Program Analysis (70020)

Overview

Herbert Wiklicky

Department of Computing
Imperial College London

herbert@doc.ic.ac.uk
h.wiklicky@imperial.ac.uk

Spring 2026

Lectures: 13 January until 26 February 2026

Lecture Theatre 144 on Tuesday (4pm-6pm)
and Thursday (2pm-4pm).

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Tutorials typically Tuesdays, first hour.

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Coursework Tests 27 January and 17 February

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Material and Notes on

https://www.doc.ic.ac.uk/∼herbert/teaching.html

Scientia, Panopto, etc.

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Examination: Week 11, 16–20 March 2026

Program Analysis

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Unfortunately, the achieving the aims of (static) program analysis tend to be computationally extremely hard.

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The approach is to find terminating algorithms for program analysis while not always finding a “meaningful” solution.

Fermat's Program – Terminates?

```
try ← true;
x ← 1;
while try do
    y ← 1;
    while y ≤ x && try do
        z ← 1;
        while z ≤ y && try do
            try ← x^3 + y^3 ≠ z^3
            z ← z + 1;
        end while
        y ← y + 1;
    end while
    x ← x + 1;
end while
```

Collatz Problem – Unknown

Take an integer x and compute a sequence of updates:

while $x \neq 1$ **do**
    if $x \bmod 2 = 0$ **then**
        $x \leftarrow x/2;$
    else
        $x \leftarrow 3 \times x + 1$
    end if
end while

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Take an integer x and compute a sequence of updates:

```
while x ≠ 1 do
    if x mod 2 = 0 then
        x ← x/2;
    else
        x ← 3 × x + 1
    end if
end while
```

Currently it is unknown whether this terminates for all x.

Techniques

Some techniques used in program analysis include:

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A First Example

Consider the following fragment in *some* procedural language.

$m \leftarrow 2$;
while $n > 1$ **do**
    $m \leftarrow m \times n$;
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end while
stop

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end while
stop

$[m \leftarrow 2]^1$;
while $[n > 1]^2$ **do**
    $[m \leftarrow m \times n]^3$;
    $[n \leftarrow n - 1]^4$
end while
$[\textbf{stop}]^5$

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end while
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We annotate a program such that it becomes clear about what program point we are talking about.

A Parity Analysis

Claim: This program fragment always returns an **even** m, idependently of the initial values of m and n.

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A program analysis, so-called parity analysis, can determine this by propagating the even/odd or *parity* information *forwards* form the start of the program.

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- **odd** — the value is known to be odd
- **unknown** — the parity of the value is unknown

For both variables m and n we record its parity at each stage of the computation (beginning of each statement).

A First Example

Executing the program with *abstract* values, parity, for m and n.

$m \leftarrow 2$;
while $n > 1$ **do**
    $m \leftarrow m \times n$;
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A First Example

Executing the program with *abstract* values, parity, for m and n.

$m \leftarrow 2$; ▷ unknown(m) – unknown(n)
while $n > 1$ **do**
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m ← 2;                    ▷ unknown(m) – unknown(n)
while n > 1 do            ▷ even(m) – unknown(n)
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Important: We can restart the loop!

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The first program computes 2 times the factorial for any positive value of n. Replacing ‘2’ by ‘1’ in the first statement gives:

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m ← 1;                          ▷ unknown(m) – unknown(n)
while n > 1 do                  ▷ odd(m) – unknown(n)
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stop                        ▷ unknown(m) – unknown(n)
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i.e. the factorial – but then the program analysis is unable to tell us anything about the parity of m at the end of the execution.

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- m could be **odd** — if the input $n \leq 1$.

However, even if we fix/require the input to be positive and **even** — e.g. by some suitable conditional assignment — the program analysis still might not be able to accurately predict that m will be **even** at statement **5**.

Safe Approximations

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- yes means *definitely* yes,
- no means *maybe* no.

Data Flow Analysis

The starting point for data flow analysis is a representation of the control flow graph of the program: the nodes of such a graph may represent individual statements – as in a flowchart – or sequences of statements; arcs specify how control may be passed during program execution.

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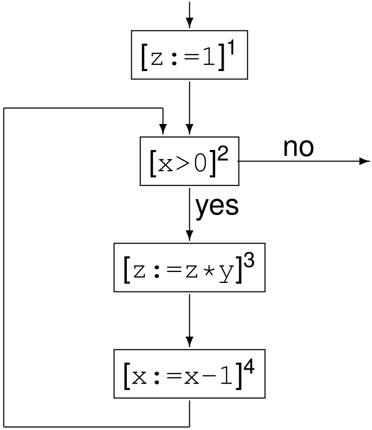
When the control flow graph is not explicitly given, we need a preliminary control flow analysis

Control Flow Information



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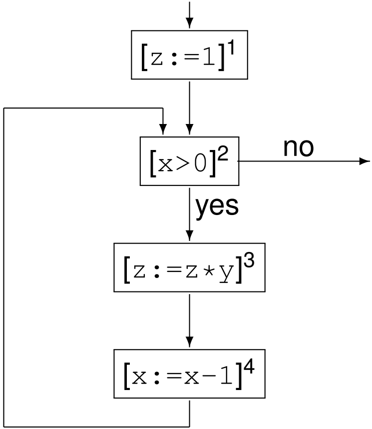

This allows us to determine the predecessors *pred* and successors *succ* of each statement, e.g. $pred(2) = \{1, 4\}$.

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$$\mathrm{RD}_{entry}(p) = \begin{cases} \mathrm{RD}_{init} & \text{if } p \text{ is initial} \\ \bigcup_{p' \in pred(p)} \mathrm{RD}_{exit}(p') & \text{otherwise} \end{cases}$$

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$$\mathrm{RD}_{exit}(p) = (\mathrm{RD}_{entry}(p) \backslash kill_{\mathrm{RD}}(p)) \cup gen_{\mathrm{RD}}(p)$$

Analysis Information

At each program point some definitions get “killed” (those which define the same variable as at the program point) while others are “generated”.

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Reaching Definitions is a forward analysis and we require the least (most precise) solutions to the set of equations.

Equations & Solutions

For our initial program fragment

$[m \leftarrow 2]^1$;
while $[n > 1]^2$ **do**
    $[m \leftarrow m \times n]^3$;
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end while
$[\textbf{stop}]^5$

Equations & Solutions

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some of the *RD* equations we get are:

$$\begin{aligned} \text{RD}_{entry}(1) &= \{(m, ?), (n, ?)\} \\ \text{RD}_{entry}(2) &= \text{RD}_{exit}(1) \cup \text{RD}_{exit}(4) \end{aligned}$$

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	RD_{entry}	RD_{exit}
1	$\{(m,?),(n,?)\}$	$\{(m,1),(n,?)\}$
2	$\{(m,1),(m,3),(n,?),(n,4)\}$	$\{(m,1),(m,3),(n,?),(n,4)\}$
3	$\{(m,1),(m,3),(n,?),(n,4)\}$	$\{(m,3),(n,?),(n,4)\}$
4	$\{(m,3),(n,?),(n,4)\}$	$\{(m,3),(n,4)\}$
5	$\{(m,1),(m,3),(n,?),(n,4)\}$	$\{(m,1),(m,3),(n,?),(n,4)\}$

Solving Equations

How can we construct solution to the data flow equations?
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OUTPUT: Reaching Definitions *RD*.

Solving Equations

How can we construct solution to the data flow equations?
Answer: Iteratively, by improving approximations/guesses.

INPUT: Control Flow Graph
i.e. initial(p), pred(p).

OUTPUT: Reaching Definitions *RD*.

METHOD: Step 1: Initialisation
Step 2: Iteration

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- *Shape Analyis — Pointer Analysis — etc.*

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- Replace the use of a variable in some expression by a constant if it is known that the value of that variable will always be a constant.
- Simplify an expression by partially evaluating it: subexpressions that contain no variables can be evaluated.

Constant Folding I

$$RD \vdash [\, x := a \,]^{\ell} \;\; \triangleright \;\; [\, x := a[y \mapsto n] \,]^{\ell}$$

$$\text{if} \begin{cases} y \in FV(a) \;\; \wedge \;\; (y, ?) \notin \mathrm{RD}_{entry}(\ell) \;\wedge \\ \forall (y', \ell') \in \mathrm{RD}_{entry}(\ell) : \\ y' = y \;\; \Rightarrow \;\; [\ldots]^{\ell'} = [\, y := n \,]^{\ell'} \end{cases}$$

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$$RD \vdash [\ x := a\ ]^{\ell} \ \triangleright\ [\ x := n\ ]^{\ell}$$

$$\text{if} \begin{cases} FV(a) = \emptyset \ \land \ a \text{ is not constant} \ \land \\ a \text{ evaluates to } n \end{cases}$$

Constant Folding II

$$\frac{RD \vdash S_1 \ \textcolor{red}{\triangleright} \ S_1'}{RD \vdash S_1 ; S_2 \ \textcolor{red}{\triangleright} \ S_1' ; S_2}$$

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$$\frac{RD \vdash S_1 \;\; \triangleright \;\; S_1'}{RD \vdash S_1 ; S_2 \;\; \triangleright \;\; S_1' ; S_2}$$

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$$\frac{RD \vdash S \ \rhd\ S'}{RD \vdash \textbf{while } [b]^\ell \textbf{ do } S \ \rhd\ \textbf{while } [b]^\ell \textbf{ do } S'}$$

An Example

To illustrate the use of the transformation consider:

$$[\, x := 10 \,]^1;\ [\, y := x + 10 \,]^2;\ [\, z := y + 10 \,]^3$$

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The (least) solution to the Reaching Definition analysis is:

$$\begin{aligned}
\mathrm{RD}_{entry}(1) &= \{(x, ?), (y, ?), (z, ?)\} \\
\mathrm{RD}_{exit}(1) &= \{(x, 1), (y, ?), (z, ?)\} \\
\mathrm{RD}_{entry}(2) &= \{(x, 1), (y, ?), (z, ?)\} \\
\mathrm{RD}_{exit}(2) &= \{(x, 1), (y, 2), (z, ?)\} \\
\mathrm{RD}_{entry}(3) &= \{(x, 1), (y, 2), (z, ?)\} \\
\mathrm{RD}_{exit}(3) &= \{(x, 1), (y, 2), (z, 3)\}
\end{aligned}$$

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We have for example the following:

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and therfore the rules for sequential composition allow us to do the following transformation:

$$\begin{aligned} RD \vdash \ & [\, x := 10 \,]^1; [\, y := x + 10 \,]^2; [\, z := y + 10 \,]^3 \triangleright \\ & [\, x := 10 \,]^1; [\, y := 10 + 10 \,]^2; [\, z := y + 10 \,]^3 \end{aligned}$$

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after which no more steps are possible.

Additional Issues

The above example shows that optimisation is in general the result of a number of successive transformations.

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$$RD \vdash S_1 \;\triangleright\; S_2 \;\triangleright\; \ldots \;\triangleright\; S_n.$$

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It could also be the case that different sequences of transformations either lead to different end results or are of very different length.

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$$\mathrm{RD}_{entry}(2) \quad = \quad \mathrm{RD}_{exit}(1) \cap \mathrm{RD}_{exit}(4)$$

instead of $\mathrm{RD}_{entry}(2) = \mathrm{RD}_{exit}(1) \cup \mathrm{RD}_{exit}(4)$.

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It requires formal (mathematical) proof whether an **analysis** (or **program transformation**) is correct with respect to some model of execution or semantics.

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This course will mostely be concerned with intutive or light-weight semantics when it comes to the “meaning” of a program and the correctness of a program analysis.

Modelling and Specification

Architecture and Structural Engineering

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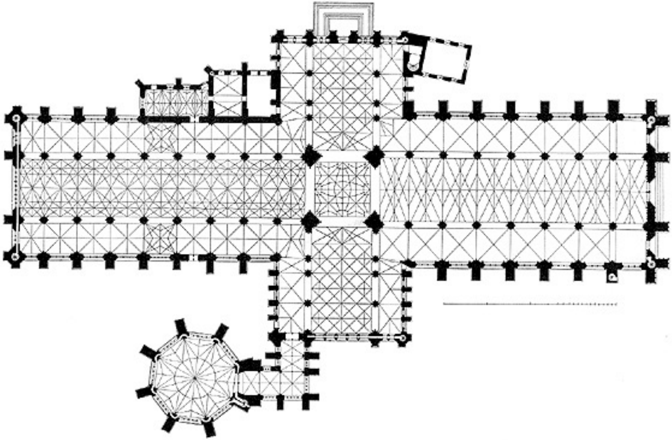

Figure: York Minster

Topics Covered – Executive Summary

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