Live Variable Analysis

A variable is *live* at the exit from a label if there exists a path from the label to a use of the variable that does not re-define the variable. The *Live Variables Analysis* will determine:

*For each program point, which variables may be live at the exit from the point.*

This analysis might be used as the basis for *Dead Code Elimination*. If the variable is not live at the exit from a label then, if the elementary block is an assignment to the variable, the elementary block can be eliminated.
A variable has even or odd parity at a label if we can guarantee that its value is even (e) or odd (o) for any execution of this label (not necessarily the same actual value). The Parity Analysis will determine:

For each program point, what is the parity of each variable.

This analysis might be used as the basis for ... (saving a bit?).

LV Analysis: Property Space

\[ \text{kill}_{LV} : \text{Block} \rightarrow \mathcal{P}(\text{Var}) \]

\[ \text{gen}_{LV} : \text{Block} \rightarrow \mathcal{P}(\text{Var}) \]

\[ \text{LV}_{entry} : \text{Lab} \rightarrow \mathcal{P}(\text{Var}) \]

\[ \text{LV}_{exit} : \text{Lab} \rightarrow \mathcal{P}(\text{Var}) \]

Important fact: Information we are interested in is in \( \mathcal{P}(\text{Var}) \).
**LV Equations and Transfer Functions**

\[ \text{LV}_{\text{exit}}(\ell) = \begin{cases} \emptyset, & \text{if } \ell \in \text{final}(S) \\ \bigcup \{ \text{LV}_{\text{entry}}(\ell') \mid (\ell', \ell) \in \text{flow}^R(S) \}, & \text{otherwise} \end{cases} \]

\[ \text{LV}_{\text{entry}}(\ell) = (\text{LV}_{\text{exit}}(\ell) \setminus \text{kill}_{\text{LV}}([B]^{\ell}) \cup \text{gen}_{\text{LV}}([B]^{\ell})) \]

where \([B]^{\ell} \in \text{blocks}(S)\)

with

\[ \begin{align*}
\text{kill}_{\text{LV}}([x := a]^{\ell}) &= \{x\} \\
\text{kill}_{\text{LV}}([\text{skip}]^{\ell}) &= \emptyset \\
\text{kill}_{\text{LV}}([b]^{\ell}) &= \emptyset \\
\text{gen}_{\text{LV}}([x := a]^{\ell}) &= \text{FV}(a) \\
\text{gen}_{\text{LV}}([\text{skip}]^{\ell}) &= \emptyset \\
\text{gen}_{\text{LV}}([b]^{\ell}) &= \text{FV}(b)
\end{align*} \]

**Parity Information**

The \(LV\) Analysis associates to lables/blocks some information, concretely the set of live variables, i.e. a set in \(P(\text{Var}_*)\).

This is modified by local *transfer functions* at each block (using \(\text{kill}_{\text{LV}}\) and \(\text{gen}_{\text{LV}}\)) and *collected* globally according to \(\text{flow}\).

For Parity we have identify the abstract properties to work with.

- Sets in \(P(\text{Var}_* \times \{e, o\})\) or maybe \(P(\text{Var}_* \times \{e, o, ?\})\), e.g. \((x, e), (x, o), (y, e) \equiv ((x, ?), (y, e))\).
- Functions in \(\text{Var}_* \rightarrow \{e, o\}\) or better \(\text{Var}_* \rightarrow \{e, o, ?\}\). e.g. \(x \mapsto ?, y \mapsto e\).

- represented as value tables, e.g. \(\begin{array}{c|c|c}
  x & y \\
  \hline
  ? & e
\end{array}\)

Questions: How to modify parity information locally and how to combine it, e.g. maybe \(\{(x, e), (x, o), (y, e)\} \cup \{(x, e), (y, e)\}\).