

Exercises

Program Analysis (CO70020)

Sheet 2

Exercise 1 Consider the following **while** program:

```
[x := 1]1;  
while [y>0]2 do [x := x-1]3;  
[x := 2]4;
```

Perform a Live Variables Analysis for this program (state equations and construct solutions).

Exercise 2 Construct/specify all elements of $\mathcal{P}(\{\mathbf{x}, \mathbf{y}, \mathbf{z}\})$, i.e. the power set of $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$. Describe the sub-set relation on $\mathcal{P}(\{\mathbf{x}, \mathbf{y}, \mathbf{z}\})$, i.e. which sub-set is a sub-set of another sub-set. What is the maximum number of sub-sets a sub-set of $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ can be included in and/or the height of $\mathcal{P}(\{\mathbf{x}, \mathbf{y}, \mathbf{z}\})$?

Exercise 3 Construct/specify all elements of $\mathcal{P}(\{\mathbf{x}, \mathbf{y}\} \times \{1, 2, 3\})$, i.e. the power set of the cartesian product $\{\mathbf{x}, \mathbf{y}\} \times \{1, 2, 3\}$. Describe the sub-set relation on $\mathcal{P}(\{\mathbf{x}, \mathbf{y}\} \times \{1, 2, 3\})$. What is the maximum number of sub-sets any sub-set of $\mathcal{P}(\{\mathbf{x}, \mathbf{y}\} \times \{1, 2, 3\})$ can be included in?

Exercise 4 Consider the following **While** program

```
while (x>0) do y:=y-1
```

Describe the possible RD solutions at every program point. What is the size of this “property space” and how many possible solution are there for the RD analysis?

Exercise 5 Consider the set of all sets of the form:

$$\{*\}, \{*, \{*\}\}, \dots, \{*, \{*, \{*, \dots\}\}\}, \dots$$

i.e. $S_1 = \{*\}$ and $S_n = \{*\} \cup \{S_{n-1}\}$ where $*$ is some element/object. Describe the element relation on this set of sets. What is the maximum number of sets any set of can be included in (be element of)?

Exercise 6 Consider the power set $\mathcal{P}(X)$ of $X = \{a, b, c, d\}$.

1. Draw the Hasse diagram.
2. Give a monotone map from $(\mathcal{P}(X), \subseteq)$ into (\mathbb{Z}, \leq) .
3. Give a monotone map from (\mathbb{Z}, \leq) into $(\mathcal{P}(X), \subseteq)$.