

# An Abstraction Technique for the Verification of Artifact-Centric Multi-Agent Systems

Francesco Belardinelli

Department of Computing  
Imperial College London, UK

joint work with F. Patrizi and A. Lomuscio  
within the EU Project ACSI (Artifact-Centric Service Interoperation)

Department of Computing, University of Liverpool  
February 14, 2012

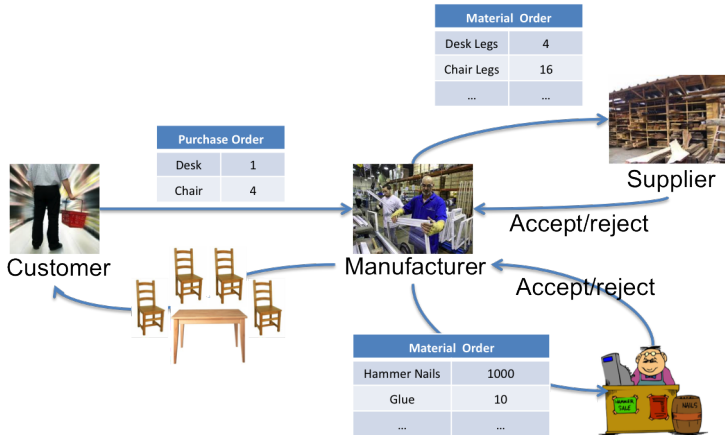
- 1 **Motivation:** Artifact Systems as *data-aware systems*
- 2 **Main task:** *formal* verification of (infinite-state) Artifact Systems
- 3 **Key contribution:** verification of *bounded* AS is decidable
- 4 Conclusion and future directions

Recent paradigm for Business Process modeling and development [CH09]

- *Artifact*: data model + lifecycle
  - ▶ (Nested) records equipped with actions
  - ▶ Actions may affect several artifacts
- *Artifact System*: set of interacting artifacts
- Data and processes are given same emphasis

# Artifact Systems

## Order-to-Cash Scenario



<i>PO</i>			
<i>id</i>	<i>prod_code</i>	<i>offer</i>	<i>status</i>

- *createPO(prod\_code)*
- *deletePO(id)*
- *addLinePO(id, prod\_code, offer)*
- ...

<i>WO</i>			
<i>id</i>	<i>po_id</i>	<i>price</i>	<i>status</i>

- *createWO(po\_id)*
- *deleteWO(id)*
- *addLineWO(id, po\_id, price)*
- ...

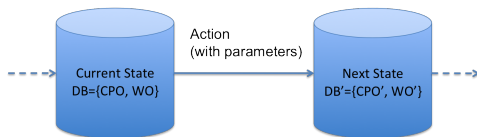
<i>MO</i>			
<i>id</i>	<i>wo_id</i>	<i>cost</i>	<i>status</i>

- *createMO(wo\_id)*
- *deleteMO(id)*
- *addLineMO(id, wo\_id, status)*
- ...

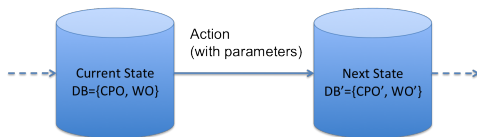
<i>Products</i>	
<i>prod_code</i>	<i>budget</i>

<i>Materials</i>	
<i>mat_code</i>	<i>cost</i>

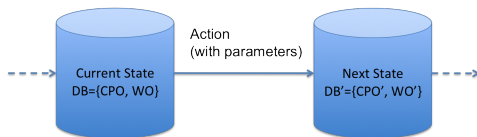
- As the process goes on, artifact actions are executed.
  - ▶ e.g., the Purchase Order is sent to the Manufacturer.



- As the process goes on, artifact actions are executed.
  - ▶ e.g., the Purchase Order is sent to the Manufacturer.
- Actions add/remove artifacts or change artifact attributes.
  - ▶ e.g., the PO status changes from *created* to *submitted*.

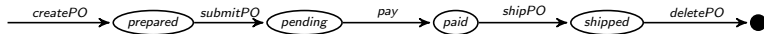


- As the process goes on, artifact actions are executed.
  - ▶ e.g., the Purchase Order is sent to the Manufacturer.
- Actions add/remove artifacts or change artifact attributes.
  - ▶ e.g., the PO status changes from *created* to *submitted*.
- The whole system can be seen as a *data-aware* dynamic system.
  - ▶ At every step, an action yields a change in the current state.

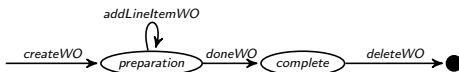




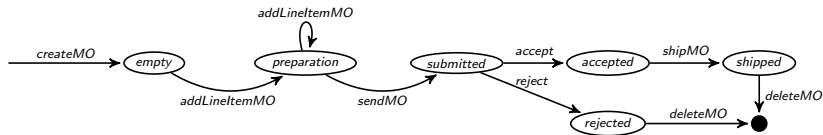
We can give a (partial) representation of AS as FSM.



(a) Purchase Order lifecycle



(b) Work Order Lifecycle



(c) Material Order lifecycle

We introduce some (basic) notions on databases to formalise data models.

We introduce some (basic) notions on databases to formalise data models.

A *database schema* is a *finite* set  $\mathcal{D} = \{P_1/a_1, \dots, P_n/a_n\}$  of predicate symbols  $P_i$  with their arity  $a_i \in \mathbb{N}$ .

- In the order-to-cash scenario

$$\mathcal{D} = \{Products/2, PO/4, WO/4, Materials/2, MO/4\}$$

We introduce some (basic) notions on databases to formalise data models.

A *database schema* is a *finite* set  $\mathcal{D} = \{P_1/a_1, \dots, P_n/a_n\}$  of predicate symbols  $P_i$  with their arity  $a_i \in \mathbb{N}$ .

- In the order-to-cash scenario

$$\mathcal{D} = \{Products/2, PO/4, WO/4, Materials/2, MO/4\}$$

A  *$\mathcal{D}$ -interpretation* on a (possibly infinite) domain  $U$  is a mapping  $D$  associating each predicate symbol  $P_i$  with a *finite*  $a_i$ -ary relation on  $U$ .

<i>PO</i>			
<i>id</i>	<i>prod_code</i>	<i>offer</i>	<i>status</i>
1	#12	\$50	prepared
2	#24	\$120	shipped
4	#45	\$80	paid
⋮	⋮	⋮	⋮

We introduce some (basic) notions on databases to formalise data models.

A *database schema* is a *finite* set  $\mathcal{D} = \{P_1/a_1, \dots, P_n/a_n\}$  of predicate symbols  $P_i$  with their arity  $a_i \in \mathbb{N}$ .

- In the order-to-cash scenario

$$\mathcal{D} = \{Products/2, PO/4, WO/4, Materials/2, MO/4\}$$

A  *$\mathcal{D}$ -interpretation* on a (possibly infinite) domain  $U$  is a mapping  $D$  associating each predicate symbol  $P_i$  with a *finite*  $a_i$ -ary relation on  $U$ .

<i>PO</i>			
<i>id</i>	<i>prod_code</i>	<i>offer</i>	<i>status</i>
1	#12	\$50	prepared
2	#24	\$120	shipped
4	#45	\$80	paid
⋮	⋮	⋮	⋮

The active domain  $adom(D)$  of each  $\mathcal{D}$ -instance  $D$  is *finite*.

Given

- a  $\mathcal{D}$ -interpretation  $D$
- an assignment  $\sigma : \text{Var} \rightarrow U$
- an FO-formula  $\varphi \in \mathcal{L}_{\mathcal{D}}$

we inductively define **satisfaction**:

$$(D, \sigma) \models P_i(t_1, \dots, t_\ell) \quad \text{iff} \quad \langle \sigma(t_1), \dots, \sigma(t_\ell) \rangle \in D(P_i)$$

$$(D, \sigma) \models t = t' \quad \text{iff} \quad \sigma(t) = \sigma(t')$$

$$(D, \sigma) \models \neg\varphi \quad \text{iff} \quad (D, \sigma) \not\models \varphi$$

$$(D, \sigma) \models \varphi \rightarrow \psi \quad \text{iff} \quad (D, \sigma) \not\models \varphi \text{ or } (D, \sigma) \models \psi$$

$$(D, \sigma) \models \forall x\varphi \quad \text{iff} \quad \text{for every } u \in \text{adom}(D), (D, \sigma_u^x) \models \varphi$$

Notice that we adopt an *active domain semantics*.

Given

- a  $\mathcal{D}$ -interpretation  $D$
- an assignment  $\sigma : Var \rightarrow U$
- an FO-formula  $\varphi \in \mathcal{L}_{\mathcal{D}}$

we inductively define **satisfaction**:

$$\begin{aligned}(D, \sigma) \models P_i(t_1, \dots, t_\ell) & \text{ iff } \langle \sigma(t_1), \dots, \sigma(t_\ell) \rangle \in D(P_i) \\(D, \sigma) \models t = t' & \text{ iff } \sigma(t) = \sigma(t') \\(D, \sigma) \models \neg\varphi & \text{ iff } (D, \sigma) \not\models \varphi \\(D, \sigma) \models \varphi \rightarrow \psi & \text{ iff } (D, \sigma) \not\models \varphi \text{ or } (D, \sigma) \models \psi \\(D, \sigma) \models \forall x\varphi & \text{ iff for every } u \in \text{adom}(D), (D, \sigma_u^x) \models \varphi\end{aligned}$$

Notice that we adopt an *active domain semantics*.

**Composition:**  $D \oplus D'$  is the  $(\mathcal{D} \cup \mathcal{D}')$ -interpretation s.t.  $D \oplus D'(P_i) = D(P_i)$  and  $D \oplus D'(P'_i) = D'(P_i)$ .

- Artifacts are manipulated by agents, e.g., customers, manufacturers, suppliers.



- Artifacts are manipulated by agents, e.g., customers, manufacturers, suppliers.
- We introduce an agent-based model for AS inspired to [FHMV95].
- An *agent* is a tuple  $i = \langle \mathcal{D}_i, L_i, Act_i, Pr_i \rangle$  where:
  - ▶  $\mathcal{D}_i$  is the *local database schema*
  - ▶  $L_i \subseteq \mathcal{D}_i(U)$  is the set of *local states*
  - ▶  $Act_i$  is the set of *local actions*
  - ▶  $Pr_i : L_i \mapsto 2^{Act_i}$  is the *local protocol function*

- Artifacts are manipulated by agents, e.g., customers, manufacturers, suppliers.
- We introduce an agent-based model for AS inspired to [FHMV95].
- An *agent* is a tuple  $i = \langle \mathcal{D}_i, L_i, Act_i, Pr_i \rangle$  where:
  - ▶  $\mathcal{D}_i$  is the *local database schema*
  - ▶  $L_i \subseteq \mathcal{D}_i(U)$  is the set of *local states*
  - ▶  $Act_i$  is the set of *local actions*
  - ▶  $Pr_i : L_i \mapsto 2^{Act_i}$  is the *local protocol function*
- The *global* database schema is such that  $\mathcal{D} = \mathcal{D}_1 \cup \dots \cup \mathcal{D}_n$ .

- Artifacts are manipulated by agents, e.g., customers, manufacturers, suppliers.
- We introduce an agent-based model for AS inspired to [FHMV95].
- An *agent* is a tuple  $i = \langle \mathcal{D}_i, L_i, Act_i, Pr_i \rangle$  where:
  - ▶  $\mathcal{D}_i$  is the *local database schema*
  - ▶  $L_i \subseteq \mathcal{D}_i(U)$  is the set of *local states*
  - ▶  $Act_i$  is the set of *local actions*
  - ▶  $Pr_i : L_i \mapsto 2^{Act_i}$  is the *local protocol function*
- The *global* database schema is such that  $\mathcal{D} = \mathcal{D}_1 \cup \dots \cup \mathcal{D}_n$ .
- Agents manipulate artifacts and have (partial) access to the information contained therein.

## Example 1: the Order-to-Cash Scenario

- Agents: Customer, Manufacturer, Supplier.

- Agents: Customer, Manufacturer, Supplier.
- Local database schemas:
  - ▶  $\mathcal{D}_C = \{Products, PO\}$
  - ▶  $\mathcal{D}_M = \{WO\}$
  - ▶  $\mathcal{D}_S = \{Materials, MO\}$

## Example 1: the Order-to-Cash Scenario

- Agents: Customer, Manufacturer, Supplier.
- Local database schemas:
  - ▶  $\mathcal{D}_C = \{Products, PO\}$
  - ▶  $\mathcal{D}_M = \{WO\}$
  - ▶  $\mathcal{D}_S = \{Materials, MO\}$
- Then  $\mathcal{D} = \{Products, PO, WO, Materials, MO\}$ .

- Agents: Customer, Manufacturer, Supplier.
- Local database schemas:
  - ▶  $\mathcal{D}_C = \{Products, PO\}$
  - ▶  $\mathcal{D}_M = \{WO\}$
  - ▶  $\mathcal{D}_S = \{Materials, MO\}$
- Then  $\mathcal{D} = \{Products, PO, WO, Materials, MO\}$ .
- Parametric actions can introduce values from an infinite domain  $U$ :
  - ▶  $createPO(id, prod\_code, offer)$  in  $Act_C$
  - ▶  $createWO(id, po\_id, price)$  in  $Act_M$
  - ▶  $createMO(id, wo\_id, cost)$  in  $Act_S$

Agents are modules that can be composed together to obtain AC-MAS.



Agents are modules that can be composed together to obtain AC-MAS.

- An *AC-MAS* is a tuple  $\mathcal{P} = \langle \mathcal{S}, U, D_0, \tau \rangle$  where
  - ▶  $\mathcal{S} \subseteq L_1 \times \dots \times L_n$  is the set of *reachable global states*
  - ▶  $U$  is the interpretation domain
  - ▶  $D_0 \in \mathcal{S}$  is the *initial global state*
  - ▶  $\tau : \mathcal{S} \times Act \mapsto 2^{\mathcal{S}}$  is the *global transition function*

Agents are modules that can be composed together to obtain AC-MAS.

- An *AC-MAS* is a tuple  $\mathcal{P} = \langle \mathcal{S}, U, D_0, \tau \rangle$  where
  - ▶  $\mathcal{S} \subseteq L_1 \times \dots \times L_n$  is the set of *reachable global states*
  - ▶  $U$  is the interpretation domain
  - ▶  $D_0 \in \mathcal{S}$  is the *initial global state*
  - ▶  $\tau : \mathcal{S} \times Act \mapsto 2^{\mathcal{S}}$  is the *global transition function*
- Temporal transition:  $D \rightarrow D'$  iff there is  $\alpha$  s.t.  $\tau(D, \alpha(\vec{u})) = D'$ .
- Epistemic relation:  $D \sim_i D'$  iff  $D_i = D'_i$  for agent  $i$ .

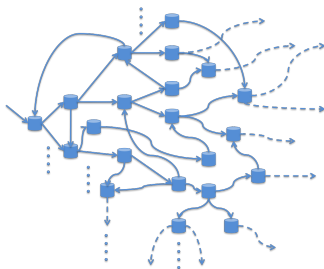
Agents are modules that can be composed together to obtain AC-MAS.

- An *AC-MAS* is a tuple  $\mathcal{P} = \langle \mathcal{S}, U, D_0, \tau \rangle$  where
  - ▶  $\mathcal{S} \subseteq L_1 \times \dots \times L_n$  is the set of *reachable global states*
  - ▶  $U$  is the interpretation domain
  - ▶  $D_0 \in \mathcal{S}$  is the *initial global state*
  - ▶  $\tau : \mathcal{S} \times Act \mapsto 2^{\mathcal{S}}$  is the *global transition function*
- Temporal transition:  $D \rightarrow D'$  iff there is  $\alpha$  s.t.  $\tau(D, \alpha(\vec{u})) = D'$ .
- Epistemic relation:  $D \sim_i D'$  iff  $D_i = D'_i$  for agent  $i$ .

AC-MAS are FO temporal epistemic structures, so a flavour of FO temporal epistemic logic can be used as specification language for AC-MAS.

## Intuition

- A transition system where each state is a  $\mathcal{D}$ -instance.
- As actions are executed, new states are generated.
- Action parameters can introduce new values.
- An infinite domain  $U$  yields potentially infinitely many distinct states.
- In general, infinite branching and infinite run-length.



- Does the system satisfy a (branching-time) *temporal epistemic* specification?  
E.g.:
  - ▶ It is always the case that every artifact can be deleted
  - ▶ There exists a way to create a certain number of artifacts
  - ▶ The manufacturer knows that a product can be shipped only after assemblage

- Does the system satisfy a (branching-time) *temporal epistemic* specification?  
E.g.:
  - ▶ It is always the case that every artifact can be deleted
  - ▶ There exists a way to create a certain number of artifacts
  - ▶ The manufacturer knows that a product can be shipped only after assemblage
- Flavour of Model Checking, but:
  - ▶ *relational* states (database instances)
  - ▶ infinite interpretation domain
  - ▶ infinite state space

How to specify system properties?

## Definition (Syntax of FO-CTLK)

$$\varphi ::= P(\vec{t}) \mid t = t' \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid \forall x\varphi \mid AX\varphi \mid A\varphi U\varphi \mid E\varphi U\varphi \mid K_i\varphi$$

How to specify system properties?

## Definition (Syntax of FO-CTLK)

$$\varphi ::= P(\vec{t}) \mid t = t' \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid \forall x\varphi \mid AX\varphi \mid A\varphi U\varphi \mid E\varphi U\varphi \mid K_i\varphi$$

We want to check FO-CTLK properties, e.g.:

- the manufacturer  $M$  knows that each  $WO$  has to match a corresponding  $PO$ :

$$AG \forall po\_id (\exists id, p, s \text{ } WO(id, po\_id, p, s) \rightarrow K_M \exists p, o, s \text{ } PO(po\_id, p, o, s))$$



How to specify system properties?

## Definition (Syntax of FO-CTLK)

$$\varphi ::= P(\vec{t}) \mid t = t' \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid \forall x\varphi \mid AX\varphi \mid A\varphi U\varphi \mid E\varphi U\varphi \mid K_i\varphi$$

We want to check FO-CTLK properties, e.g.:

- the manufacturer  $M$  knows that each  $WO$  has to match a corresponding  $PO$ :

$$AG \forall po\_id (\exists id, p, s \text{ } WO(id, po\_id, p, s) \rightarrow K_M \exists p, o, s \text{ } PO(po\_id, p, o, s))$$

Difficulty: the infinite domain  $U$  gives rise to infinitely many states!

Investigated solution: can we *simulate* the concrete values with a finite set of *abstract* symbols?

## Verification Formalism: FO-CTLK

## Semantics

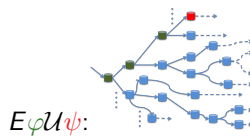
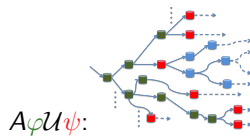
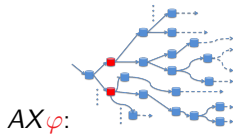
A run  $r$  is an infinite sequence  $D^0 \rightarrow D^1 \rightarrow \dots$  of states;  $r(i) = D^i$ .

## Definition (Semantics of FO-CTLK)

$(\mathcal{P}, D, \sigma) \models \varphi$	iff	$(D, \sigma) \models \varphi$ , if $\varphi$ is an FO-formula
$(\mathcal{P}, D, \sigma) \models \neg\varphi$	iff	$(\mathcal{P}, D, \sigma) \not\models \varphi$
$(\mathcal{P}, D, \sigma) \models \varphi \rightarrow \psi$	iff	$(\mathcal{P}, D, \sigma) \not\models \varphi$ or $(\mathcal{P}, D, \sigma) \models \psi$
$(\mathcal{P}, D, \sigma) \models \forall x\varphi$	iff	for all $u \in \text{adom}(D)$ , $(\mathcal{P}, D, \sigma_u^x) \models \varphi$
$(\mathcal{P}, D, \sigma) \models AX\varphi$	iff	for all runs $r$ , if $r(0) = D$ then $(\mathcal{P}, r(1), \sigma) \models \varphi$
$(\mathcal{P}, D, \sigma) \models A\varphi U\psi$	iff	for all runs $r$ , if $r(0) = D$ then there is $k \geq 0$ s.t. $(\mathcal{P}, r(k), \sigma) \models \psi$ , and for all $j$ , $0 \leq j < k$ implies $(\mathcal{P}, r(j), \sigma) \models \varphi$
$(\mathcal{P}, D, \sigma) \models E\varphi U\psi$	iff	for some run $r$ , $r(0) = D$ and there is $k \geq 0$ s.t. $(\mathcal{P}, r(k), \sigma) \models \psi$ , and for all $j$ , $0 \leq j < k$ implies $(\mathcal{P}, r(j), \sigma) \models \varphi$
$(\mathcal{P}, D, \sigma) \models K_i\varphi$	iff	for all $D'$ , $D \sim_i D'$ implies $(\mathcal{P}, D', \sigma) \models \varphi$

A formula  $\varphi$  is *true* in  $D$ , or  $(\mathcal{P}, D) \models \varphi$ , if  $(\mathcal{P}, D, \sigma) \models \varphi$  for all  $\sigma$ .

A formula  $\varphi$  is *true* in  $\mathcal{P}$ , or  $\mathcal{P} \models \varphi$ , if  $(\mathcal{P}, D_0) \models \varphi$ .



- *Model Checking for AC-MAS:*

*Given  $\mathcal{P}$  and  $\varphi$ , does  $(\mathcal{P}, D_0, \sigma) \models \varphi$  for some  $\sigma$ ?*

- *Model Checking for AC-MAS:*

*Given  $\mathcal{P}$  and  $\varphi$ , does  $(\mathcal{P}, D_0, \sigma) \models \varphi$  for some  $\sigma$ ?*

- Similar to Model Checking but technically more challenging:
  - ▶ Relational states
  - ▶ Infinite state-space

- *Model Checking for AC-MAS:*

*Given  $\mathcal{P}$  and  $\varphi$ , does  $(\mathcal{P}, D_0, \sigma) \models \varphi$  for some  $\sigma$ ?*

- Similar to Model Checking but technically more challenging:
  - ▶ Relational states
  - ▶ Infinite state-space

### Theorem

*The MC problem for AC-MAS is undecidable.*

- BUT decidable over finite interpretation domains:
  - ▶ by reduction to standard propositional case (*propositionalise* FO facts).

- Here we devise a notable case of decidability

- Here we devise a notable case of decidability
- If all  $\mathcal{D}$ -instances of the AC-MAS are **bounded**, then, though **infinite-state**, model-checking is decidable.

## Definition ( $b$ -bounded (Artifact) System)

Given a bound  $b \in \mathbb{N}$  s.t.  $b \geq |\text{adom}(D_0)|$ , an AC-MAS  $\mathcal{P}$  is  $b$ -bounded if for every  $D \in \mathcal{P}$ ,  $|\text{adom}(D)| \leq b$ .



- Here we devise a notable case of decidability
- If all  $\mathcal{D}$ -instances of the AC-MAS are **bounded**, then, though **infinite-state**, model-checking is decidable.

## Definition ( $b$ -bounded (Artifact) System)

Given a bound  $b \in \mathbb{N}$  s.t.  $b \geq |\text{adom}(D_0)|$ , an AC-MAS  $\mathcal{P}$  is  $b$ -bounded if for every  $D \in \mathcal{P}$ ,  $|\text{adom}(D)| \leq b$ .

- Practical approach: verify implementation, rather than design.

- Here we devise a notable case of decidability
- If all  $\mathcal{D}$ -instances of the AC-MAS are **bounded**, then, though **infinite-state**, model-checking is decidable.

## Definition ( $b$ -bounded (Artifact) System)

Given a bound  $b \in \mathbb{N}$  s.t.  $b \geq |\text{adom}(D_0)|$ , an AC-MAS  $\mathcal{P}$  is  $b$ -bounded if for every  $D \in \mathcal{P}$ ,  $|\text{adom}(D)| \leq b$ .

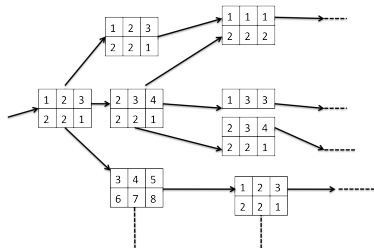
- Practical approach: verify implementation, rather than design.
- Idea: actual machines have bounded memory.

# Verification of Bounded AC-MAS

As a consequence of the domain  $U$  being infinite, we still have:

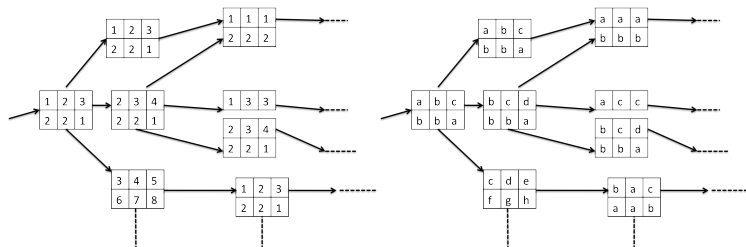
- infinite branching;
- infinite state-space.

E.g., with at most 2 tuples:



QUESTION:

- Can we model-check a bounded system?
  - Non-trivial! we cannot *construct* the (infinite) model.



- The concrete AC-MAS is abstracted by *replacing* the infinite interpretation domain  $\mathbb{N}$  with a finite one ( $\{a, b, c, d, e, f, g, h\}$ ).

- The cardinality of the new domain  $U'$  depends on
  - ▶ the (memory) bound  $b$
  - ▶ the AC-MAS  $\mathcal{P}$
  - ▶ the specification  $\varphi$  to check

- The cardinality of the new domain  $U'$  depends on
  - ▶ the (memory) bound  $b$
  - ▶ the AC-MAS  $\mathcal{P}$
  - ▶ the specification  $\varphi$  to check
- The resulting finite-state system can be model-checked by standard techniques

- The cardinality of the new domain  $U'$  depends on
  - ▶ the (memory) bound  $b$
  - ▶ the AC-MAS  $\mathcal{P}$
  - ▶ the specification  $\varphi$  to check
- The resulting finite-state system can be model-checked by standard techniques
- BUT how did we get rid of an infinite number of elements and transitions?

- The cardinality of the new domain  $U'$  depends on
  - ▶ the (memory) bound  $b$
  - ▶ the AC-MAS  $\mathcal{P}$
  - ▶ the specification  $\varphi$  to check
- The resulting finite-state system can be model-checked by standard techniques
- BUT how did we get rid of an infinite number of elements and transitions?
- We apply an *abstraction process* based on two formal notions:
  - 1 *Isomorphism* between  $\mathcal{D}$ -instances;
  - 2 *Bisimulation* between AC-MAS.



## Data Abstraction

## Isomorphic instances

## Definition (Isomorphism)

Two  $\mathcal{D}$ -instances  $D$  and  $D'$  are *C-isomorphic*, or  $D \simeq_C D'$ , iff there is a bijection  $\iota : \text{adom}(D) \cup C \mapsto \text{adom}(D') \cup C$  s.t.

- (i)  $\iota$  is the identity on  $C$
- (ii) for every  $\vec{u} \in U^*$ ,  $\vec{u} \in D(P_i)$  iff  $\iota(\vec{u}) \in D'(P_i)$

In words: instances obtained by uniformly renaming the elements not in  $C$ .  
E.g., for  $C = \{1\}$ ,  $\iota(1) = 1$ ,  $\iota(2) = a$ ,  $\iota(3) = b$ ,  $\iota(4) = c$ .

1	2	3
2	4	3

1	a	b
a	c	b

# Data Abstraction

## Isomorphic instances (cont.)

Isomorphic instances have a notable (well-known) property:

### Lemma

If  $D \simeq D'$  then for every FO-formula  $\varphi$  s.t.  $\text{con}(\varphi) \subseteq C$ ,

$$D \models \varphi \Leftrightarrow D' \models \varphi$$

1	blue	orange
blue	green	orange

- The *coloured instance* satisfies  $\varphi$  iff all the instances isomorphic to it do
- The *coloured instance* stands for infinitely many isomorphic instances (*isomorphism type*): same values iff same colours
- Observation: for a given bound  $b$ , there are only finitely many isomorphism types

### Definition (Bisimilarity)

Two AC-MAS  $\mathcal{P}_1$  and  $\mathcal{P}_2$  are *C-bisimilar*, or  $\mathcal{P}_1 \approx_C \mathcal{P}_2$ , iff there exists a *bisimulation relation*  $\approx_C$  s.t.  $D_{10} \approx_C D_{20}$ , and if  $D_1 \approx_C D_2$  then

- (i)  $D_1 \simeq_C D_2$
- (ii) if  $D_1 \rightarrow D'_1$  then there is  $D'_2$  s.t.  $D_2 \rightarrow D'_2$  and  $D'_1 \approx_C D'_2$
- (iii) if  $D_2 \rightarrow D'_2$  then there is  $D'_1$  s.t.  $D_1 \rightarrow D'_1$  and  $D'_1 \approx_C D'_2$
- (iv) Similarly, (ii) and (iii) hold for the epistemic relation  $\sim_i$  for every agent  $i$

# Data Abstraction

## Bisimilar AC-MAS

### Definition (Bisimilarity)

Two AC-MAS  $\mathcal{P}_1$  and  $\mathcal{P}_2$  are *C-bisimilar*, or  $\mathcal{P}_1 \approx_C \mathcal{P}_2$ , iff there exists a *bisimulation relation*  $\approx_C$  s.t.  $D_{10} \approx_C D_{20}$ , and if  $D_1 \approx_C D_2$  then

- (i)  $D_1 \simeq_C D_2$
- (ii) if  $D_1 \rightarrow D'_1$  then there is  $D'_2$  s.t.  $D_2 \rightarrow D'_2$  and  $D'_1 \approx_C D'_2$
- (iii) if  $D_2 \rightarrow D'_2$  then there is  $D'_1$  s.t.  $D_1 \rightarrow D'_1$  and  $D'_1 \approx_C D'_2$
- (iv) Similarly, (ii) and (iii) hold for the epistemic relation  $\sim_i$  for every agent  $i$

Intuitively, the following diagrams commute:

$$\begin{array}{ccccccc}
 D_1 & \longrightarrow & D'_1 & & D_1 & \sim_i & D'_1 \\
 \{ & & \} & & \{ & & \} \\
 D_2 & \longrightarrow & D'_2 & & D_2 & \sim_i & D'_2
 \end{array}$$

**However**, bisimulation alone is not sufficient to preserve FO-CTLK formulas.

## Definition (Uniformity)

An AC-MAS  $\mathcal{P}$  is *C-uniform* iff for  $D, D', D'' \in \mathcal{S}$  and  $D''' \in \mathcal{D}(U)$ :

- 1  $D \rightarrow D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \rightarrow D'''$ ;
- 2  $D \sim_i D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \sim_i D'''$ .

## Definition (Uniformity)

An AC-MAS  $\mathcal{P}$  is *C-uniform* iff for  $D, D', D'' \in \mathcal{S}$  and  $D''' \in \mathcal{D}(U)$ :

- ①  $D \rightarrow D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \rightarrow D'''$ ;
- ②  $D \sim_i D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \sim_i D'''$ .

- Intuitively, the behaviour of uniform AC-MAS is independent from data not explicitly named.
  - ▶ Suppose that  $P(a) \rightarrow P(b)$
  - ▶ Further,  $P(a) \oplus P'(b) \simeq P(c) \oplus P'(d)$
  - ▶ Hence,  $P(c) \rightarrow P(d)$

## Definition (Uniformity)

An AC-MAS  $\mathcal{P}$  is *C-uniform* iff for  $D, D', D'' \in \mathcal{S}$  and  $D''' \in \mathcal{D}(U)$ :

- ①  $D \rightarrow D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \rightarrow D'''$ ;
- ②  $D \sim_i D'$  and  $D \oplus D' \simeq_C D'' \oplus D'''$  imply  $D'' \sim_i D'''$ .

- Intuitively, the behaviour of uniform AC-MAS is independent from data not explicitly named.
  - ▶ Suppose that  $P(a) \rightarrow P(b)$
  - ▶ Further,  $P(a) \oplus P'(b) \simeq P(c) \oplus P'(d)$
  - ▶ Hence,  $P(c) \rightarrow P(d)$
- Uniform AC-MAS cover a vast number of interesting cases.

Bisimilarity together with uniformity is sufficient to preserve FO-CTLK formulas.

## Theorem

Consider two bisimilar uniform AC-MAS  $\mathcal{P}_1$  and  $\mathcal{P}_2$ , and an FO-CTLK formula  $\varphi$ .

If

$$\textcircled{1} \quad |U_2| \geq \max_{D \in \mathcal{P}_1} |\text{adom}(D)| + |C| + |\text{var}(\varphi)|$$

$$\textcircled{2} \quad |U_1| \geq \max_{D \in \mathcal{P}_2} |\text{adom}(D)| + |C| + |\text{var}(\varphi)|$$

then

$$\mathcal{P}_1 \models \varphi \quad \text{iff} \quad \mathcal{P}_2 \models \varphi$$



We verify the actual, bounded implementations of AC-MAS.

We verify the actual, bounded implementations of AC-MAS.

Consider

- an AC-MAS  $\mathcal{P}_1$  on a domain  $U_1$  s.t.
  - 1  $U_1$  is infinite
  - 2  $\mathcal{P}_1$  is  $b$ -bounded, i.e., for all  $D \in \mathcal{P}_1$ ,  $|adom(D)| \leq b$

We verify the actual, bounded implementations of AC-MAS.

Consider

- an AC-MAS  $\mathcal{P}_1$  on a domain  $U_1$  s.t.
  - 1  $U_1$  is infinite
  - 2  $\mathcal{P}_1$  is  $b$ -bounded, i.e., for all  $D \in \mathcal{P}_1$ ,  $|\text{adom}(D)| \leq b$
- an FO-CTLK formula  $\varphi$ .

We verify the actual, bounded implementations of AC-MAS.

Consider

- an AC-MAS  $\mathcal{P}_1$  on a domain  $U_1$  s.t.
  - ①  $U_1$  is infinite
  - ②  $\mathcal{P}_1$  is  $b$ -bounded, i.e., for all  $D \in \mathcal{P}_1$ ,  $|\text{adom}(D)| \leq b$
- an FO-CTLK formula  $\varphi$ .
- Then, there exists a finite abstraction  $\mathcal{P}_2$  of  $\mathcal{P}_1$  s.t.
  - ①  $\mathcal{P}_2$  is uniform and bisimilar to  $\mathcal{P}_1$
  - ②  $|U_2| \geq 2b + |C| + |\text{var}(\varphi)|$

We verify the actual, bounded implementations of AC-MAS.

Consider

- an AC-MAS  $\mathcal{P}_1$  on a domain  $U_1$  s.t.
  - 1  $U_1$  is infinite
  - 2  $\mathcal{P}_1$  is  $b$ -bounded, i.e., for all  $D \in \mathcal{P}_1$ ,  $|\text{adom}(D)| \leq b$
- an FO-CTLK formula  $\varphi$ .
- Then, there exists a finite abstraction  $\mathcal{P}_2$  of  $\mathcal{P}_1$  s.t.
  - 1  $\mathcal{P}_2$  is uniform and bisimilar to  $\mathcal{P}_1$
  - 2  $|U_2| \geq 2b + |C| + |\text{var}(\varphi)|$
- In particular,

$$\mathcal{P}_1 \models \varphi \quad \text{iff} \quad \mathcal{P}_2 \models \varphi$$

- **Problem:** the result in the previous slide assumes that  $\mathcal{P}_1$  is given and then builds  $\mathcal{P}_2$ .

- **Problem:** the result in the previous slide assumes that  $\mathcal{P}_1$  is given and then builds  $\mathcal{P}_2$ .
- BUT  $\mathcal{P}_1$  is infinite, so we may not be able to construct  $\mathcal{P}_2$ .

- **Problem:** the result in the previous slide assumes that  $\mathcal{P}_1$  is given and then builds  $\mathcal{P}_2$ .
- BUT  $\mathcal{P}_1$  is infinite, so we may not be able to construct  $\mathcal{P}_2$ .
- We need a methodology to obtain the abstract  $\mathcal{P}_2$  without going through the concrete  $\mathcal{P}_1$ .



- **Problem:** the result in the previous slide assumes that  $\mathcal{P}_1$  is given and then builds  $\mathcal{P}_2$ .
- BUT  $\mathcal{P}_1$  is infinite, so we may not be able to construct  $\mathcal{P}_2$ .
- We need a methodology to obtain the abstract  $\mathcal{P}_2$  without going through the concrete  $\mathcal{P}_1$ .
- To do so, we need to specify the form of actions.

We give an example of uniform AC-MAS consistent with GSM [HDD<sup>+</sup>11].

For each agent  $i$  we define  $Act_i$  as the set of *local (parametric) actions* of the form  $\omega(\vec{x}) \doteq \langle \pi(\vec{y}), \psi(\vec{z}) \rangle$  s.t.

- $\omega(\vec{x})$  is the *operation signature* and  $\vec{x} = \vec{y} \cup \vec{z}$  is the set of *operation parameters*
- $\pi(\vec{y})$  is the *operation precondition*, i.e., an FO-formula over  $\mathcal{D}_i$
- $\psi(\vec{z})$  is the *operation postcondition*, i.e., an FO-formula over  $\mathcal{D} \cup \mathcal{D}'$

We call the AC-MAS specified in this way *Artifact System Programs*.

Now,  $D \rightarrow D'$  iff for some  $\alpha(\vec{x}) \in Act$  there is an execution  $\alpha(\vec{u}) = \langle \pi(\vec{v}), \psi(\vec{w}) \rangle$   
and

- $adom(D') \subseteq adom(D) \cup \vec{w} \cup con(\psi)$
- $D \models \pi(\vec{v})$ , i.e., the action is *enabled*
- $D \oplus D' \models \psi(\vec{w})$

Specification of actions affecting the MO in the order-to-cash scenario:

- $createMO(id, wo\_id, cost) = \langle \pi(wo\_id, cost), \psi(id, wo\_id, cost) \rangle$  where:
  - ▶  $\pi(wo\_id, cost) \equiv$   
 $\exists po\_id, p (WO(wo\_id, po\_id, p, completed) \wedge$   
 $\exists pr, o (PO(po\_id, pr, o, pending) \wedge$   
 $Materials(pr, cost))) \wedge$   
 $\phi_{b-1}$
  - ▶  $\psi(id, wo\_id, cost) \equiv$   
 $MO'(id, wo\_id, cost, preparation) \wedge$   
 $\forall id', w, c, s (MO(id', w, c, s) \rightarrow id \neq id') \wedge$   
 $\phi_b$

where  $\phi_k ::= \forall x_1, \dots, x_{k+1} \bigvee_{i \neq j} (x_i = x_j)$  says that there are at most  $k$  objects in the active domain.

The specification of  $createMO$  guarantees that the bound  $b$  is not violated by action execution.

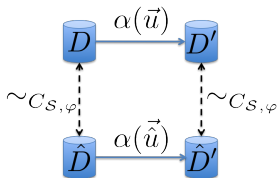
- AS programs are uniform.

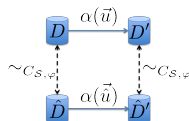
## Finite Abstraction of AS Programs

- AS programs are uniform.
- If
  - the AS program  $\mathcal{P}_{Act, U_1}$  is  $b$ -bounded
  - the finite domain  $U_2$  is s.t.  $|U_2| \geq 2b + |C_{AS}| + N_{AS}$
 then
  - the induced AS program  $\mathcal{P}_{Act, U_2}$  is a finite abstraction of  $\mathcal{P}_{Act, U_1}$

## Lemma

If  $D \simeq_C \hat{D}$  then every concrete transition  $D \rightarrow D'$  has an abstract counterpart  $\hat{D} \rightarrow \hat{D}'$  s.t.  $D' \simeq_C \hat{D}'$ .





Given a concrete execution  $\alpha(\vec{u}) = \langle \pi(\vec{v}), \psi(\vec{w}) \rangle$ , there exist  $\vec{v}, \vec{w}, \hat{D}'$  s.t.

- (i)  $\hat{D} \models \pi(\vec{v})$
- (ii)  $\hat{D} \oplus \hat{D}' \models \psi(\vec{w})$
- (iii)  $D' \sim_C \hat{D}'$

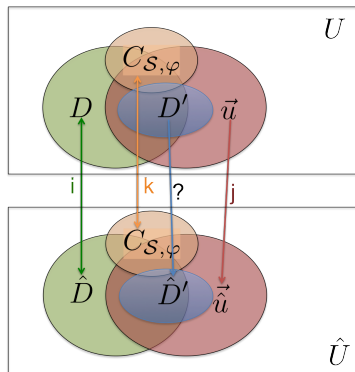
- there exist  $\hat{D}'$  and  $\vec{u}$ , and a  $C$ -isomorphism between

$$\{D, D', \vec{u}\} \text{ and } \{\hat{D}, \hat{D}', \vec{u}\}$$

- This is enough, as  $\pi$  and  $\varphi$  are invariant w.r.t.  $C$ -isomorphic instances.

## Finite Abstraction of AS Programs

If-Part (Intuition) Cont.

How to define an  $C$ -isomorphism between  $\{D, D', \vec{u}\}$  and  $\{\hat{D}, \hat{D}', \vec{\hat{u}}\}$ :

- 1 obtain  $\vec{\hat{u}}$  by renaming the elements in  $\vec{u}$  according to  $\iota$ ,  $k$ , and preserving (in)equalities –  $\hat{U}$  contains enough elements to do so;
- 2 obtain  $\hat{D}'$  by renaming the elements in  $D'$  according to  $\iota$  and  $j$ .



- If
  - ▶ the AS program  $\mathcal{P}_{Act, U_1}$  is  $b$ -bounded
  - ▶ the finite domain  $U_2$  is s.t.  $|U_2| \geq 2b + |C_{AS}| + N_{AS}$ ,then
  - ▶ the induced AS program  $\mathcal{P}_{Act, U_2}$  is a finite abstraction of  $\mathcal{P}_{Act, U_1}$ .

- If
  - ▶ the AS program  $\mathcal{P}_{Act, U_1}$  is  $b$ -bounded
  - ▶ the finite domain  $U_2$  is s.t.  $|U_2| \geq 2b + |C_{AS}| + N_{AS}$ ,then
  - ▶ the induced AS program  $\mathcal{P}_{Act, U_2}$  is a finite abstraction of  $\mathcal{P}_{Act, U_1}$ .
- In particular, if  $|U_2| \geq 2b + |C_{AS}| + \max\{N_{AS}, |\text{var}(\varphi)|\}$ , then

$$\mathcal{P}_{Act, U_1} \models \varphi \quad \text{iff} \quad \mathcal{P}_{Act, U_2} \models \varphi$$

# Application to the General Case

## Preservation Theorem

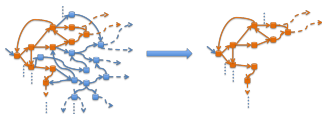
- What if  $\mathcal{P}$  is unbounded? (apart from undecidability)

# Application to the General Case

## Preservation Theorem

- What if  $\mathcal{P}$  is unbounded? (apart from undecidability)

Observation: for fixed  $b \in \mathbb{N}$ , the  $b$ -abstraction  $\hat{\mathcal{P}}_b$  corresponds to an (infinite) fragment of  $\mathcal{P}$



Preservation theorem for the *existential fragment* FO $\exists$ -ECTLK.

$$\varphi ::= \phi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x \varphi \mid EX \varphi \mid E\varphi U \varphi \mid \bar{K}_i \varphi$$

## Theorem

Given an AS program  $\mathcal{P}$ ,  $b \geq |\text{adom}(D_0)|$ , and an FO $\exists$ -ECTLK formula  $\varphi$ ,

$$\hat{\mathcal{P}}_b \models \varphi \Rightarrow \mathcal{P} \models \varphi$$

Observe we can iterate on  $b$ .

## Results...

- We are able to model check AC-MAS wrt full FO-CTLK...
- ...however, our results hold only for *uniform* systems.
- This class includes many interesting systems (AS programs).

## Results...

- We are able to model check AC-MAS wrt full FO-CTLK...
- ...however, our results hold only for *uniform* systems.
- This class includes many interesting systems (AS programs).

## ... and Future Work

- Techniques for finite abstraction.
- Abstraction techniques for finite-state systems are effective on the abstract system?
- How to perform the boundedness check.



D. Cohn and R. Hull.

Business Artifacts: A Data-Centric Approach to Modeling Business Operations and Processes.  
*IEEE Data Eng. Bull.*, 32(3):3–9, 2009.



R. Fagin, J. Y. Halpern, Y. Moses, and M. Y. Vardi.

*Reasoning About Knowledge*.  
The MIT Press, 1995.



Rick Hull, Elio Damaggio, Riccardo De Masellis, Fabiana Fournier, Manmohan Gupta, Fenno (Therry) Heath III, Stacy Hobson, Mark H. Linehan, Sridhar Maradugu, Anil Nigam, Piyawadee Sukaviriya, and Roman Vaculin.

Business Artifacts with Guard-Stage-Milestone Lifecycles: Managing Artifact Interactions with Conditions and Events.  
In *Proc. of DEBS*, 2011.  
To appear.