Quantum Computation (CO484)
Quantum Cryptography with No Cloning

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Cloning of Qubits?

Is it possible to create a second copy of a general qubit $|\psi\rangle$ using a unitary operation $U$. 

**Theorem (No Cloning Theorem)**

There exists no unitary transformation $U$ such that $U |\psi\rangle |0\rangle = |\psi\rangle |\psi\rangle$ for all qubits $|\psi\rangle \in \mathbb{C}^2$. 
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*The exists no unitary transformation $U$ such that*

$$U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle$$

*for all qubits $|\psi\rangle \in \mathbb{C}^2$.***
Argument

Consider two qubits $|\psi\rangle$ and $|\phi\rangle$. Then by linearity:

$$U(\alpha |\psi\rangle + \beta |\phi\rangle) |0\rangle = \alpha U(|\psi\rangle) |0\rangle + \beta U(|\phi\rangle) |0\rangle$$

but also if $U$ is a cloning operator:

$$U(\alpha |\psi\rangle + \beta |\phi\rangle) |0\rangle = (\alpha |\psi\rangle + \beta |\phi\rangle)(\alpha |\psi\rangle + \beta |\phi\rangle)$$

Only for $\alpha = 0$ or $\beta = 0$ we have

$$\alpha |\psi\rangle|\psi\rangle + \beta |\phi\rangle|\phi\rangle = \alpha^2 |\psi\rangle|\psi\rangle + \beta^2 |\phi\rangle|\phi\rangle + \alpha\beta |\psi\rangle|\phi\rangle + \alpha\beta |\phi\rangle|\psi\rangle$$
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\[
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+ \alpha \beta |\psi\rangle |\phi\rangle + \alpha \beta |\phi\rangle |\psi\rangle
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Approximate Cloning?

Is it not even possible to \textit{approximately} clone a qubit.
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Is it not even possible to **approximately** clone a qubit.

Consider **two** qubits $|\psi\rangle$ and $|\phi\rangle$ with $0 < \langle \psi | \phi \rangle < 1$ such that

$$
U(|\psi\rangle \otimes |0\rangle) \approx |\psi\rangle \otimes |\psi\rangle \quad \text{and} \quad U(|\phi\rangle \otimes |0\rangle) \approx |\phi\rangle \otimes |\phi\rangle
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By unitarity – $U$ preserving inner products – we get

$$\langle \psi | 0 \rangle \langle 0 | 0 \rangle = \langle \psi | \phi \rangle \approx \langle \psi | \psi \rangle$$
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By unitarity – $U$ preserving inner products – we get

$$(|\psi\rangle |0\rangle) \dagger (|\phi\rangle |0\rangle) = \langle \psi | \phi \rangle \langle 0 | 0 \rangle = \langle \psi | \phi \rangle \approx \langle \psi | \phi \rangle^2$$

Thus $\langle \psi | \phi \rangle \approx 0$ or $\langle \psi | \phi \rangle \approx 1$. 
Communication on Insecure Channels

\[
\text{ENC}(T, K_A) = M = \text{DEC}(M, K_B) = T = \text{DEC}\left(\text{ENC}(T, K_A), K_B\right) = T
\]
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Alice \rightarrow Bob
Communication on Insecure Channels

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Eve
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\end{align*}
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One-Time-Pad or Vernam Cipher

Gilbert Sandford Vernam, 1917

**Step 0.** Alice and Bob share a common, random key $K$. 
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Step 1. Alice calculates $M = T \oplus K$. 

$K = K_A = K_B$

$\text{ENC}(T, K) = \text{DEC}(T, K) = T \oplus K$.

Caveat: Never ever reuse random key $K$!
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\[ T \oplus 1 1 1 0 1 0 = K \]

\[ K \oplus 1 1 1 0 1 0 = T \]
Example

\[
\begin{array}{c}
T & 0 & 1 & 1 & 0 & 1 & 1 \\
K & \oplus & 1 & 1 & 1 & 0 & 1 & 0 \\
\end{array}
\]
Example

\[
\begin{array}{cccccc}
T & 0 & 1 & 1 & 0 & 1 & 1 \\
K & \oplus & 1 & 1 & 1 & 0 & 1 & 0 \\
\hline
M & 1 & 0 & 0 & 0 & 0 & 1
\end{array}
\]

\[
\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow
\]

\[
\begin{array}{cccccc}
M & 1 & 0 & 0 & 0 & 0 & 1
\end{array}
\]
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\[ \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \]

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Quantum Key Distribution

The **problem** with One-Time-Pads is **Key Distribution**.
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Quantum Key Distribution aims to exploit quantum features in order to protect the keys, utilising:

- No-Cloning. The message cannot be duplicated.
- Measurement. Observing the message changes it.

These quantum techniques aim to address two security aims:

- **Authentication.** Is sender really Alice?
- **Intrusion Detection.** Is Eve eavesdropping?
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BB84
Charles Bennett and Gilles Brassard 1984

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\[
\leftrightarrow = \{ |\downarrow\rangle, |\leftrightarrow\rangle \} = \{ (1, 0)^T, (0, 1)^T \}\]

\[
\otimes = \{ |\nwarrow\rangle, |\nearrow\rangle \} = \{ \frac{1}{\sqrt{2}} (-1, 1)^T, \frac{1}{\sqrt{2}} (1, 1)^T \}\]
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\end{align*}
\]

Interpretation of messages in both basis

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\leftrightarrow$</th>
<th>$\star$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$</td>
<td>\leftrightarrow\rangle$</td>
</tr>
<tr>
<td>1</td>
<td>$</td>
<td>\downarrow\rangle$</td>
</tr>
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Measuring in Wrong Base

As long as Alice and Bob send and receive qubits in the same basis, Bob will always measure the same qubit Alice has sent.
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This is due to the following obvious facts that:

$$
|\leftarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\leftrightarrow\rangle) \\
|\leftrightarrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\leftrightarrow\rangle)
$$

$$
|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\leftarrow\rangle + |\leftrightarrow\rangle) \\
|\downarrow\rangle = \frac{1}{\sqrt{2}}(|\leftarrow\rangle - |\leftrightarrow\rangle)
$$
BB84 Protocol

Step 1.a Alice chooses $n$ random bits to send (e.g. to be used as One-Time-Pad).
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Step 1.b Alice randomly chooses \( n \) times whether to use \( \leftrightarrow \) or \( \rightarrow \) to encode each bit.

Step 2.a Alice encodes the bits accordingly in the bases and sends the qubits to Bob.

Step 2.b Bob randomly chooses \( n \) times whether to use \( \leftrightarrow \) or \( \rightarrow \) to measure the qubits he got and measures them.

Step 3. Over the classical channel Alice and Bob compare which basis they used for each bit. If they agree they keep it otherwise they drop it.

Step 4.a Bob choose a part (e.g. half) of the transmitted bits (drops them) and compares them openly with Alice.

Step 4.b If these test bits do not agree (subject to transmission errors) Alice and Bob conclude that Eve was eavesdropping and abandon transmission.
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Example

\[ K_A \mid 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \]
Example

\[
\begin{array}{c|cccccccccccc}
K_A & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
B_A & \uparrow \downarrow & \uparrow \downarrow & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \uparrow \downarrow & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \boldsymbol{\uparrow} \boldsymbol{\downarrow} & \uparrow \downarrow \\
\end{array}
\]
### Example

<table>
<thead>
<tr>
<th>$K_A$</th>
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<th>0</th>
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<tbody>
<tr>
<td>$B_A$</td>
<td>←</td>
<td>→</td>
<td>←</td>
<td>→</td>
<td>←</td>
<td>→</td>
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<td>→</td>
<td>←</td>
<td>→</td>
<td>←</td>
<td>→</td>
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</tr>
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</table>

The diagram illustrates the relationship between $K_A$ and $B_A$ with arrows indicating transitions. The table shows the binary values for $K_A$ and $B_A$.
Example

\[
\begin{array}{cccccccccccccc}
K_A & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
B_A & \uparrow & \uparrow & \times & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\
& \leftrightarrow & \uparrow & \leftrightarrow & \uparrow & \leftrightarrow & \uparrow & \leftrightarrow & \uparrow & \leftrightarrow & \uparrow & \leftrightarrow & \uparrow & \leftrightarrow \\
B_B & \times & \uparrow & \times & \times & \uparrow & \times & \times & \uparrow & \times & \times & \uparrow & \times & \times \\
\end{array}
\]
## Example

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The idea is to use a \textbf{non-orthogonal} basis to encode 0 and 1, e.g.

\[ B = \{|\leftrightarrow\rangle, |\swarrow\rangle\} = \{(1, 0)^T, \frac{1}{\sqrt{2}}(1, 1)^T\} \]
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\[ B = \{ |\leftrightarrow\rangle, |\nearrow\rangle \} = \{ (1, 0)^T, \frac{1}{\sqrt{2}} (1, 1)^T \} \]

**Step 1.** Alice chooses \( n \) random bits and encodes them, e.g. 0 \( \equiv |\leftrightarrow\rangle \) and 1 \( \equiv |\nearrow\rangle \) and send these qubits to Bob.
Charles Bennett 1992

The idea is to use a **non-orthogonal** basis to encode 0 and 1, e.g.

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B = \{|\leftrightarrow\rangle, |\swarrow\rangle\} = \{(1, 0)^T, \frac{1}{\sqrt{2}}(1, 1)^T\}
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**Step 1.** Alice chooses \(n\) random bits and encodes them, e.g. \(0 \equiv |\leftrightarrow\rangle\) and \(1 \equiv |\swarrow\rangle\) and send these qubits to Bob.

**Step 2.** Bob measures these qubits in randomly chosen base \(\leftrightarrow\) or \(\swarrow\).
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**Step 1.** Alice chooses \( n \) random bits and encodes them, e.g. 0 \( \equiv |\leftrightarrow\rangle \) and 1 \( \equiv |\uparrow\downarrow\rangle \) and send these qubits to Bob.

**Step 2.** Bob measures these qubits in randomly chosen base \( \leftrightarrow \) or \( \uparrow\downarrow \).

**Step 3.** Bob tells Alice over an open classical which qubits he considers ambiguous in order to drop them.
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B = \{ |\leftrightarrow\rangle, |\swarrow\rangle \} = \{(1, 0)^T, \frac{1}{\sqrt{2}} (1, 1)^T\}
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**Step 1.** Alice chooses \( n \) random bits and encodes them, e.g. 0 \( \equiv |\leftrightarrow\rangle \) and 1 \( \equiv |\swarrow\rangle \) and send these qubits to Bob.

**Step 2.** Bob measures these qubits in randomly chosen base \( |\leftrightarrow\rangle \) or \( |\swarrow\rangle \).

**Step 3.** Bob tells Alice over an open classical which qubits he considers **ambiguous** in order to drop them.

Again – as in BB84 – some bits can be sacrificed to see if an extensive number of “transmission errors” indicates that Eve was eavesdropping and abandon transmission.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\uparrow\downarrow$. If Bob observes $\uparrow$, Bob knows that Alice sent 1 $\equiv |1\rangle$. If Bob observes $\downarrow$, Bob drops this bit. In the average three quarters of the qubits have to be discarded.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\leftrightarrow$. If Bob observes $\uparrow\downarrow$ Bob knows that Alice sent 1 $\equiv |\uparrow\rangle$.

$\uparrow\downarrow$ Bob drops this bit.

Using $\leftrightarrow$. If Bob observes $|\uparrow\rangle$ Bob knows that Alice sent 0 $\equiv |\downarrow\rangle$.

$|\uparrow\rangle$ Bob drops this bit.

In the average three quarters of the qubits have to be discarded.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\leftrightarrow$. If Bob observes $\vert\leftrightarrow\rangle$ Bob knows that Alice sent 1 $\equiv \vert\checkmark\rangle$. $\vert\leftrightarrow\rangle$ Bob drops this bit.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\leftarrow\rightarrow$. If Bob observes

$|\uparrow\rangle$ Bob knows that Alice sent 1 \(\equiv |✓\rangle\).

$|\leftrightarrow\rangle$ Bob drops this bit.

Using $\leftarrow\rightarrow\leftarrow\rightarrow$. If Bob observes
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\uparrow\downarrow$. If Bob observes

$|\uparrow\downarrow\rangle$ Bob knows that Alice sent 1 $\equiv |\check\uparrow\rangle$.

$|\leftrightarrow\rangle$ Bob drops this bit.

Using $\leftrightarrow$. If Bob observes

$|\leftrightarrow\rangle$ Bob knows that Alice sent 0 $\equiv |\leftrightarrow\rangle$.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\leftrightarrow$. If Bob observes

$|\uparrow\rangle$ Bob knows that Alice sent 1 $\equiv |\nearrow\rangle$.  
$|\leftrightarrow\rangle$ Bob drops this bit.

Using $\swarrow$. If Bob observes

$|\swarrow\rangle$ Bob knows that Alice sent 0 $\equiv |\leftarrow\rangle$.  
$|\swarrow\rangle$ Bob drops this bit.
Ambiguous Bits

When Bob measures the qubits received from Alice he will conclude that certain observations are inconclusive.

Using $\uparrow\downarrow$. If Bob observes

$|\uparrow\downarrow\rangle$ Bob knows that Alice sent 1 $\equiv |\checkmark\rangle$.

$|\leftrightarrow\rangle$ Bob drops this bit.

Using $\leftrightarrow$. If Bob observes

$|\leftrightarrow\rangle$ Bob knows that Alice sent 0 $\equiv |\leftrightarrow\rangle$.

$|\downarrow\leftrightarrow\rangle$ Bob drops this bit.

In the average three quarters of the qubits have to be discarded.
Example

| $K_A$ | 0 0 1 0 1 0 1 0 1 0 1 1 1 1 0 |

$\sqrt{\sqrt{\sqrt{\sqrt{K_0}}}}$
Example

\[ K_A \] 

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Example

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### Example

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Artur Ekert 1991

The idea is to distribute a key $K$ via pairs of entangled states, for example the **Bell states**:

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
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This protocol is inspired by the Einstein-Podolsky-Rosen (EPR, 1935) Gedanken-Experiment.
EPR Protocol

Step 1. A random sequence of entangled 2-qubit states – e.g. \( \sqrt{2} (|00\rangle + |11\rangle) \) – is created. For each such state one of the qubits is given to Alice and Bob, respectively.

Step 2. Bob and Alice measure each of their qubits in a randomly chosen base.

Step 3. Over the classical channel Alice and Bob compare which basis they used for each bit. If they agree they keep it otherwise they drop it. As in BB84 too many “transmission errors” indicate that Eve was eavesdropping and the transmission is abandoned. Ekert proposed a more sophisticated eavesdropping detection (Bell’s theorem).
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Step 2. Bob and Alice measure each of their qubits in a randomly chosen base \( \begin{array}{c} \downarrow \uparrow \end{array} \) or \( \begin{array}{c} \times \end{array} \).

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Example

\[ B_A \mid \begin{array}{ccccccccc}
\uparrow & & & & \uparrow & & & & \uparrow \\
\swarrow & & & & \swarrow & & & & \swarrow
\end{array} \]
Example

\[ B_A \quad \text{obs} \]

\[ \begin{array}{c|cccccccccccccc}
\times & \times & \uparrow & \uparrow & \times & \times & \downarrow & \downarrow & \times & \times & \downarrow & \downarrow & \times & \times \\
\downarrow & \downarrow & \uparrow & \uparrow & \times & \times & \downarrow & \downarrow & \times & \times & \downarrow & \downarrow & \times & \times \\
\end{array} \]
Example

<table>
<thead>
<tr>
<th>$B_A$</th>
<th>$\uparrow\downarrow$</th>
<th>$\uparrow\downarrow$</th>
<th>$\leftrightarrow$</th>
<th>$\uparrow$</th>
<th>$\leftarrow$</th>
<th>$\uparrow$</th>
<th>$\downarrow$</th>
<th>$\leftrightarrow$</th>
<th>$\uparrow$</th>
<th>$\downarrow$</th>
<th>$\leftarrow$</th>
<th>$\uparrow$</th>
<th>$\downarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$obs$</td>
<td>$\uparrow\downarrow$</td>
<td>$\uparrow\downarrow$</td>
<td>$\leftrightarrow$</td>
<td>$\uparrow$</td>
<td>$\leftarrow$</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
<td>$\leftrightarrow$</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
<td>$\leftarrow$</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$K_A$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Example

<table>
<thead>
<tr>
<th>$B_A$</th>
<th>obs</th>
<th>0 1 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$</td>
<td>0 1 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>$B_B$</td>
<td>0 1 0 1 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>
### Example

<table>
<thead>
<tr>
<th>$B_A$</th>
<th>$B_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$obs$</td>
<td>$obs$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$K_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 0 1 0 0 1 0 0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>

### Diagram

- $B_A$ and $B_B$ are represented by arrows pointing in various directions.
- $K_A$ is shown as a sequence of numbers, indicating transitions between states.
### Example

<table>
<thead>
<tr>
<th>$B_A$</th>
<th>$B_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>obs</strong></td>
<td><strong>obs</strong></td>
</tr>
<tr>
<td>$K_A$</td>
<td>$K_B$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$B_A$</th>
<th>$B_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_A$</td>
<td>0 1 0 1 0 0 1 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>$K_B$</td>
<td>0 0 0 0 0 0 0 1 0 0 0 0 1 0</td>
<td></td>
</tr>
</tbody>
</table>
**Example**

<table>
<thead>
<tr>
<th></th>
<th>$B_A$</th>
<th>$B_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$obs$</td>
<td>[Diagram]</td>
<td>[Diagram]</td>
</tr>
<tr>
<td>$K_A$</td>
<td>0 1 0 1 0 0 1 0</td>
<td>0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$K_B$</td>
<td>0 0 0 0 1 0 0 0 0 1 0</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{array}{ccccccccccc}
\sqrt{} & \sqrt{} & \sqrt{} & \sqrt{} & \sqrt{} & \sqrt{} & \sqrt{} & \sqrt{} & \sqrt{}
\end{array}
\]
### Example

<table>
<thead>
<tr>
<th>$B_A$</th>
<th>$B_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>obs</td>
<td>obs</td>
</tr>
<tr>
<td>$K_A$</td>
<td>0 1 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>$K_B$</td>
<td>0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0</td>
</tr>
<tr>
<td>$K$</td>
<td>$\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$</td>
</tr>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
</tbody>
</table>