-suitable* iff the formula $\alpha_{\mathfrak{C}_1} \wedge \bar{D}\alpha_{\mathfrak{C}_2}$ is consistent.

We now introduce the frame underlying the quasimodel for ϕ.

Definition 6.5. (Frame)
Let $A^+ = A \cup \{D\}$. A *frame* $\mathcal{F}$ is a tuple $\langle \langle \mathbb{N}_j, <_j\rangle_{j\in J}, \{\prec_l\}_{l\in A^+}\rangle$ such that:

(i) for $j \in J$, each $\mathbb{N}_j$ is a copy of the natural numbers with the strict total order $<_j$;

(ii) the pair $\langle \bigcup_{j\in J} \mathbb{N}_j, \bigcup_{l\in A^+} \prec_l\rangle$ is a set of disjoint intransitive trees[1].

A frame is *synchronous* if for every $l \in A^+$, $n_j \prec_l n'_{j'}$ implies $n = n'$. Further, we introduce state functions mapping points in $\mathcal{F}$ to consistent state candidates.

Definition 6.6. (State Function)
A *state function* for ϕ over $\mathcal{F}$ is a map $\mathfrak{f}$ associating with each $n_j \in \mathcal{F}$ a consistent state candidate $\mathfrak{f}(n_j) = \mathfrak{C}_{n_j}$ for ϕ such that:

(i) the domain of $\mathfrak{f}$ is not empty;

(ii) if $\mathfrak{f}$ is defined on n_j then $\mathfrak{f}$ is defined on $n + 1_j$;

(iii) if $\mathfrak{f}$ is defined on n_j and $n_j \prec_l n'_{j'}$ then $\mathfrak{f}$ is defined on $n'_{j'}$.

In what follows we often do not distinguish between a state n_j and its associated state candidate $\mathfrak{f}(n_j) = \mathfrak{C}_{n_j}$.

Finally, we provide the definition of *objects*, which correspond to the *runs* in [25, 29]. We use this denomination to avoid confusion with the runs in QIS.

[1]The pair $\langle U, R\rangle$ is an intransitive tree iff (i) there is a root $u_0 \in U$ such that $u_0 R^* u$ for every $u \in U$, where R^* is the reflexive and transitive closure of R; (ii) for every $u \in U$ the set $\{u' \in U \mid u' R^* u\}$ is finite and linearly ordered by R^*; (iii) every $u \in U$ but the root u_0 has exactly one predecessor; (iv) the root u_0 is irreflexive.

Definition 6.7. (Object)
Let $\mathfrak{f}$ be a state function for ϕ over $\mathcal{F}$. An *object* in $\langle \mathcal{F}, \mathfrak{f} \rangle$ is a map ρ associating with every $n_j \in Dom(\mathfrak{f})$ a type $\rho(n_j)$ in T_{n_j} such that:

(i) the pair $(\rho(n_j), \rho(n+1_j))$ is $\bigcirc$-suitable;

(ii) if $n_j \prec_l n'_{j'}$ then $\rho(n_j)$ and $\rho(n'_{j'})$ are l-suitable;

(iii) $\chi \mathcal{U} \psi \in \rho(n_j)$ iff there is $n'_j \geq_j n_j$ such that $\psi \in \rho(n'_j)$ and $\chi \in \rho(n''_j)$ for all $n_j \leq_j n''_j <_j n'_j$;

(iv) if $\neg K_i \psi \in \rho(n_j)$ then for some $n'_{j'}$, $n_j \prec_i n'_{j'}$ and $\psi \notin \rho(n'_{j'})$;

(v) if $\neg D \psi \in \rho(n_j)$ then for some $n'_{j'}$, $n_j \prec_D n'_{j'}$ and $\psi \notin \rho(n'_{j'})$;

(vi) if $\neg C \psi \in \rho(n_j)$ then for some $n'_{j'}$, $(n_j, n'_{j'}) \in (\bigcup_{l \in A^+} \prec_l)^*$ and $\psi \notin \rho(n'_{j'})$.

A map ρ associating with every $n_j \in Dom(\mathfrak{f})$ a type $\rho(n_j) \in T_{n_j}$ such that only conditions (i) and (iii) hold is a *temporal* object. Similarly, a map ρ associating with every $n_j \in Dom(\mathfrak{f})$ a type $\rho(n_j) \in T_{n_j}$ such that only conditions (ii) and (iv)-(vi) hold is an *epistemic* object. Now we have all the elements to give the definition of quasimodels.

Definition 6.8. (Quasimodel)
A *quasimodel* for ϕ is a tuple $\mathfrak{Q} = \langle \mathcal{F}, \mathfrak{f}, \mathcal{O} \rangle$ such that $\mathfrak{f}$ is a state function over $\mathcal{F}$ and

(i) $\phi \in \mathfrak{t}$, for some $\mathfrak{t} \in T_{n_j}$ and $T_{n_j} \in \mathfrak{C}_{n_j}$;

(ii) every pair $(\mathfrak{C}_{n_j}, \mathfrak{C}_{n+1_j})$ is $\bigcirc$-suitable, and if $n_j \prec_l n'_{j'}$ then $\mathfrak{C}_{n_j}$ and $\mathfrak{C}_{n'_{j'}}$ are l-suitable;

(iii) for every $\mathfrak{t} \in T_{n_j}$ there exists an object $\rho \in \mathcal{O}$ such that $\rho(n_j) = \mathfrak{t}$;

(iv) for every $c \in con\phi$ the function ρ_c such that $\rho_c(n_j) = \mathfrak{t}$ for $\langle \mathfrak{t}, c \rangle \in T^{con}_{n_j}$ is an object in $\mathcal{O}$.

As the first step in the completeness proof we show that for monodic formulas satisfability in quasimodels implies satisfability in Kripke models.

Lemma 6.1. If a monodic formula $\phi \in \mathcal{L}^1_m$ has a quasimodel $\mathfrak{Q}$, then ϕ is satisfiable in a Kripke model.

Proof:
The proof of this lemma is similar to those of Lemmas 11.72 and 12.9 in [6].

First of all, for every monodic formula $\psi \in \mathcal{L}^1_m$ of the form $K_i\chi$, $D\chi$, $C\chi$, $\bigcirc\chi$, or $\chi_1 \mathcal{U} \chi_2$, if ψ is a sentence then we introduce a propositional variable p_ψ and p_ψ is the *surrogate* of ψ, if x is the only free variable in ψ then we introduce a unary predicative letter P^1_ψ and the formula $P^1_\psi(x)$ is the *surrogate* of ψ. Given a formula $\phi \in \mathcal{L}^1_m$ we denote by $\overline{\phi}$ the formula obtained from ϕ by substituting all its modal subformulas that are not within the scope of another modal operator with their surrogates.

Since every state candidate $\mathfrak{C}$ in the quasimodel $\mathfrak{Q}$ is consistent and the system QKT^1_m is based on first-order logic, the formula $\overline{\alpha}_{\mathfrak{C}}$ is consistent with first-order (non-modal) logic. As a consequence, by completeness of first-order logic, there is a first-order structure $\mathcal{I} = \langle I, \mathcal{D} \rangle$ where $\mathcal{D}$ is a non-empty set of individuals and I is an interpretation on $\mathcal{D}$, which satisfies $\overline{\alpha}_{\mathfrak{C}}$, that is, $I^\sigma \models \overline{\alpha}_{\mathfrak{C}}$ for some assignment σ to $\mathcal{D}$.

Now consider a cardinal $\kappa \geq \aleph_0$ greater than the cardinality of the set $\mathcal{O}$ of all objects in $\mathfrak{Q}$ and define $\mathcal{D} = \{\langle \rho, \xi \rangle \mid \rho \in \mathcal{O}, \xi < \kappa\}$. By the theory of first-order logic, we can assume without loss of generality that $\mathcal{D}$ is the domain of the first-order structure $\mathcal{I}_{n_j} = \langle I_{n_j}, \mathcal{D} \rangle$ satisfying $\overline{\alpha}_{\mathfrak{C}_{n_j}}$, that is, all the structures $\mathcal{I}_{n_j}$ share a common domain $\mathcal{D}$. Moreover, we can assume that for every $\mathfrak{t} \in T_{n_j}$, $\langle \rho, \xi \rangle \in \mathcal{D}$, $\rho(n_j) = \mathfrak{t}$ iff $I^{\sigma}_{n_j} \models \bar{\mathfrak{t}}[x]$ for $\sigma(x) = \langle \rho, \xi \rangle$, and $I_{n_j}(c) = \langle \rho, 0 \rangle$ for every $c \in con\phi$.

Let us now define the Kripke model $\mathcal{M}$. Let $\mathcal{F} = \langle \langle \mathbb{N}_j, <_j \rangle_{j \in J}, \{\prec_l\}_{l \in A^+} \rangle$ be the frame of the quasimodel $\mathfrak{Q}$, we define $\mathcal{M}$ as $\langle \langle \mathbb{N}_j, <_j \rangle_{j \in J}, \{R_i\}_{i \in A}, \mathcal{D}, I \rangle$ where each sequence $\mathbb{N}_j$ of naturals in $\mathcal{F}$ belongs also to $\mathcal{M}$; each relation R_i is the reflexive, symmetric and transitive closure of $\prec_i \cup \prec_D$; $\mathcal{D}$ is defined as above; and the interpretation I is obtained by gluing together the various I_{n_j}.

By induction on the length of $\psi \in sub_x\phi$ we can show that for every assignment σ,

$$I^{\sigma}_{n_j} \models \overline{\psi} \quad \text{iff} \quad (\mathcal{M}^{\sigma}, n_j) \models \psi$$

The base of induction follows by the definition of I. The step for propositional connectives and quantifiers follows by the induction hypothesis and equations $\overline{\psi_1 \to \psi_2} = \overline{\psi_1} \to \overline{\psi_2}$, $\overline{\neg\psi_1} = \neg\overline{\psi_1}$, $\overline{\forall x \psi_1} = \forall x \overline{\psi_1}$. To deal with modal operators we state the following remark; the relevant cases directly follow.

Remark 6.1. For every $\rho \in \mathcal{O}$ and $n_j \in \mathbb{N}_j$

$$\begin{array}{rll}
(i) \bigcirc \psi \in \rho(n_j) & \text{iff} & \psi \in \rho(n+1_j) \\
(ii)\ K_i\psi \in \rho(n_j) & \text{iff} & \text{for every } n'_{j'}, (n_j, n'_{j'}) \in R_i \text{ implies } \psi \in \rho(n'_{j'}) \\
(iii)\ D\psi \in \rho(n_j) & \text{iff} & \text{for every } n'_{j'}, (n_j, n'_{j'}) \in \bigcap_{i \in A} R_i \text{ implies } \psi \in \rho(n'_{j'}) \\
(iv)\ C\psi \in \rho(n_j) & \text{iff} & \text{for every } n'_{j'}, (n_j, n'_{j'}) \in \left(\bigcup_{i \in A} R_i\right)^* \text{ implies } \psi \in \rho(n'_{j'})
\end{array}$$

The proof of this remark is similar to the one of Lemma 12.10 in [6].

To complete the proof of Lemma 6.1 we remark that by definition of quasimodel $\phi \in \mathfrak{t}$, for some $\mathfrak{t} \in T_{n_j}$ and $T_{n_j} \in \mathfrak{C}_{n_j}$, therefore we obtain that ϕ is satisfied in the Kripke model $\mathcal{M}$. $\square$

Note that if $\mathfrak{Q}$ is a synchronous quasimodel for ϕ, then the Kripke model built from $\mathfrak{Q}$ in Theorem 6.1 is also synchronous.

Now it is left to prove the existence of such a quasimodel for ϕ.

Lemma 6.2. If $\phi \in \mathcal{L}^1_m$ is a consistent monodic formula, then there exists a (synchronous) quasimodel for ϕ.

In the proof we use the following partial results. These lemmas, which we state without proof, are modifications of Lemmas 11.73 and 12.11 in [6].

Lemma 6.3. Let $\mathfrak{C}$ be a consistent state candidate, then we can construct an infinite sequence $\{\mathfrak{C}_n\}_{n \in \mathbb{N}}$ of state candidates such that $\mathfrak{C} = \mathfrak{C}_0$ and

(i) every pair $(\mathfrak{C}_n, \mathfrak{C}_{n+1})$ is $\bigcirc$-suitable;

(ii) for every $\mathfrak{t} \in T_n$ there exists a temporal object ρ such that $\rho(n) = \mathfrak{t}$;

(iii) for every $c \in con\phi$ the function ρ_c such that $\rho_c(n) = \mathfrak{t}$, for $\langle \mathfrak{t}, c\rangle \in T_n^{con}$, is a temporal object.

Lemma 6.4. If $\mathfrak{C}$ is a consistent state candidate, then we can construct a structure $\mathcal{W} = \langle W, \prec_1, \ldots, \prec_m, \prec_D\rangle$ such that W is a non-empty set of state candidates, and the pair $\langle W, \bigcup_{l\in A^+} \prec_l\rangle$ is a tree. Furthermore,

(i) $\mathfrak{C} \prec_l \mathfrak{C}'$ only if $\mathfrak{C}$ and $\mathfrak{C}'$ are l-suitable;

(ii) for every $\mathfrak{t} \in T$, $w \in W$, there exists an epistemic object ρ such that $\rho(w) = \mathfrak{t}$;

(iii) for every $c \in con\phi$ the function ρ_c such that $\rho_c(w) = \mathfrak{t}$, for $\langle \mathfrak{t}, c\rangle \in T_w^{con}$, is an epistemic object.

We can now prove Lemma 6.2.

Proof:
Let π_ϕ be the disjunction of all formulas $\alpha_{\mathfrak{C}}$ for all state candidates $\mathfrak{C}$ for ϕ. We denote by $\overline{\pi}_\phi$ the formula obtained from π_ϕ by substituting all its modal subformulas that are not within the scope of another modal operator with their surrogates. Note that $\overline{\pi}_\phi$ is true in every first-order model, so by completeness of first-order logic we have that $\vdash \pi_\phi$. Since ϕ is consistent, also $\phi \wedge \pi_\phi$ is consistent. Then there is a consistent state candidate $\mathfrak{C} = \langle T, T^{con}\rangle$ such that $\phi \in \mathfrak{t}$ for some $\mathfrak{t} \in T$.

We define the structure $\langle \mathcal{F}, \mathfrak{f}\rangle$ underlying the quasimodel $\mathfrak{Q}$ in steps. At step $2n$+1 we extend the structure with a chain $\mathbb{N}_{\mathfrak{C}'}$ of state candidates for every state candidate $\mathfrak{C}'$ introduced at step $2n$. At stage $2n$+2 we provide every state candidate introduced at step $2n$+1 with a tree of state candidates as shown in Lemma 6.4.

We start with the base of induction. Define $\mathcal{F}_0 = \langle\langle \mathbb{N}_j, <_j\rangle_{j\in J_0}, \{\prec_l^0\}_{l\in A^+}\rangle$ where J_0 is empty and for every $l \in A^+$, $\prec_l^0$ is also empty. The function $\mathfrak{f}_0$ is empty as well. We also consider a set U_0 which contains only the state candidate $\mathfrak{C}$ defined above, and assume $U_{-1} = \emptyset$.

At step $2n$+1 the frame $\mathcal{F}_{2n+1}$ is defined as the tuple $\langle\langle \mathbb{N}_j, <_j\rangle_{j\in J_{2n+1}}, \{\prec_l^{2n+1}\}_{l\in A^+}\rangle$ such that $J_{2n+1} = J_{2n} \cup \{U_{2n} \setminus U_{2n-1}\}$ and for each $l \in A^+$, $\prec_l^{2n+1} = \prec_l^{2n}$. Further, for every $u \in U_{2n} \setminus U_{2n-1}$ by Lemma 6.3 there exists a sequence $\{u_k\}_{k\in\mathbb{N}}$ of state candidates such that $u_0 = u$. Thus, the state function $\mathfrak{f}_{2n}$ is extended to $\mathfrak{f}_{2n+1}$ such that $\mathfrak{f}_{2n+1}(n_u) = u_n$ for $u \in U_{2n} \setminus U_{2n-1}$, and $\mathfrak{f}_{2n+1}$ is equal to $\mathfrak{f}_{2n}$ on all the other $u \in U_{2n-1}$. Finally, $U_{2n+1} = \bigcup_{j\in J_{2n+1}} \mathbb{N}_j$.

For defining $\mathcal{F}_{2n+2}$ we take $J_{2n+2} = J_{2n+1}$. Moreover, by Lemma 6.4 for every $u \in U_{2n+1} \setminus U_{2n}$ there is a structure $\langle W_u, \{\prec_l\}_{l\in A^+}\rangle$ such that the pair $\langle W_u, \bigcup_{l\in A^+} \prec_l\rangle$ is a tree. We define $\prec_l^{2n+2}$ as $\prec_l^{2n+1} \cup \prec_l$ for each $l \in A^+$. Finally, $\mathfrak{f}_{2n+2} = \mathfrak{f}_{2n+1}$ and $U_{2n+2} = U_{2n+1} \cup \bigcup_{u\in U_{2n+1}\setminus U_{2n}} W_u$.

Now consider the quasimodel $\mathfrak{Q} = \langle \mathcal{F}, \mathfrak{f}, \mathcal{O}\rangle$ where $\mathcal{F} = \langle\langle \mathbb{N}_j, <_j\rangle_{j\in J}, \{\prec_l\}_{l\in A^+}\rangle$ such that $J = \bigcup_{k\in\mathbb{N}} J_k$ and $\prec_l = \bigcup_{k\in\mathbb{N}} \prec_l^k$ for $l \in A^+$, $\mathfrak{f} = \bigcup_{k\in\mathbb{N}} \mathfrak{f}_k$, and $\mathcal{O}$ is the set of all objects on $\langle \mathcal{F}, \mathfrak{f}\rangle$. By Lemmas 6.3 and 6.4 and by construction of $\mathfrak{Q}$ we can show that $\mathcal{O}$ is non-empty and the objects in $\mathcal{O}$ satisfy the constraints on quasimodels. Since $\phi \in \mathfrak{t}$ for some $\mathfrak{t} \in \mathfrak{C}$ and $\mathfrak{C} \in \mathfrak{Q}$, we have that $\mathfrak{Q}$ is a quasimodel for ϕ.

Furthermore, if we want to obtain a synchronous quasimodel from the construction above we modify the step $2n$+1 for $n \geq 1$ as follows. For every $u \in U_{2n} \setminus U_{2n-1}$ by construction there exists a structure $\langle W_{u'}, \{\prec_l\}_{l\in A^+}\rangle$ for some $u' \in U_{2n-1}$ such that $u \in W_{u'}$. Moreover, for some $j \in J_{2n}$, $m \in \mathbb{N}$, $u' = m_j$. Now, by Lemma 6.3 there exists a sequence $\{u_k\}_{k\in\mathbb{N}}$ of state candidates such that $u_0 = u$, but now we define the state function $\mathfrak{f}_{2n+1}$ such that $\mathfrak{f}_{2n+1}((m+k)_u) = u_k$ for $k \in \mathbb{N}$. It it not difficult to show that by this construction the quasimodel $\mathfrak{Q}$ for ϕ is synchronous. This completes the proof of Lemma 6.1. □

By combining Lemmas 6.2 and 6.1 we can state the main result of this paper.

Theorem 6.1. (Completeness)
The system QKT_m^1 is complete with respect to the class $\mathcal{QIS}$ of quantified interpreted systems.

Assume that $\nvdash \phi$, then $\neg\phi$ is consistent and by Lemmas 6.2 and 6.1 there is a Kripke model $\mathcal{M}$ satisfying $\neg\phi$. By Lemma 5.1 the QIS $g(\mathcal{M})$ does not validate ϕ, therefore $\mathcal{QIS} \not\models \phi$. The following result can be proved similarly.

Theorem 6.2. (Completeness)
The system QKT_m^1 is complete with respect to the class $\mathcal{QIS}^{sync}$ of synchronous QIS.

7. Conclusions and Further Work

In this paper we analysed a quantified version of interpreted systems, the typical formalism for temporal epistemic logic in multi-agent systems, and proved completeness for the system QKT_m^1 defined on the monodic fragment of the first-order language $\mathcal{L}_m$ that includes linear-time modalities and epistemic operators for group knowledge. This result makes use of previous contributions on the axiomatisation of first-order epistemic and temporal logic [25, 29]. Further, we showed that a wide range of specifications on message passing systems can be expressed in the monodic fragment of $\mathcal{L}_m$.

Still, further work is needed in this line of research. The present paper deals with the class $\mathcal{QIS}$ of all quantified interpreted systems and the class $\mathcal{QIS}^{sync}$ of synchronous QIS. In the axiomatisation QKT_m^1 for these classes there is no interaction between temporal and epistemic operators, but interaction is essential to express epistemic concepts such as *perfect recall* and *no learning*. These refinements have been widely studied at the propositional level [8, 10], but it is not clear to which extent these results apply to the first order.

Finally, another issue not tackled in this paper is decidability. We believe that by combining the techniques in [15, 28] it is likely to find decidable monodic fragments of first-order temporal epistemic logic. However, this topic demands further investigations.

Acknowledgements: The research described in this paper was partly supported by EU Marie Curie Fellowship "First-order Modal Logics for the Specification and Verification of Multi-Agent Systems", by the EPSRC research project "Methods for reliability and control for autonomous underwater vehicles", and by the research project "Logica Modale e Conoscenza" funded by the Scuola Normale Superiore, Pisa.

References

[1] Belardinelli, F., Lomuscio, A.: A Complete First-Order Logic of Knowledge and Time, *KR* (G. Brewka, J. Lang, Eds.), AAAI Press, 2008, ISBN 978-1-57735-384-3.

[2] Belardinelli, F., Lomuscio, A.: Quantified epistemic logics for reasoning about knowledge in multi-agent systems, *Artificial Intelligence*, **173**(9-10), 2009, 982–1013.

[3] Blackburn, P., van Benthem, J., Wolter, F., Eds.: *Handbook of Modal Logic*, Elsevier, 2007.

[4] Cohen, P., Levesque, H.: Communicative Actions for Artificial Agents, *Proceedings of the First International Conference on Multi-Agent Systems (ICMAS'95)*, AAAI Press, 1995.

[5] Fagin, R., Halpern, J. Y., Moses, Y., Vardi, M. Y.: *Reasoning about Knowledge*, MIT Press, Cambridge, 1995, ISBN 0-262-06162-7.

[6] Gabbay, D., Kurucz, A., Wolter, F., Zakharyaschev, M.: *Many-Dimensional Modal Logics: Theory and Applications*, vol. 148 of *Studies in Logic*, Elsevier, 2003, ISBN 0-444-50826-0.

[7] Gammie, P., van der Meyden, R.: MCK: Model Checking the Logic of Knowledge, *Proceedings of 16th International Conference on Computer Aided Verification (CAV'04)*, 3114, Springer-Verlag, 2004.

[8] Halpern, J., Meyden, R., Vardi, M. Y.: Complete Axiomatisations for Reasoning about Knowledge and Time, *SIAM Journal on Computing*, **33**(3), 2003, 674–703.

[9] Halpern, J., Moses, Y.: Knowledge and common knowledge in a distributed environment, *Journal of the ACM*, **37**(3), 1990, 549–587, A preliminary version appeared in *Proc. 3rd ACM Symposium on Principles of Distributed Computing*, 1984.

[10] Halpern, J., Moses, Y.: A Guide to completeness and complexity for modal logics of knowledge and belief, *Artificial Intelligence*, **54**, 1992, 319–379.

[11] Halpern, J. Y., Vardi, M. Y.: The complexity of reasoning about knowledge and time 1: lower bounds, *Journal of Computer and System Sciences*, **38**(1), 1989, 195–237.

[12] Hodkinson, I.: Monodic Packed Fragment with Equality is Decidable, *Studia Logica*, **72**, 2002, 185–197.

[13] Hodkinson, I.: Complexity of monodic guarded fragments over linear and real time, *Annals of Pure and Applied Logic*, **138**, 2006, 94–125.

[14] Hodkinson, I. M., Kontchakov, R., Kurucz, A., Wolter, F., Zakharyaschev, M.: On the Computational Complexity of Decidable Fragments of First-Order Linear Temporal Logics., *TIME*, IEEE Computer Society, 2003, ISBN 0-7695-1912-1.

[15] Hodkinson, I. M., Wolter, F., Zakharyaschev, M.: Decidable fragment of first-order temporal logics, *Ann. Pure Appl. Logic*, **106**(1-3), 2000, 85–134.

[16] Hodkinson, I. M., Wolter, F., Zakharyaschev, M.: Decidable and Undecidable Fragments of First-Order Branching Temporal Logics., *LICS*, IEEE Computer Society, 2002, ISBN 0-7695-1483-9.

[17] Hoek, W., Meyer, J., Treur, J.: Formal Semantics of Temporal Epistemic Reflection, *Logic Program Synthesis and Transformation — Meta-Programming in Logic, 4th International Workshops (LOPSTR'94 and META'94)*, 883, Springer Verlag, Pisa, 1994.

[18] Lamport, L.: Time, clocks, and the ordering of events in a distributed system, *Commun. ACM*, **21**(7), 1978, 558–565, ISSN 0001-0782.

[19] Meyden, R.: Axioms for Knowledge and Time in Distributed Systems with Perfect Recall, *Proceedings, Ninth Annual IEEE Symposium on Logic in Computer Science*, IEEE Computer Society Press, Paris, France, 1994.

[20] Meyer, J.-J. C., Hoek, W.: *Epistemic Logic for AI and Computer Science*, vol. 41 of *Cambridge Tracts in Theoretical Computer Science*, Cambridge University Press, 1995.

[21] Parikh, R., Ramanujam, R.: Distributed Processes and the Logic of Knowledge., *Logic of Programs* (R. Parikh, Ed.), 193, Springer, 1985, ISBN 3-540-15648-8.

[22] Raimondi, F., Lomuscio, A.: Automatic verification of multi-agent systems by model checking via ordered binary decision diagrams, *J. Applied Logic*, **5**(2), 2007, 235–251.

[23] Rao, A., Georgeff, M.: Deliberation and its Role in the Formation of Intentions, *Proceedings of the 7th Conference on Uncertainty in Artificial Intelligence* (B. D. D'Ambrosio, P. Smets, P. Bonissone, Eds.), Morgan Kaufmann Publishers, San Mateo, CA, USA, 1991, ISBN 1-55860-203-8.

[24] Reynolds, M.: Axiomatising first-order temporal logic: until and since over linear time., *Studia Logica*, **57**(2/3), 1996, 279–302.

[25] Sturm, H., Wolter, F., Zakharyaschev, M.: Monodic Epistemic Predicate Logic., *JELIA* (M. Ojeda-Aciego, I. P. de Guzmán, G. Brewka, L. M. Pereira, Eds.), 1919, Springer, 2000, ISBN 3-540-41131-3.

[26] Sturm, H., Wolter, F., Zakharyaschev, M.: Common Knowledge and Quantification, *Economic Theory*, **19**, 2002, 157–186.

[27] Wolter, F.: First Order Common Knowledge Logics., *Studia Logica*, **65**(2), 2000, 249–271.

[28] Wolter, F., Zakharyaschev, M.: Decidable Fragments of First-Order Modal Logics., *J. Symb. Log.*, **66**(3), 2001, 1415–1438.

[29] Wolter, F., Zakharyaschev, M.: Axiomatizing the monodic fragment of first-order temporal logic., *Ann. Pure Appl. Logic*, **118**(1-2), 2002, 133–145.

[30] Wooldridge, M.: Computationally grounded theories of agency, in: *Proceedings of ICMAS, International Conference of Multi-Agent Systems* (E. Durfee, Ed.), IEEE Press, 2000, 13–22.

[31] Wooldridge, M.: *Reasoning about Rational Agents*, MIT Press, 2000.