

Program Analysis (70020)

Overview

Herbert Wiklicky

Department of Computing
Imperial College London

`herbert@doc.ic.ac.uk`
`h.wiklicky@imperial.ac.uk`

Autumn 2022

Lectures 11 October and 24 November 2022

Lecture Theatre 145 on Tuesday (2pm-4pm)
and Thursday (9am-11am).

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Material and Notes on

<https://www.doc.ic.ac.uk/~herbert/teaching.html>

Scientia, Panopto (COMP97146/97128), etc.

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Examination: Week 11, **12-16 December**

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Unfortunately, the achieving the aims of (static) program analysis tend to be computationally extremely hard.

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The approach is to find terminating algorithms for program analysis while not always finding a “meaningful” solution.

Fermat's Program – Terminates

```
1: try ← true;
2: x ← 1;
3: while try do
4:   y ← 1;
5:   while  $y \leq x$  && try do
6:     z ← 1;
7:     while  $z \leq y$  && try do
8:       try ←  $x^3 + y^3 \neq z^3$ 
9:       z ← z + 1;
10:    end while
11:    y ← y + 1;
12:  end while
13:  x ← x + 1;
14: end while
```

Collatz Problem – Unknown

Take an integer x and compute a sequence of updates:

```
1: while  $x \neq 1$  do  
2:   if  $x \bmod 2 = 0$  then  
3:      $x \leftarrow x/2$ ;  
4:   else  
5:      $x \leftarrow 3 \times x + 1$   
6:   end if  
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7: end while
```

Currently it is unknown whether this terminates for **all** x .

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A First Example

Consider the following fragment in *some* procedural language.

```
1:  $m \leftarrow 2$ ;  
2: while  $n > 1$  do  
3:    $m \leftarrow m \times n$ ;  
4:    $n \leftarrow n - 1$   
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```

```
 $[m \leftarrow 2]^1$ ;  
while  $[n > 1]^2$  do  
   $[m \leftarrow m \times n]^3$ ;  
   $[n \leftarrow n - 1]^4$   
end while  
 $[\text{stop}]^5$ 
```

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Consider the following fragment in *some* procedural language.

1: $m \leftarrow 2$;	$[m \leftarrow 2]^1$;
2: while $n > 1$ do	while $[n > 1]^2$ do
3: $m \leftarrow m \times n$;	$[m \leftarrow m \times n]^3$;
4: $n \leftarrow n - 1$	$[n \leftarrow n - 1]^4$
5: end while	end while
6: stop	$[\mathbf{stop}]^5$

We annotate a program such that it becomes clear about what **program point** we are talking about.

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Claim: This program fragment always returns an **even** m , independently of the initial values of m and n .

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We can **statically** determine that in any circumstances the value of m at the last statement will be **even** for any input n .

A **program analysis**, so-called parity analysis, can determine this by propagating the even/odd or *parity* information *forwards* from the start of the program.

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For both variables **m** and **n** we record its parity at each stage of the computation (beginning of each statement).

A First Example

Executing the program with *abstract* values, parity, for m and n .

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▷ unknown(m) – unknown(n)

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Important: We can restart the loop!

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The first program computes 2 times the factorial for any positive value of n . Replacing '2' by '1' in the first statement gives:

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i.e. the factorial – but then the program analysis is unable to tell us anything about the parity of m at the end of the execution.

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- ▶ m could be **odd** — if the input $n \leq 1$.

However, even if we fix/require the input to be positive and **even** — e.g. by some suitable conditional assignment — the program analysis still might not be able to accurately predict that m will be **even** at statement **5**.

Safe Approximations

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- ▶ **yes** means *definitely* yes,
- ▶ **no** means *maybe* no.

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The starting point for **data flow analysis** is a representation of the control flow graph of the program: the nodes of such a graph may represent individual statements – as in a flowchart – or sequences of statements; arcs specify how control may be passed during program execution.

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The data flow analysis is usually specified as a set of **equations** which associate analysis information with program points which correspond to the nodes in the control flow graph. This information may be propagated *forwards* through the program (e.g. parity analysis) or *backwards*.

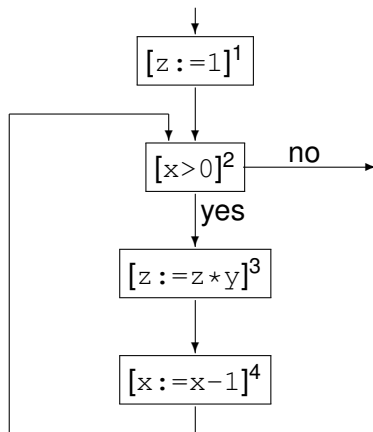
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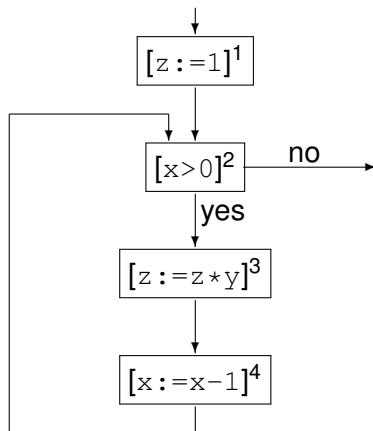
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When the control flow graph is not explicitly given, we need a preliminary **control flow analysis**

Control Flow Information



Control Flow Information



This allows us to determine the predecessors *pred* and successors *succ* of each statement, e.g. $pred(2) = \{1, 4\}$.

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$$RD_{exit}(p) = (RD_{entry}(p) \setminus kill_{RD}(p)) \cup gen_{RD}(p)$$

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At each program point some definitions get “killed” (those which define the same variable as at the program point) while others are “generated”.

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A suitable representation for properties are sets of pairs, where each pair contains a variable x and a program point p : the meaning of the pair (x, p) is that the assignment to x at point p is the current one.

Analysis Information

At each program point some definitions get “killed” (those which define the same variable as at the program point) while others are “generated”.

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Reaching Definitions is a forward analysis and we require the least (most precise) solutions to the set of equations.

Equations & Solutions

For our initial program fragment

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[ $m \leftarrow 2$ ]1;  
while [ $n > 1$ ]2 do  
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some of the *RD* equations we get are:

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	RD_{entry}	RD_{exit}
1	$\{(m, ?), (n, ?)\}$	$\{(m, 1), (n, ?)\}$
2	$\{(m, 1), (m, 3), (n, ?), (n, 4)\}$	$\{(m, 1), (m, 3), (n, ?), (n, 4)\}$
3	$\{(m, 1), (m, 3), (n, ?), (n, 4)\}$	$\{(m, 3), (n, ?), (n, 4)\}$
4	$\{(m, 3), (n, ?), (n, 4)\}$	$\{(m, 3), (n, 4)\}$
5	$\{(m, 1), (m, 3), (n, ?), (n, 4)\}$	$\{(m, 1), (m, 3), (n, ?), (n, 4)\}$

Solving Equations

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METHOD: Step 1: Initialisation
Step 2: Iteration

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- ▶ *Shape Analysis — Pointer Analysis — etc.*

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To illustrate the ideas we shall show how Reaching Definitions can be used to perform Constant Folding.

There are two ingredients to this:

- ▶ Replace the use of a variable in some expression by a constant if it is known that the value of that variable will always be a constant.
- ▶ Simplify an expression by partially evaluating it: subexpressions that contain no variables can be evaluated.

Constant Folding I

$$RD \vdash [x := a]^\ell \triangleright [x := a[y \mapsto n]]^\ell$$
$$\text{if } \left\{ \begin{array}{l} y \in FV(a) \wedge (y, ?) \notin RD_{\text{entry}}(\ell) \wedge \\ \forall (y', \ell') \in RD_{\text{entry}}(\ell) : \\ y' = y \Rightarrow [\dots]^{\ell'} = [y := n]^{\ell'} \end{array} \right.$$

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$$RD \vdash [x := a]^\ell \triangleright [x := n]^\ell$$
$$\text{if } \left\{ \begin{array}{l} FV(a) = \emptyset \wedge a \text{ is not constant} \wedge \\ a \text{ evaluates to } n \end{array} \right.$$

Constant Folding II

$$\frac{RD \vdash S_1 \triangleright S'_1}{RD \vdash S_1; S_2 \triangleright S'_1; S_2}$$

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An Example

To illustrate the use of the transformation consider:

$$[x := 10]^1; [y := x + 10]^2; [z := y + 10]^3$$

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The (least) solution to the Reaching Definition analysis is:

$$RD_{entry}(1) = \{(x, ?), (y, ?), (z, ?)\}$$

$$RD_{exit}(1) = \{(x, 1), (y, ?), (z, ?)\}$$

$$RD_{entry}(2) = \{(x, 1), (y, ?), (z, ?)\}$$

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and therefore the rules for sequential composition allow us to do the following transformation:

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after which no more steps are possible.

Additional Issues

The above example shows that optimisation is in general the result of a number of successive transformations.

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This could be costly because once S_1 has been transformed into S_2 we might have to *re-compute* the Reaching Definition analysis before the next transformation step can be done.

It could also be the case that different sequences of transformations either lead to different end results or are of very different length.

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It requires formal (mathematical) proof whether an **analysis** (or **program transformation**) is **correct** with respect to some model of execution or semantics.

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This course will mostly be concerned with intuitive or light-weight semantics when it comes to the “meaning” of a program and the correctness of a program analysis.

Topics Covered – Executive Summary

- ▶ Data Flow Analysis

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- ▶ Monotone Frameworks

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