First-order Modal Languages for the Specification of Multi-agent Systems

Joint work with Alessio Lomuscio
Department of Computing
Imperial College London

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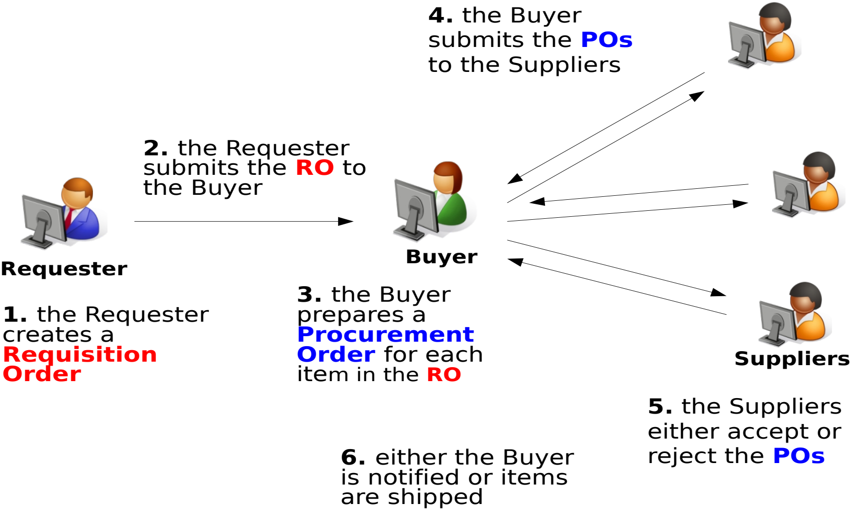
Background

- Propositional modal languages (temporal, epistemic, deontic, coalition logic) are a success story on the applications of logic to computer science.
- Today a number of tools and techniques based on (some of) these formalisms are available for the specification and verification of distributed and multi-agent systems.
  - NuSMV
  - MCMAS
  - PRISM
  - SPIN
  - UPAAL
  - etc...
- Still, as increasing complex scenarios (web services, e-commerce, ACSI project) are tackled by the verification community, the demand for ever more expressive languages is becoming more pressing.

The Equipment Purchase Scenario



Extending to First-order

First-order logic is typically considered the most "convenient" setting to formalise natural language:

- each of the Requisition Order, Procurement Order can be formalised as a database, i.e., a first-order structure;
- we are interested in model checking formulas like

$$AG\ \forall id, itm, p(RO(id, itm, p, \text{closed}) \rightarrow \exists s\ K_{Buyer} PO(p, s, itm, \text{accepted}))$$

  a Requisition Order can be in state *closed* only if the Buyer knows that the corresponding Procurement Order has been accepted by some Supplier.

Extending to First-order

However, first-order (modal) logic is known to be highly undecidable:

- [HWZ00] Let $\mathcal{F}$ be either $\{\langle \mathbb{N}, < \rangle\}$ or $\{\langle \mathbb{Z}, < \rangle\}$. Then the set $\mathcal{TL}^2 \cap \mathcal{TL}^{mo} \cap TL(\mathcal{F})$ is not recursively enumerable.
- [HWZ00] Let $\mathcal{F}$ be one of the following classes of temporal frames: $\{\langle \mathbb{N}, < \rangle\}$, $\{\langle \mathbb{Z}, < \rangle\}$, the class of all strict linear orders. Then $\mathcal{TL}^2 \cap \mathcal{TL}^{mo} \cap TL_{fin}(\mathcal{F})$ is not recursively enumerable.
- [WZ01] The two variable monadic fragment of $\mathbf{QK}^*$ (no matter with constants or expanding domains) is not recursively enumerable.
- [W00] For any first-order common knowledge logic L:
  - The fragment of L with monadic predicates only (without equality and function symbols) is not recursively enumerable.
  - The fragment of L with local constant symbols and the equality symbol only (without predicates and proper function symbols) is not recursively enumerable.

The Monodic Fragment

Definition (Monodic fragment)

Let $\mathcal{L}$ be a first-order modal language, the monodic fragment $\mathcal{L}^1$ of $\mathcal{L}$ is the set of formulas $\phi \in \mathcal{L}$ such that any subformula of ϕ of the form $\Box\psi$, for some modal operator $\Box$, contains at most one free variable.

Examples:

- agent i knows that every resource is universally available until it is requested,

$$\forall y(Resource(y) \rightarrow K_i(\forall z Available(y,z)\mathcal{U}\exists x Request(x,y)) \qquad \checkmark$$

- for every process agent i knows that it will eventually try to access every resource,

$$\forall x K_i(Process(x) \rightarrow \forall y(Resource(y) \rightarrow F\ Access(x,y)) \qquad 6$$

The monodic fragment contains all *de dicto* formulas, so the limitation is only on *de re* formulas.

- [HWZ00] Let $\mathcal{TL}' \subseteq \mathcal{TL}_1$ and suppose there is an algorithm which is capable of deciding, for any $\mathcal{TL}'$-sentence ϕ, whether an arbitrary state candidate for ϕ is realizable. Let $\mathcal{F}$ be one of the following classes of flows of time: $\{\langle \mathbb{N}, < \rangle\}$, $\{\langle \mathbb{Z}, < \rangle\}$, $\{\langle \mathbb{Q}, < \rangle\}$, the class of all finite strict linear orders, any first-order-definable class of strict linear orders. Then the satisfability problem for the $\mathcal{TL}'$-sentences in $\mathcal{F}$, and so the decision problem for the fragment $TL(\mathcal{F}) \cap \mathcal{TL}'$, is decidable.
- [WZ01] For $L = \{\mathbf{QK}^*, \mathbf{QK}, \mathbf{QT}, \mathbf{QK4}, \mathbf{QS4}\}$ the fragments $L \cap \mathcal{ML}_1^2$, $L \cap \mathcal{ML}_1^m$, and $L \cap \mathbf{MGF}_1$ are decidable.
- [SWZ00] Let L be any of $\mathbf{K}_n^C$, $\mathbf{T}_n^C$, $\mathbf{KD}_n^C$, $\mathbf{K4}_n^C$, $\mathbf{S4}_n^C$, $\mathbf{KD45}_n^C$, $\mathbf{S5}_n^C$. Then the following fragments are decidable: the monodic fragment, the two-variable fragment, the guarded fragment.
  However,
- Let L be any of the logics above. Then the fragment L_1 in the language $\mathcal{QCL}_1^=$ is not recursively enumerable.

- In this talk we show that the axiomatisations for temporal epistemic logic available at the propositional level can be lifted to the monodic fragment.
- This work builds on previous contributions:
  - on the monodic fragment of first-order temporal [WZ02] and epistemic logic [SWZ00, SWZ02];
  - on propositional temporal epistemic logic [HvdMV03].
- We consider well-known properties of multi-agent systems including *perfect recall*, *synchronicity*, *no learning*, and having a *unique initial state*.

Let $A = \{1, \ldots, i, \ldots, m\}$ be a set of agents.

Definition (terms and formulas in $\mathcal{L}_m$)

$$
\begin{aligned}
t &::= x \mid c \\
\phi &::= P^k(t_1, \ldots, t_k) \mid \neg\psi \mid \psi \to \psi' \mid \forall x \psi \mid \bigcirc\psi \mid \psi \mathcal{U} \psi' \mid K_i \psi
\end{aligned}
$$

The language $\mathcal{L}_m$ combines at first order:

- the *LTL* operators *next* $\bigcirc$ and *until* $\mathcal{U}$, and
- the epistemic operators K_i for each agent $i \in A$.

The language $\mathcal{L}C_m$ extends $\mathcal{L}_m$ with the common knowledge operator C.

Interpreted systems are the typical formalism for reasoning about knowledge in multi-agent systems [FHMV95].

- Let $\mathcal{S} \subseteq L_e \times L_1 \times \ldots \times L_m$ be the set of global states, where L_i is the set of local states for each agent i in A and for the environment e.

Definition (QIS)

A *quantified interpreted system* is a tuple $\mathcal{P} = \langle R, \mathcal{D}, I \rangle$ such that

- R is a non-empty set of runs $r : \mathbb{N} \to \mathcal{S}$
- $\mathcal{D}$ is a non-empty set of individuals
- I is a first-order interpretation of individual constants and predicate symbols.

$\mathcal{QIS}_m$ is the class of all quantified interpreted systems $\mathcal{P}$ with m agents.

Quantified Interpreted Systems

Consider a formula $\phi \in \mathcal{L}_m$ (resp. $\mathcal{LC}_m$), a point $(r, n) \in \mathcal{P}$, and an assignment $\sigma : Var \to \mathcal{D}$:

$(\mathcal{P}^\sigma, r, n) \models P^k(t_1, \ldots, t_k)$	iff	$\langle I^\sigma(t_1), \ldots, I^\sigma(t_k)\rangle \in I(P^k, r, n)$
$(\mathcal{P}^\sigma, r, n) \models \neg\psi$	iff	$(\mathcal{P}^\sigma, r, n) \not\models \psi$
$(\mathcal{P}^\sigma, r, n) \models \psi \to \psi'$	iff	$(\mathcal{P}^\sigma, r, n) \not\models \psi$ or $(\mathcal{P}^\sigma, r, n) \models \psi'$
$(\mathcal{P}^\sigma, r, n) \models \forall x \psi$	iff	for all $a \in \mathcal{D}$, $(\mathcal{P}^{\sigma^x_a}, r, n) \models \psi$
$(\mathcal{P}^\sigma, r, n) \models \bigcirc\psi$	iff	$(\mathcal{P}^\sigma, r, n+1) \models \psi$
$(\mathcal{P}^\sigma, r, n) \models \psi \mathcal{U} \psi'$	iff	there is $n' \geq n$ such that $(\mathcal{P}^\sigma, r, n') \models \psi'$ and for all $n \leq n'' < n'$, $(\mathcal{P}^\sigma, r, n'') \models \psi$
$(\mathcal{P}^\sigma, r, n) \models K_i\psi$	iff	$r'_i(n') = r_i(n)$ implies $(\mathcal{P}^\sigma, r', n') \models \psi$
$(\mathcal{P}^\sigma, r, n) \models C\psi$	iff	for all $k \in \mathbb{N}$, $(\mathcal{P}^\sigma, r, n) \models E^k\psi$

where $E^0\psi = \bigwedge_{i \in A} K_i\psi = E\psi$, and $E^{k+1}\psi = EE^k\psi$.

The definitions of *truth* and *validity* in a class of QIS are standard.

A QIS $\mathcal{P}$ satisfies

synchronicity	iff	time is part of the local state of each agent, i.e., $r_i(n) = r'_i(n')$ implies $n = n'$
perfect recall	iff	each agent remembers everything that has happened to her during the run (i.e., local states are histories)
no learning	iff	no agent aquires new knowledge during the run
unique initial state	iff	all runs begin from the same global state, i.e., for all $r, r' \in \mathcal{R}$, $r(0) = r'(0)$

The superscripts *sync*, *pr*, *nl*, *uis* denote specific subclasses of $\mathcal{QIS}_m$.

For instance, $\mathcal{QIS}_m^{pr,sync}$ contains the synchronous QIS with perfect recall.

The system QKT_m combines at first order:

- the linear temporal logic *LTL*, and
- the multi-modal epistemic logic $S5_m$.

Classical first-order logic	classical propositional tautologies $\phi \rightarrow \psi, \phi \Rightarrow \psi$ $\forall x \phi \rightarrow \phi[x/t]$ $\phi \rightarrow \psi[x/t] \Rightarrow \phi \rightarrow \forall x \psi$, x not free in ϕ
Axioms for Time	$\bigcirc(\phi \rightarrow \psi) \rightarrow (\bigcirc\phi \rightarrow \bigcirc\psi)$ $\bigcirc\neg\phi \leftrightarrow \neg \bigcirc \phi$ $\phi\mathcal{U}\psi \leftrightarrow \psi \vee (\phi \wedge \bigcirc(\phi\mathcal{U}\psi))$ $\phi \Rightarrow \bigcirc\phi$ $\chi \rightarrow \neg\psi \wedge \bigcirc\chi \Rightarrow \chi \rightarrow \neg(\phi\mathcal{U}\psi)$
Axioms for Knowledge	$K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$ $K_i\phi \rightarrow \phi$ $K_i\phi \rightarrow K_iK_i\phi$ $\neg K_i\phi \rightarrow K_i\neg K_i\phi$ $\phi \Rightarrow K_i\phi$
Barcan formulas	$\bigcirc\forall x\phi \leftrightarrow \forall x \bigcirc \phi$ $K_i\forall x\phi \leftrightarrow \forall x K_i\phi$

More Systems

Consider the following axioms for common knowledge:

C1	$C\phi \leftrightarrow (\phi \wedge EC\phi)$
C2	$\phi \rightarrow (\psi \wedge E\phi) \Longrightarrow \phi \rightarrow C\psi$

The system QKTC_m extends QKT_m with C1 and C2.

Consider the following axioms on the interaction between time and knowledge:

KT1	$K_i \bigcirc \phi \rightarrow \bigcirc K_i\phi$
KT2	$K_i\phi \wedge \bigcirc(K_i\psi \wedge \neg K_i\chi) \rightarrow \bar{K}_i((K_i\phi)\mathcal{U}((K_i\psi)\mathcal{U}\neg\chi))$
KT3	$(K_i\phi)\mathcal{U}K_i\psi \rightarrow K_i((K_i\phi)\mathcal{U}K_i\psi)$
KT4	$\bigcirc K_i\phi \rightarrow K_i \bigcirc \phi$
KT5	$K_i\phi \leftrightarrow K_j\phi$

We use $1, \ldots, 5$ as superscripts to denote the systems obtained by adding to QKT_m (resp. QKTC_m) any combination of KT1-KT5.

For instance, the system $\mathrm{QKT}_m^{2,3}$ extends QKT_m with KT2 and KT3.

Soundness Results

A QIS $\mathcal{P}$ satisfies any of KT1-KT5 if $\mathcal{P}$ satisfies the corresponding condition:

Axiom	*Condition on QIS*
KT1	perfect recall, synchronicity
KT2	perfect recall
KT3	no learning
KT4	no learning, synchronicity
KT5	all agents share the same knowledge

Theorem (Completeness)

The systems in the first and second column are sound and complete w.r.t. the corresponding classes of QIS in the third column.

Systems		QIS
QKT_m	$QKTC_m$	$\mathcal{QIS}_m$, $\mathcal{QIS}_m^{sync}$, $\mathcal{QIS}_m^{uis}$, $\mathcal{QIS}_m^{sync,uis}$
QKT_m^1		$\mathcal{QIS}_m^{pr}$, $\mathcal{QIS}_m^{pr,uis}$
QKT_m^2		$\mathcal{QIS}_m^{pr,sync}$, $\mathcal{QIS}_m^{pr,sync,uis}$
QKT_m^3		$\mathcal{QIS}_m^{nl}$
QKT_m^4		$\mathcal{QIS}_m^{nl,sync}$
$QKT_m^{2,3}$		$\mathcal{QIS}_m^{nl,pr}$
$QKT_m^{1,4}$		$\mathcal{QIS}_m^{nl,pr,sync}$
$QKT_1^{2,3}$		$\mathcal{QIS}_1^{nl,uis}$, $\mathcal{QIS}_1^{nl,pr,uis}$
$QKT_m^{1,4,5}$	$QKTC_m^{1,4,5}$	$\mathcal{QIS}_m^{nl,sync,uis}$, $\mathcal{QIS}_m^{nl,pr,sync,uis}$

- All completeness results available at the propositional level [HvdMV03] can be lifted to the monodic fragment of $\mathcal{L}_m$ (resp. $\mathcal{LC}_m$).
- In the general case there is no complete axiomatisation of the validities on $\mathcal{QIS}_m^{nl,uis}$ and $\mathcal{QIS}_m^{nl,pr,uis}$.

  This is already the case at the propositional level.
- However, for $m = 1$, $\mathcal{QIS}_1^{nl,pr,uis} = \mathcal{QIS}_1^{nl,uis} = \mathcal{QIS}_1^{nl,pr}$.
  Hence $\mathrm{QKT}_1^{2,3}$ is a complete axiomatisation for $\mathcal{QIS}_1^{nl,uis}$ and $\mathcal{QIS}_1^{nl,pr,uis}$.

Idea of the Completeness Proof

Two ingredients are needed in order to prove completeness:

- quasimodels, introduced in [HWZ00];
- monodic-friendly Kripke models, a Kripke-style semantics for $\mathcal{L}^1_m$ (resp. $\mathcal{L}C^1_m$).

Lemma

For every monodic formula ϕ,
satisfiability in mf-Kripke models $\Rightarrow$ satisfiability in QIS.
satisfiability in quasimodels $\Rightarrow$ satisfiability in mf-Kripke models.

Synchronicity, *perfect recall*, *no learning*, and *unique initial state* are preserved.

Lemma

a monodic ϕ is consistent w.r.t. a system S $\Rightarrow$ ϕ is satisfiable in a quasimodel for S.

As a result,

Theorem

The system S is complete for the corresponding class of QIS.

Definition (mf-Kripke model)

A *monodic-friendly Kripke model* is a tuple $\mathcal{M} = \langle \mathcal{R}, \{\sim_i\}_{i \in A, a \in \mathcal{D}}, \mathcal{D}, I \rangle$ where

- $\mathcal{R}$ is a non-empty set of indexes $r, r', \ldots$
- $\mathcal{D}$ is a non-empty set of individuals
- I is a first-order interpretation of individual constants and predicate symbols
- for $i \in A$, $a \in \mathcal{D}$, $\sim_{i,a}$ is an equivalence relation on the points (r, n), for $r \in \mathcal{R}$ and $n \in \mathbb{N}$

$$(\mathcal{M}^\sigma, r, n) \models K_i\psi[y] \quad \textit{iff} \quad (r, n) \sim_{i,\sigma(y)} (r', n') \textit{ implies } (\mathcal{M}^\sigma, r', n') \models \psi$$

$$(\mathcal{M}^\sigma, r, n) \models C\psi[y] \quad \textit{iff} \quad \text{for all } k \in \mathbb{N}, (\mathcal{M}^\sigma, r, n) \models E^k\psi[y]$$

- The class of mf-Kripke models with m agents is denoted by $\mathcal{K}_m$.
- We can consider mf-Kripke models satisfying *synchronicity*, *perfect recall*, *no learning*, or having a *unique initial state*.
- mf-Kripke models do not necessarily satisfy BF and axiom K!!

Monodic-friendly Kripke models are necessary to prove completeness for systems encompassing either perfect recall or no learning.

We are only able to prove that

Lemma

For every monodic formula ϕ,
satisfiability in quasimodels $\Rightarrow$ satisfiability in mf-Kripke models.

not in general Kripke models.

However,

Lemma

Let $\mathcal{M}$ be a mf-Kripke model satisfying BF and axiom K.
There exists a map $g : \mathcal{K}_m \rightarrow \mathcal{QIS}_m$ such that for every ϕ,

$$\mathcal{M} \text{ satisfies } \phi \ \Leftrightarrow \ g(\mathcal{M}) \text{ satisfies } \phi$$

- The map g preserves any of *synchronicity*, *perfect recall*, *no learning*, or having a *unique initial state*.

Quasimodels

- A quasimodel for a monodic formula $\phi \in \mathcal{L}^1_m$ (resp. $\mathcal{L}C^1_m$) is a relational structure whose points are sets of sets of subformulas of ϕ.
- Quasimodels have been introduced in [HWZ00], and then applied to first-order temporal [WZ02] and epistemic logic [SWZ00, SWZ02].
- Here we combine quasimodels with the techniques in [HvdMV03].

Quasimodels

- Fix a monodic ϕ in $\mathcal{L}^1_m$ (resp. $\mathcal{LC}^1_m$), $sub_x\phi$ is the set of subformulas of ϕ containing at most the free variable x. Also, $sub_x\phi$ is closed under negation.
- For $k \in \mathbb{N}$ we define the closures $cl_k\phi$ and $cl_{k,i}\phi$ by mutual recursion.

Definition

Let $cl_0\phi = sub_x\phi$.
For $k \geq 0$, $cl_{k+1}\phi = \bigcup_{i \in Ag} cl_{k,i}\phi$.
For $k \geq 0$, $i \in Ag$,
$cl_{k,i}\phi = cl_k\phi \cup \{K_i(\psi_1 \vee \ldots \vee \psi_n), \neg K_i(\psi_1 \vee \ldots \vee \psi_n) \mid \psi_1, \ldots, \psi_n \in cl_k\phi\}$.

- Let ι be any finite sequence $i_1, \ldots, i_k$ of agents such that $i_n \neq i_{n+1}$.
- If ι is the empty sequence ϵ then $cl_\iota\phi = cl_{ad(\phi)}\phi$. If $\iota = \iota'\sharp i$, then $cl_\iota\phi = cl_{k,i}\phi$ for $k = ad(\phi) - |\iota|$.

Definition (Type)

A *ι-type* $\mathfrak{t}$ is any boolean saturated subset of $cl_\iota\phi$.

Quasimodels

- Types represent individuals in a particular point. Thus, points are represented by collections of types, i.e., *state candidates*.
  To each state candidate $\mathfrak{C}$ is associated a formula $\alpha_{\mathfrak{C}} \in \mathcal{L}^1_m$.
- Now we need a way of identifying the same individual across points.

Definition (Object)

Let a *frame* $\mathcal{F}$ be a tuple $\langle \mathcal{R}, \{\sim_{i,a}\}_{i \in A, a \in \mathcal{D}}, \mathcal{D} \rangle$ where $\mathcal{R}$, $\sim_{i,a}$ and $\mathcal{D}$ are defined as for mf-Kripke models, and whose points are state candidates.

For $a \in \mathcal{D}$, an *object* in $\mathcal{F}$ is a map ρ_a associating with every $(r, n) \in \mathcal{F}$ a type $\rho_a(r, n) \in \mathfrak{C}_{r,n}$ such that:

1. $\rho_a(r, n) \wedge \bigcirc \rho_a(r, n+1)$ is consistent
2. if $(r, n) \sim_{i,a} (r', n')$ then $\rho_a(r, n) \wedge \bar{K}_i \rho_a(r', n')$ is consistent
3. $\chi \mathcal{U} \psi \in \rho_a(r, n)$ iff there is $n' \geq n$ such that $\psi \in \rho_a(r, n')$ and $\chi \in \rho_a(r, n'')$ for all $n \leq n'' < n'$
4. if $\neg K_i \psi \in \rho_a(r, n)$ then there is some (r', n') such that $(r, n) \sim_{i,a} (r', n')$ and $\neg \psi \in \rho_a(r', n')$.

We can now provide the definition of *quasimodels*.

Definition (Quasimodel)

A *quasimodel* for ϕ is a tuple $\mathfrak{Q} = \langle \mathcal{R}, \{\sim_{i,\rho}\}_{i \in A, \rho \in \mathcal{O}}, \mathcal{O} \rangle$ such that $\langle \mathcal{R}, \{\sim_{i,\rho}\}_{i \in A, \rho \in \mathcal{O}}, \mathcal{O} \rangle$ is a frame, and

1. $\phi \in \mathfrak{t}$ for some $\mathfrak{t} \in \mathfrak{C}_{r,n}$
2. $\alpha_{\mathfrak{C}_{r,n}} \wedge \bigcirc \alpha_{\mathfrak{C}_{r,n+1}}$ is consistent
3. if $(r, n) \sim_{i,\rho} (r', n')$ then $\rho(r, n) \wedge \bar{K}_i \rho(r', n')$ is consistent
4. for all $\mathfrak{t} \in \mathfrak{C}_{r,n}$ there exists an object $\rho \in \mathcal{O}$ such that $\rho(r, n) = \mathfrak{t}$
5. for all $c \in con\phi$, the function ρ^c such that $\rho^c(r, n) = \mathfrak{t}^c \in \mathfrak{C}_{r,n}$ is an object in $\mathcal{O}$.

Lemma

For every monodic formula ϕ,
satisfiability in quasimodels $\Rightarrow$ satisfiability in QIS.

It's only left to prove that

Lemma

a monodic ϕ is consistent w.r.t. a system $S \Rightarrow \phi$ is satisfiable in a quasimodel for S.

The proof of this lemma depends on the particular system S.

Conclusions and Future Work

In this talk:

- we presented a number of classes of QIS satisfying *perfect recall*, *synchronicity*, *no learning*, and having a *unique initial state*.
- we proved that the axiomatisations available at the propositional level can be lifted to the monodic fragment of $\mathcal{L}_m$ (resp. $\mathcal{L}C_m$).

In future work we aim to:

- analyze CTL and epistemic modalities interpreted on QIS.
- explore the issues pertaining to the decidability of the logics here discussed.

Thank you!

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