Verifying Auctions as Artifact Systems: Decidability via Finite Abstraction

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based on work with Alessio Lomuscio
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Model Checking in one slide

Model checking: technique(s) to **automatically** verify that a system design S satisfies a property P **before** deployment.

More formally, given

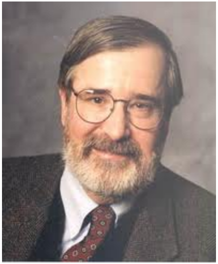
- a model $\mathcal{M}_S$ of system S
- a formula ϕ_P representing property P

we check that

$$\mathcal{M}_S \models \phi_P$$

Turing Award 2007

www.acm.org/press-room/news-releases-2008/turing-award-07



(a) E. Clarke
(CMU, USA)

(b) A. Emerson
(U. Texas, USA)

(c) J. Sifakis
(IMAG, F)

- Jury justification

  "For their roles in developing model checking into a highly effective verification technology, widely adopted in the hardware and software industries."

Overview

1. Motivation and Background:
   - Artifact Systems as *data-aware* systems
   - Parallel English (ascending bid) Auctions as Artifact Systems (eBay, etc.)

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   - model checking is appropriate for control-intensive applications...
   - ...but less suited for data-intensive applications (data typically range over infinite domains) [1].

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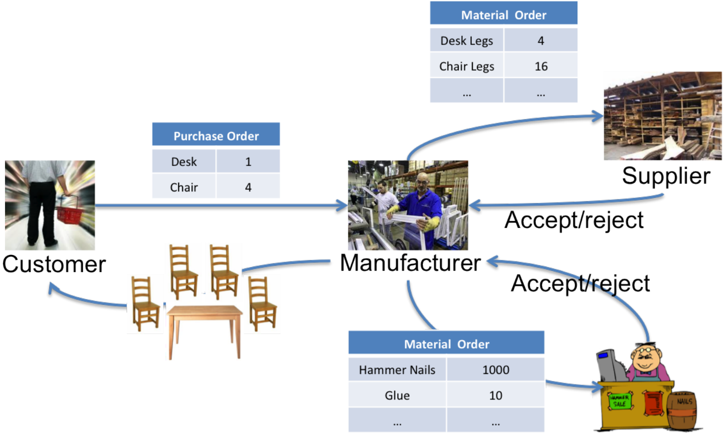
   - Parallel English (ascending bid) Auctions as Artifact Systems (eBay, etc.)
2. Main task: *Formal* verification of *infinite-state* AS
   - model checking is appropriate for control-intensive applications...
   - ...but less suited for data-intensive applications (data typically range over infinite domains) [1].
3. Key contribution:
   - Verification of *bounded* and *uniform* AS is decidable
   - Verification of Parallel English Auctions is decidable

Artifact Systems

- Recent paradigm in Service-Oriented Computing [2].
- Motto: let's give *data* and *processes* the same relevance!
- *Artifact*: data model + lifecycle
  - (nested) records equipped with actions
  - actions may affect several artifacts
  - evolution stemming from the interaction with other artifacts/external actors
- *Artifact System*: interacting artifacts, representing services, manipulated by agents.
- *Auctions as Artifact Systems*

Artifact Systems

Order-to-Cash Scenario



Purchase Order	
Desk	1
Chair	4

Material Order	
Desk Legs	4
Chair Legs	16
...	...

Material Order	
Hammer Nails	1000
Glue	10
...	...

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Artifact Systems

Parallel English (ascending bid) Auctions

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- each bidder is rational;
- he has an *intrinsic value* for each item being auctioned;
- and he keeps this information private from other bidders and the auctioneer.

Artifact Systems

Auction Data Model

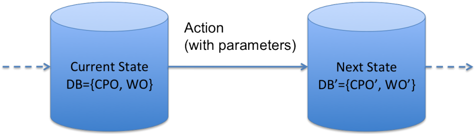
Bidding					
item	*base_price*	bid_1	...	bid_ℓ	*status*

- $init_A(item, base\_price)$
- $bid_i(item, bid)$
- $time\_out(item)$
- $skip_A$
- $skip_i$
- ...

$trueValue_i$	
item	*true_value*

- $init_i(item, true\_value)$
- ...

- Agents operate on artifacts.
  - e.g., the bidder sends a new bid to the auctioneer.
- Actions add/remove artifacts or change artifact attributes.
  - e.g., the auctioneer puts a new item on auction.
- The whole system can be seen as a *data-aware* dynamic system.
  - at every step, an action yields a change in the current state.



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3. If not, can we identify *relevant* fragments that are reasonably well-behaved?
4. How can we implement this?

Challenges

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$\Rightarrow$ The model checking problem cannot be tackled by standard techniques.

Artifact Systems

Results

1. *Artifact-centric multi-agent systems* (AC-MAS) as a formal model for AS.
   Intuition: databases (?) that evolve in time and are manipulated by agents.

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2. FO-CTLK as a specification language:

$$AG\ \forall it, \vec{bd}, s(\exists! bp\ Bidding(it, \vec{bd}, bp, s) \land \exists^{\leq 1} tv\ trueValue_i(it, tv))$$

   for each item there is exactly one base price, while bidders associate at most one true value to each item (possibly none).

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4. Case study: modelling and veryfing auctions as AC-MAS.

Semantics: Databases

The data model of AS is given as a particular kind of database.

- a *database schema* is a *finite* set $\mathcal{D} = \{P_1/a_1, \ldots, P_n/a_n, Q_1/b_1, \ldots, Q_m/b_m\}$ of (typed) relation symbols R_i with arity $c_i \in \mathbb{N}$.

- an *instance* on a domain U is a mapping D associating
  - each symbol P_i with a *finite* a_i-ary relation on U
  - each symbol Q_i with a (possibly infinite) b_i-ary relation on U

- the *active domain* $adom(D)$ is the set of all $u \in U$ appearing in some $D(P_i)$.

- the *disjoint union* $D \oplus D'$ is the $(\mathcal{D} \cup \mathcal{D}')$-interpretation s.t.
  (i) $D \oplus D'(R_i) = D(R_i)$
  (ii) $D \oplus D'(R_i') = D'(R_i)$

We consider untyped languages; the extension to types is not problematic.

Agents have partial access (*views*) to the artifact system.

- An *agent* is a tuple $A_i = \langle \mathcal{D}_i, Act_i, Pr_i \rangle$ where
  - $\mathcal{D}_i$ is the *local database schema*
  - Act_i is the set of *local actions* $\alpha(\vec{x})$ with parameters $\vec{x}$
  - $Pr_i : \mathcal{D}_i(U) \mapsto 2^{Act_i(U)}$ is the *local protocol function*

- the setting is reminiscent of the *interpreted systems semantics* for MAS [3],...

- ...but here the local state of each agent is relational.

Intuitively, agents manipulate artifacts and have (partial) access to the information contained in the global db schema $\mathcal{D} = \mathcal{D}_1 \cup \cdots \cup \mathcal{D}_\ell$.

Example 1: Parallel English (ascending bid) Auction

- Agents: Auctioneer, Bidder_1, ..., Bidder_ℓ

- local db schema $\mathcal{D}_A$ for the auctioneer
  - *Bidding(item, base_price,* bid_1, ..., bid_ℓ, *status)*
  - $<$ on $\mathbb{Q}$
- local db schema $\mathcal{D}_i$ for the bidders
  - *Bidding(item, base_price,* bid_1, ..., bid_ℓ, *status)*
  - $TValue_i$*(item, true_value)*
  - $<$ on $\mathbb{Q}$
- then, $\mathcal{D} = \{Bidding, TValue_1, \ldots, TValue_\ell, <\}$

- Actions introduce values from an infinite domain $U = Items \cup \mathbb{Q} \cup \{active, term\}$:
  - $init_A$*(item, base_price), time out(item),* $skip_A$ belong to Act_A
  - $init_i$*(item, true_value),* bid_i*(item, bid),* $skip_i$ belong to Act_i

- The protocol function specifies the preconditions for actions:
  - e.g., $bid_i(item, bid) \in Pr_i(D)$ whenever *item* appears in $D(TValue_i)$, the highest bid bid_j in *Bidding*, $j \neq i$, for *item* is $<$ *true_value* for bidder B_i, $bid_j < bid \leq true\_value$, and $D(status) = active$ for *item*.
  - the *skip* actions are always enabled.

Agents are modules that can be composed together to obtain AC-MAS.

- *Global states* are tuples $s = \langle D_0, \ldots, D_\ell \rangle \in \mathcal{D}(U)$.
- An *AC-MAS* is a tuple $\mathcal{P} = \langle Ag, s_0, \rightarrow \rangle$ where
  - $Ag = \{A_0, \ldots, A_\ell\}$ is a *finite set of agents*
  - $s_0 \in \mathcal{D}(U)$ is the *initial global state*
  - $s \xrightarrow{\alpha(\vec{u})} s'$ is the *transition relation*
- *Epistemic relation*: $s \sim_i s'$ iff $D_i = D'_i$
- An AC-MAS $\mathcal{P}$ is *rigid* iff for all states s, s', symbols Q, and agents A_i, $A_j \in Ag$, $D_i(Q) = D'_j(Q)$.
- AC-MAS are infinite-state systems in general

AC-MAS are first-order temporal epistemic structures.
$\Rightarrow$ FO-CTLK can be used as a specification language.

Example 2: the Auction AC-MAS

The *Auction AC-MAS* $\mathcal{A} = \langle Ag, s_0, \rightarrow \rangle$ is defined as

- $Ag = \{A, B_1, \ldots, B_\ell\}$
- s_0 is the *empty interpretation* of $\mathcal{D} = \{Bidding, TValue_1, \ldots, TValue_\ell, <\}$ but for $<$
- $\rightarrow$ is the *transition relation* s.t. $s \xrightarrow{\alpha(\vec{u})} s'$ whenever
  - $\alpha_i = bid_i(item, bid')$ and s' modifies s by replacing any tuple $(item, \ldots, bid_i, \ldots, status)$ in $D_s(Bidding)$ with $(item, \ldots, bid_i', \ldots, status)$
  - $\alpha_A = timeout(item)$ and the value of $status$ in $D_{s'}(Bidding)$ for $item$ is $term$
  - $\ldots$

Notice:

- the auction AC-MAS $\mathcal{A}$ is **rigid**
- actions preserve the consistency of the underlying database
- the active domain $adom(s_0)$ is empty

Syntax: FO-CTLK

- Data call for First-order Logic.
- Evolution calls for Temporal Logic.
- Agents (operating on artifacts) call for Epistemic Logic.

The specification language FO-CTLK:

$$\varphi \quad ::= \quad R(t_1, \ldots, t_c) \mid t = t' \mid \neg\varphi \mid \varphi \rightarrow \varphi \mid \forall x \varphi \mid AX\varphi \mid A\varphi U\varphi \mid E\varphi U\varphi \mid K_i\varphi$$

Alternation of free variables and modal operators is enabled.

Semantics of FO-CTLK

Formal definition

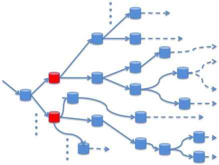
An AC-MAS $\mathcal{P}$ satisfies an FO-CTLK-formula φ in a state s for an assignment σ, iff

$$
\begin{array}{lll}
(\mathcal{P}, s, \sigma) \models R(\vec{t}) & \text{iff} & \langle \sigma(t_1), \ldots, \sigma(t_c) \rangle \in D_s(R) \\
(\mathcal{P}, s, \sigma) \models t = t' & \text{iff} & \sigma(t) = \sigma(t') \\
(\mathcal{P}, s, \sigma) \models \neg\varphi & \text{iff} & (\mathcal{P}, s, \sigma) \not\models \varphi \\
(\mathcal{P}, s, \sigma) \models \varphi \rightarrow \psi & \text{iff} & (\mathcal{P}, s, \sigma) \not\models \varphi \text{ or } (\mathcal{P}, s, \sigma) \models \psi \\
(\mathcal{P}, s, \sigma) \models \forall x \varphi & \text{iff} & \text{for all } u \in adom(s),\ (\mathcal{P}, s, \sigma^x_u) \models \varphi \\
(\mathcal{P}, s, \sigma) \models AX\varphi & \text{iff} & \text{for all runs } r,\ r(0) = s \text{ implies } (\mathcal{P}, r(1), \sigma) \models \varphi \\
(\mathcal{P}, s, \sigma) \models A\varphi U \varphi' & \text{iff} & \text{for all runs } r,\ r(0) = s \text{ implies } (\mathcal{P}, r(k), \sigma) \models \varphi' \text{ for some } k \geq 0, \\
 & & \text{and } (\mathcal{P}, r(k'), \sigma) \models \varphi \text{ for all } 0 \leq k' < k \\
(\mathcal{P}, s, \sigma) \models E\varphi U \varphi' & \text{iff} & \text{there exists } r \text{ s.t. } r(0) = s,\ (\mathcal{P}, r(k), \sigma) \models \varphi' \text{ for some } k \geq 0, \\
 & & \text{and } (\mathcal{P}, r(k'), \sigma) \models \varphi \text{ for all } 0 \leq k' < k \\
(\mathcal{P}, s, \sigma) \models K_i\varphi & \text{iff} & \text{for all states } s',\ s \sim_i s' \text{ implies } (\mathcal{P}, s', \sigma) \models \varphi
\end{array}
$$

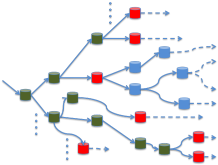
- Active-domain semantics, but...
  - ...we can refer to individuals that no longer exist
  - the number of states is infinite in general

Semantics of FO-CTLK

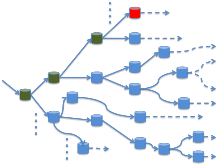
Intuition



(d) $AX\varphi$



(e) $A\varphi U\psi$



(f) $E\varphi U\psi$

Verification of AC-MAS

How do we verify FO-CTLK specifications on auctions?

- the true value of items for each bidder is secret to all other bidders and to the auctioneer:

$$AG\ \forall item\ \neg\exists true\_value \bigvee_{j \neq i \vee j = A} K_j\ TValue_i(item, true\_value)$$

- for each bidder, each bid is less or equal to her true value:

$$AG\ \forall it, \vec{x}, bd_i, \vec{y}, tv(Bidding(it, \vec{x}, bd_i, \vec{y}) \wedge TValue_i(it, tv) \rightarrow bd_i \leq tv)$$

- each bidder can raise her bid unless she has already hit her true value:

$$\begin{aligned} & AG\ \forall it, \vec{x}, bd_i, \vec{y}(Bidding(it, \vec{x}, bd_i, \vec{y}) \rightarrow \\ & \rightarrow (TValue_i(it, bd_i) \vee EF\ \exists \vec{x}', bd_i', \vec{y}'(bd_i' > bd_i \wedge Bidding(it, \vec{x}', bd_i', \vec{y}')))) \end{aligned}$$

<u>Problem</u>: the infinite domain U may generate infinitely many states!

<u>Investigated solution</u>: can we *simulate* the concrete values from U with a finite set of *abstract* symbols?

Abstraction: Isomorphism and Bisimulation

- two states s, s' are *isomorphic*, or $s \simeq s'$, if there is a bijection

$$\iota : adom(s) \cup C \mapsto adom(s') \cup C$$

such that

  - ι is the identity on C
  - for every $\vec{u}$ in $adom(s)$, $A_i \in Ag$, $\vec{u} \in D_i(R) \Leftrightarrow \iota(\vec{u}) \in D'_i(R)$

	$D(R)$	
A_1	a	b
A_2	b	c
A_3	d	e

$\simeq$

	$D'(R)$	
A_1	1	2
A_2	2	c
A_3	4	5

  - $$\begin{aligned} \iota : a &\mapsto 1 \\ b &\mapsto 2 \\ c &\mapsto c \\ d &\mapsto 4 \\ e &\mapsto 5 \end{aligned}$$

Abstraction: Isomorphism and Bisimulation

- two states s, s' are *bisimilar*, or $s \approx s'$, if
    1. $s \simeq s'$
    2. if $s \longrightarrow t$ then there is t' s.t. $s' \longrightarrow t'$, $s \oplus t \simeq s' \oplus t'$, and $t \approx t'$

$$
\begin{tikzcd}
\boxed{s} \arrow[r] \arrow[d, phantom, "\approx"] & \boxed{t} \\
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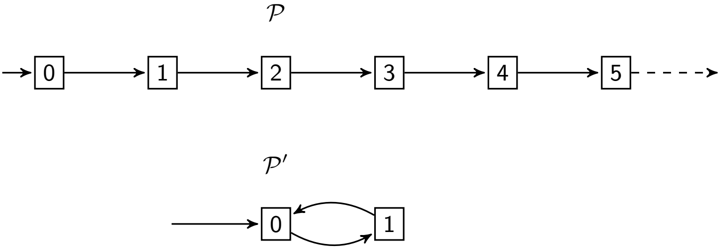
\end{tikzcd}$$

  3. the other direction holds as well
  4. similar conditions for the epistemic relation $\sim_i$

Abstraction: Isomorphism and Bisimulation

However, bisimulation is not sufficient to preserve FO-CTLK formulas:



$$\phi \quad = \quad AG\ \forall x\ (P(x) \rightarrow AX\ AG\ \neg P(x))$$

Uniformity

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- More formally, an AC-MAS $\mathcal{P}$ is *uniform* iff for $s, t, s' \in \mathcal{S}$ and $t' \in \mathcal{D}(U)$:

1. $s \rightarrow t$ and $s \oplus t \simeq s' \oplus t'$ imply $s' \rightarrow t'$

s	
a	b
b	c
d	e

$\longrightarrow$

t	
a	f
f	c

s'	
1	2
2	c
4	5

t'	
1	6
6	c

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$\longrightarrow$

t'	
1	6
6	c

2. Also, rigid AC-MAS must satisfy a condition akin to density of $<$ on $\mathbb{Q}$.

- Uniform AC-MAS cover a number of interesting cases [2, 4], including the auction AC-MAS $\mathcal{A}$.

Bisimulation and Equivalence w.r.t. FO-CTLK

Theorem

Consider

- bisimilar *and* uniform *AC-MAS* $\mathcal{P}$ *and* $\mathcal{P}'$
- *an FO-CTLK formula* φ

If

1. $|U'| \geq 2 \cdot \sup_{s \in \mathcal{P}}\{|adom(s)|\} + |C| + |vars(\varphi)|$
2. $|U| \geq 2 \cdot \sup_{s' \in \mathcal{P}'}\{|adom(s')|\} + |C| + |vars(\varphi)|$

then

$$\mathcal{P} \models \varphi \quad \textit{iff} \quad \mathcal{P}' \models \varphi$$

Can we apply this result to finite abstraction?

Abstraction

Abstractions are defined in an agent-based, modular way.

- Let $A = \langle \mathcal{D}, Act, Pr \rangle$ be an agent defined on the domain U.
  Given a domain U', the *abstract agent* $A' = \langle \mathcal{D}, Act, Pr' \rangle$ on U' is s.t.
  - Pr' is the smallest function s.t. if $\alpha(\vec{u}) \in Pr(D)$, $D' \in \mathcal{D}'(U')$ and $D' \simeq D$ for some witness ι, then $\alpha(\vec{u}') \in Pr'(D')$ where $\vec{u}' = \iota'(\vec{u})$ for some constant-preserving bijection ι' extending ι to $\vec{u}$.

- Let $\mathcal{P} = \langle Ag, s_0, \rightarrow \rangle$ be an AC-MAS.
  The *abstraction* $\mathcal{P}' = \langle Ag', s'_0, \rightarrow' \rangle$ of $\mathcal{P}$ is an AC-MAS s.t.
  - Ag' be the set of abstract agents on U'
  - $s'_0 \simeq s_0$
  - $\rightarrow'$ is the smallest function s.t. if $s \xrightarrow{\alpha(\vec{u})} t$, and $s \oplus t \simeq s' \oplus t'$ for some witness ι, then $s' \xrightarrow{\alpha(\iota'(\vec{u}))} t'$ for some constant-preserving bijection ι' extending ι to $\vec{u}$.

The abstraction of a rigid AC-MAS is not necessarily rigid!

Abstraction

- Let $N_{Ag} = \sum_{A_i \in Ag} \max_{\{\alpha(\vec{x}) \in Act_i\}} |\vec{x}|$ be the sum of the maximum numbers of parameters contained in the action types of each agent

Lemma

Consider

- *a* uniform *and* rigid *AC-MAS* $\mathcal{P}$
- *a set* $U' \supseteq C$ *s.t.* $|U'| \geq 2 \sup_{s \in \mathcal{P}} |adom(s)| + |C| + N_{Ag}$

Then, there exists an abstraction $\mathcal{P}'$ *of* $\mathcal{P}$ *that is uniform and bisimilar to* $\mathcal{P}$*.*

How can we define finite abstractions?

Bounded Models and Finite Abstractions

- An AC-MAS $\mathcal{P}$ is *b-bounded* iff for all $s \in \mathcal{P}$, $|adom(s)| \leq b$.
- Bounded systems can still be infinite!

Theorem

Consider

- *a* b-bounded, uniform *and* rigid *AC-MAS* $\mathcal{P}$ *on an infinite domain* U
- *an FO-CTLK formula* φ

Given a finite $U' \supseteq C$ *s.t.*

$$|U'| \geq 2b + |C| + \max\{|vars(\varphi)|, N_{Ag}\}$$

there exists a finite abstraction $\mathcal{P}'$ *of* $\mathcal{P}$ *s.t.*

- $\mathcal{P}'$ *is uniform and bisimilar to* $\mathcal{P}$

In particular,

$$\mathcal{P} \models \varphi \quad \textit{iff} \quad \mathcal{P}' \models \varphi$$

$\Rightarrow$ Under specific circumstances, we can model check an infinite-state system by verifying its finite abstraction.

Finite Abstract Auction I

- Notice: the auction AC-MAS $\mathcal{A}$ is bounded by $b = |Items|(2|Ag| - 1) + 2$
- Consider a finite $U' \geq 2b + vars(\phi)$
- Abstract agents <u>A</u>uctioneer A' and <u>B</u>idders B'_i
  - the local db schemas $\mathcal{D}'_A$ and $\mathcal{D}'_i$ are the same as for A and B_i
  - the sets of actions Act'_A and Act'_i are the same as for A and B_i
  - the protocol function Pr'_A is the same as for A
  - as to Pr'_i, $bid_i(item, bid) \in Pr'_i(D')$ whenever $item$ appears in $D'(TValue_i)$, the highest bid bid_j in $Bidding$, $j \neq i$, for $item$ is $<$ $true\_value$ for bidder B_i, and bid is an abstract value that does not represent any bid in D', and for $item$, $D'(status) = active$.

Finite Abstract Auction II

The abstract auction AC-MAS $\mathcal{A}' = \langle Ag', s'_0, \tau' \rangle$ is defined as

- $Ag' = \{A', B'_1, \ldots, B'_\ell\}$
- s'_0 is the empty interpretation of $\mathcal{D}$
- $\rightarrow'$ mimics $\rightarrow$
  - e.g., if $\alpha_i = bid_i(item, bid)$, then $s \xrightarrow{\alpha(\vec{u})}{}' \, t$ whenever t is the db instance that modifies s by replacing any tuple $(item, \ldots, bid_i, \ldots, status)$ in $D_s(Bidding)$ with $(item, \ldots, bid'_i, \ldots, status)$, where the value $bid' \in U'$ has been found as above. In particular, $bid < bid' \leq true\_value$ in t.

- By the assumption that $U' \geq 2b + vars(\phi)$ and Theorem 3 we have that $\mathcal{A}'$ is a finite abstraction of $\mathcal{A}$. In particular,
  - $\mathcal{A}'$ is uniform and bisimilar to $\mathcal{A}$ (but not rigid) and

$$\mathcal{A} \models \varphi \quad \text{iff} \quad \mathcal{A}' \models \varphi$$

Extensions

1. Non-uniform AC-MAS: for *sentence-atomic* FO-CTL the results above still hold.

$$AG\ \forall it, \vec{bd}, s(\exists! bp\ Bidding(it, \vec{bd}, bp, s) \wedge \exists^{\leq 1} tv\ \ TValue_i(it, tv))$$

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2. Non-uniform and unbounded AC-MAS: one-way preservation result for FO-ACTLK$^-$.

Theorem

For every AC-MAS $\mathcal{P}$ and $\varphi \in$ FO-ACTLK$^-$, there exists a finite abstraction $\mathcal{P}'$ s.t.

$$\mathcal{P}' \models \varphi \quad \Rightarrow \quad \mathcal{P} \models \varphi$$

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3. *Model checking bounded* AC-MAS w.r.t. FO-CTL is undecidable.

4. Complexity result:

Theorem

The model checking problem for finite AC-MAS w.r.t. FO-CTLK is EXPSPACE-complete in the size of the formula and data.

Results

and main limitations

- We are able to model check AC-MAS w.r.t. full FO-CTLK...
- ...however, our results hold only for *rigid*, *uniform* and *bounded* systems.
- This class includes many interesting systems (AS programs, [2, 4]).
- The model checking problem is EXPSPACE-complete.

Next Steps

- Techniques for finite abstraction.
- Model checking techniques for finite-state systems are effective on the abstract system?
- How to perfom the boundedness check.

Merci!

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