A Logic for Global and Local Announcements

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Outline

1. Background: logics for (public, semi-private, private) announcements [vDHvdHK15]

   In PAL announcements are
   - **public**: all agents listen to (and are aware of) the announcement
   - **global**: how the new information is processed depends on the model (i.e., public announcements are model transformers)

2. Goal: to generalise PAL by weaking **publicity** and **globality**
   - **privacy**: announcements to any subset $A \subseteq Ag$ of agents
   - **locality**: announcements are **pointed model** transformers

3. Dynamic Epistemic Logic: action models allow private announcements, but
   - updated indistinguishability relations are not necessarily equivalences
   - updated models might be strictly larger ...
   - ... several problems are undecidable

4. GLAL: an extension of PAL supporting both **private** and **local** announcements
   - updated indistinguishability relations **are** equivalences
   - updated models are normally "smaller" ...
   - ... the model checking and satisfaction problems are decidable

Let Ag be a set of agents and AP a set of propositional atoms.

Definition (GLAL)

Formulas ϕ in $\mathcal{L}_{glal}$ are defined by the following BNF:

$$\psi \quad ::= \quad p \mid \neg\psi \mid \psi \wedge \psi \mid C_A\psi \mid [\psi]^+_A\psi \mid [\psi]^-_A\psi$$

- $K_a\phi$ is introduced as $C_{\{a\}}\phi$
- $E_A\phi$ is introduced as $\bigwedge_{a\in A} K_a\phi$
- $[\psi]^+_A\phi$ ::= after **globally** announcing ψ to the agents in A, ϕ is true
- $[\psi]^-_A\phi$ ::= after **locally** announcing ψ to the agents in A, ϕ is true

$$\mathcal{L}_{pl} \subseteq \mathcal{L}_{el} \subseteq \mathcal{L}_{pal^+} \subseteq \mathcal{L}_{glal}$$

Formulas in GLAL are interpreted on (multi-modal) Kripke models.

Definition (Frame)

A **frame** is a tuple $\mathcal{F} = \langle W, \{R_a\}_{a \in Ag} \rangle$ where

- W is a set of **possible worlds**
- for every agent $a \in Ag$, $R_a \subseteq 2^{W \times W}$ is an **equivalence relation** on W.

A **model** is a pair $\mathcal{M} = \langle \mathcal{F}, V \rangle$ where $V : AP \to 2^W$ is an assignment to atoms.

- $R_A^C = (\bigcup_{a \in A} R_a)^*$ is the reflexive and transitive closure of $\bigcup_{a \in A} R_a$
- $R(w) = \{w' \in W \mid R(w, w')\}$ is the R-equivalence class of $w \in W$

Satisfaction & Refinements

The **satisfaction set** $[\![\varphi]\!]_{\mathcal{M}} \subseteq W$ is defined as

$$
\begin{array}{lcl}
[\![p]\!]_{\mathcal{M}} & = & V(p) \\
[\![\neg\psi]\!]_{\mathcal{M}} & = & W \setminus [\![\psi]\!]_{\mathcal{M}} \\
[\![\psi \wedge \psi']\!]_{\mathcal{M}} & = & [\![\psi]\!]_{\mathcal{M}} \cap [\![\psi']\!]_{\mathcal{M}} \\
[\![C_A\psi]\!]_{\mathcal{M}} & = & \{w \in W \mid \text{for all } w' \in R^C_A(w), w' \in [\![\psi]\!]_{\mathcal{M}}\} \\
[\![[\psi]^-_A\psi']\!]_{\mathcal{M}} & = & \{w \in W \mid \text{if } w \in [\![\psi]\!]_{\mathcal{M}} \text{ then } w \in [\![\psi']\!]_{\mathcal{M}^-_{(w,\psi,A)}}\} \\
[\![[\psi]^+_A\psi']\!]_{\mathcal{M}} & = & \{w \in W \mid \text{if } w \in [\![\psi]\!]_{\mathcal{M}} \text{ then } w \in [\![\psi']\!]_{\mathcal{M}^+_{(w,\psi,A)}}\}
\end{array}
$$

where **refinements** $\mathcal{M}^-_{(w,\psi,A)} = \langle W^-, \{R^-_a\}_{a\in Ag}, V^-\rangle$ and $\mathcal{M}^+_{(w,\psi,A)} = \langle W^+, \{R^+_a\}_{a\in Ag}, V^+\rangle$ have

- $W^- = W^+ = W$ and $V^- = V^+ = V$
- for every agent $b \notin A$, $R^-_b = R^+_b = R_b$; while for $a \in A$,

$$
R^-_a(v) = \begin{cases} R_a(v) \cap [\![\psi]\!]_{\mathcal{M}} & \text{if } v \in R_a(w) \cap [\![\psi]\!]_{\mathcal{M}} \\ R_a(v) \cap [\![\neg\psi]\!]_{\mathcal{M}} & \text{if } v \in R_a(w) \cap [\![\neg\psi]\!]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}
$$

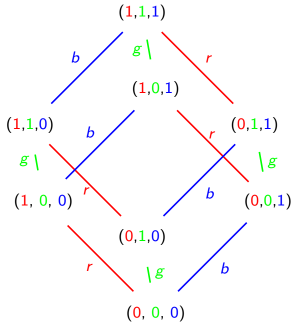
$$
R^+_a(v) = \begin{cases} R_a(v) \cap [\![\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_A(w) \cap [\![\psi]\!]_{\mathcal{M}} \\ R_a(v) \cap [\![\neg\psi]\!]_{\mathcal{M}} & \text{if } v \in R^C_A(w) \cap [\![\neg\psi]\!]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}
$$

Remark

- for every agent $a \in Ag$, R^-_a and R^+_a are equivalence relations
- $[\psi]^+_A$ and $[\psi]^-_A$ are interpreted as local (pointed model) transformers
- the difference between global and local announcements collapse whenever A is a singleton

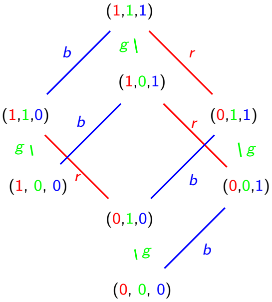
Examples: the Muddy Children Puzzle

The model $\mathcal{M}$ for 3 children (red, blue, and green), where no child knows whether she is muddy, can be represented as follows:



Examples: the Muddy Children Puzzle

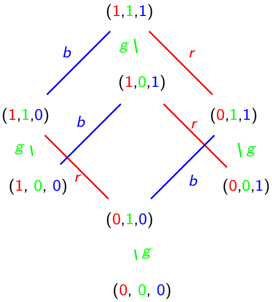
- Suppose that only red is muddy, i.e., the actual world is $(1, 0, 0)$
- then, the father **locally** announces to red, green, and blue that at least one child is muddy: $\alpha := m_r \vee m_b \vee m_g$
- the updated model $\mathcal{M}^{-}_{(100,\alpha,rgb)}$ is as follows:



- only the indistinguishability relation for red is updated
- now everybody knows that at least one child is muddy: $(\mathcal{M}, 100) \vDash [\alpha]^{-}_{rgb} E_{rgb}\alpha$
- the father's announcement does not make α common knowledge: $(\mathcal{M}, 100) \nvDash [\alpha]^{-}_{rgb} C_{rgb}\alpha$
- In general, for every world $s \neq 000$, $(\mathcal{M}, s) \nvDash [\alpha]^{-}_{rgb} C_{rgb}\alpha$

Examples: the Muddy Children Puzzle

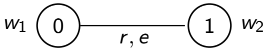
- Suppose that the father **globally** announces to red and blue that at least one child is muddy
- the updated model $\mathcal{M}^{+}_{(100,\alpha,rb)}$ is as follows:



- now the indistinguishability relations for both red and blue are updated and ...
  ...they acquire common knowledge that at least one child is muddy: $(\mathcal{M}, 100) \vDash [\alpha]^{+}_{rb} C_{rb}\alpha$
- but again the father's announcement is not enough to make α common knowledge amongst all children: $(\mathcal{M}, 100) \nvDash [\alpha]^{-}_{rb} C_{rgb}\alpha$

Examples: Communication Scenario

Consider communication between sender s and receiver r over a reliable channel that is listened to by eavesdropper e:



Examples: Communication Scenario

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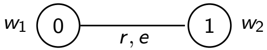
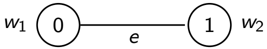


After s has communicated to r the value of the bit, we obtain the updated model $\mathcal{N}_{(w_1, bit=0, r)}$:

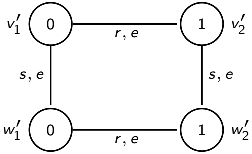


Hence, receiver r learns the value of the bit: $(\mathcal{N}, w_1) \models [bit = 0]_r K_r(bit = 0)$

On the other hand, eavesdropper e learns that r knows it: $(\mathcal{N}, w_1) \models [bit = 0]_r K_e Kw_r(bit = 0)$

Examples: Communication Scenario

Compare model $\mathcal{N}$ above with the following **bisimilar** model $\mathcal{N}'$,



Examples: Communication Scenario

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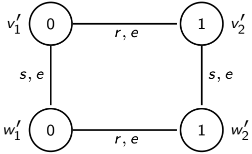
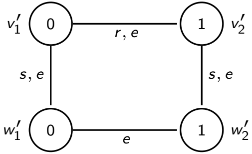


However, after communicating to r the value of the bit, the updated model $\mathcal{N}'_{(w_1', bit=0, r)}$ is not bisimilar to $\mathcal{N}_{(w_1, bit=0, r)}$:



In particular, in w_1' eavesdropper e does not learn that r knows the value of the bit: $(\mathcal{N}', w_1') \not\models [bit = 0]_r K_e Kw_r(bit = 0)$.

$\Rightarrow$ GLAL is not preserved under standard modal bisimulations.

Comparison with PAL

GLAL is at least as expressive as PAL:

Proposition

For all formulas ϕ, ψ in PAL, $(\mathcal{M}, w) \vDash [\phi]\psi$ iff $(\mathcal{M}, w) \vDash [\phi]^+_{Ag}\psi$.

By this result we can define a truth-preserving embedding τ from PAL to GLAL.

Proposition

For all formulas ϕ in PAL, $(\mathcal{M}, w) \vDash \phi$ iff $(\mathcal{M}, w) \vDash \tau(\phi)$.

Actually, by the example above,

Theorem

GLAL is strictly more expressive than PAL, and therefore than epistemic logic.

Comparison with Attentive Announcements

- **Attention-based Announcements** [BDH$^+$16]: agents process the new information only if they are paying attention.
- whether they pay attention is handled by a designated set of atoms.
- close relationship with GLAL: in $(\mathcal{N}', w_1')$ although r processes the new information, agent s is uncertain about this fact.
- consider adding an 'attention atom' h_r for receiver r such that h_r is true in w_1' and w_2' but false in v_1' and v_2'.
- then, the announcement of $bit = 0$ to r in $(\mathcal{N}', w_1')$ corresponds to the attention-based announcement wherein sender s is uncertain as to whether r is paying attention.

Differences:

- [BDH$^+$16] models truly private announcements [GG97] (equivalence relations **are not** preserved), whereas our proposal considers semi-private announcements that **do** preserve equivalence relations.
- Our announcements are not necessarily public.

Comparison with Semi-Private Announcements

- **Semi-Private Announcements** [GG97, vD00, vdHP06, BvDM08]: after announcing semi-privately ϕ to coalition A, all agents in A know ϕ, and the agents in $Ag \setminus A$ know that all agents in A know whether ϕ.

- In GLAL agents in $Ag \setminus A$ do not necessarily know that all agents in A know whether ϕ.

- Semi-private announcements can be modeled by refinement $\mathcal{M}^{sp}_{(w,\psi,A)}$ according to which $W^{sp} = W$, $V^{sp} = V$, and for $a \in A$,

$$R^{sp}_a(v) = \begin{cases} R_a(v) \cap [[\psi]]_{\mathcal{M}} & \text{if } v \in R^C_{Ag}(w) \cap [[\psi]]_{\mathcal{M}} \\ R_a(v) \cap [[\neg\psi]]_{\mathcal{M}} & \text{if } v \in R^C_{Ag}(w) \cap [[\neg\psi]]_{\mathcal{M}} \\ R_a(v) & \text{otherwise} \end{cases}$$

- The two frameworks are not directly comparable.

Validities

No complete axiomatisation, but some interesting validities.

- Truthfully announcing a propositional formula $\phi \in \mathcal{L}_{pl}$ entails the knowledge thereof:

$$\models [\phi]^-_A E_A \phi$$
$$\models [\phi]^+_A C_A \phi$$

- Differently w.r.t. PAL, announcements in GLAL cannot be rewritten as simpler formulas. Nonetheless, the following are validities in GLAL:

$$\begin{array}{rcl} [\phi]^-_A p & \leftrightarrow & \phi \to p \\ [\phi]^-_A \neg\psi & \leftrightarrow & \phi \to \neg[\phi]^-_A \psi \\ [\phi]^-_A (\psi \wedge \psi') & \leftrightarrow & [\phi]^-_A \psi \wedge [\phi]^-_A \psi' \end{array}$$

- Further, epistemic operators and nested announcements commute with announcement operators if they refer to the same coalition (but not in general):

$$\begin{array}{rcl} [\phi]^+_A C_A \psi & \leftrightarrow & \phi \to C_A [\phi]^+_A \psi \\ [\phi]^-_A E_A \psi & \leftrightarrow & \phi \to E_A [\phi]^-_A \psi \\ [\phi]^-_A [\phi']^-_A \psi & \leftrightarrow & \left[\phi \wedge [\phi]^-_A \phi'\right]^-_A \psi \\ [\phi]^+_A [\phi']^+_A \psi & \leftrightarrow & \left[\phi \wedge [\phi]^+_A \phi'\right]^+_A \psi \end{array}$$

- Operators $[\phi]^+_A$ and $[\phi]^-_A$ are normal modalities.
  None of schemes T, S4 and B hold.

A New Notion of Bisimulation

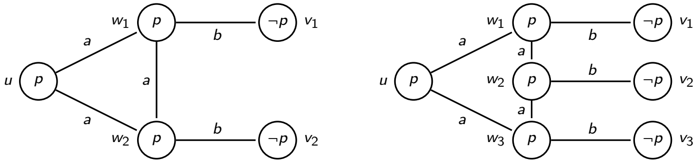
We remarked that GLAL is not preserved under modal bisimulation.

- define $R_A(w, v)$ as: $R_a(w, v)$ iff $a \in A$.

Definition (±-Simulation)

Given models $\mathcal{M}$ and $\mathcal{M}'$, a ±-**simulation** is a relation $\mathbf{S} \subseteq W \times W'$ such that $\mathbf{S}(w, w')$ implies

- **Atoms** $w \in V(p)$ iff $w' \in V'(p)$, for every $p \in AP$
- **Forth** for every $A \subseteq Ag$ and $v \in W$, if $R_A(w, v)$ then for some $v' \in W'$, $R'_A(w', v')$ and $\mathbf{S}(v, v')$
- **Reach** for every $v, v' \in W$, $a \in Ag$, if $\mathbf{S}(v, v')$ then $R_a(w, v)$ iff $R'_a(w', v')$



Theorem

If states s and s' are bisimilar, then for every formula ψ in GLAL, $(\mathcal{M}, s) \models \psi$ iff $(\mathcal{M}', s') \models \psi$.

Model Checking and Satisfiability

Definition (Model Checking and Satisfiability)

- **Model Checking Problem**: given a finite pointed model $(\mathcal{M}, w)$, and formula ϕ in GLAL, determine whether $(\mathcal{M}, w) \vDash \phi$.
- **Satisfiability Problem**: given a formula ϕ in GLAL, determine whether $(\mathcal{M}, w) \vDash \phi$ for some pointed model $(\mathcal{M}, w)$.

Theorem

The model checking problem for GLAL is PTIME-complete.

Model refinements can be computed in polynomial time.

Theorem

The satisfiability problem for GLAL is decidable.

Decision procedure inspired by tableaux for epistemic logic.

Conclusions

Contributions:

- GLAL: a logic for global and local announcements
- strictly more expressive than PAL
- alternative to action models to represent private announcements
- however, not preserved under standard modal bisimulation
- but we have a novel, truth-preserving notion of bisimulation
- the model checking problem is no harder than for epistemic logic
- the satisfiability problem is decidable.

Future Work:

- axiomatisation
- closer comparison with DEL
- more elaborate form of communication (asynchronous, FIFO, LIFO, etc.)
- real-life scenarios and applications

Questions?

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