Exercises
Program Analysis (CO70020)
Sheet 5

Exercise 1 Consider the following imperative language with statements of the form:

\[ S ::= x := a \mid \text{skip} \mid S_1 ; S_2 \mid \text{if} \; b \; \text{then} \; S_1 \; \text{else} \; S_2 \mid \text{while} \; b \; \text{do} \; S \mid \text{choose} \; S_1 \mid S_2 \mid \ldots \mid S_n \mid \text{combine} \; S_1 \mid S_2 \mid \ldots \mid S_n \]

In the choose statement only one of the \( n \geq 1 \) statements \( S_i \) is actually selected to be executed. The combine executes all of the \( n \) statements \( S_i \) in some sequence. In both statements the choices are made non-deterministically.

Define a Live Variable Analysis \( LV \), similar to the one for the simple while language, for this extended language. Define an appropriate labelling for statements/blocks and give a definition for the flow \( \text{flow} \) (together with init and final).

Exercise 2 Consider the following expression from which labels have been stripped:

\[
(\text{let} \; g = (\text{fn} \; f \Rightarrow (\text{if} \; f \; 3 \; \text{then} \; 10 \; \text{else} \; 5)) \newline \in \; (g \; (\text{fn} \; y \Rightarrow (y > 2)) \; ))
\]

Label the expression and give a brief and informal description of its execution: what does it evaluate to?

Write down the constraints for a 0-CFA and provide the least solution that satisfies the constraints.

Exercise 3 Consider the following extraction function for \( n \in \mathbb{N} \):

\[
\beta(n) = \begin{cases} 
\min \text{bits to represent } n & \text{if } n < 2^8 \\
\text{overflow} & \text{otherwise}
\end{cases}
\]

which allows for a Bit-Size analysis for “small” integers via Abstract Interpretation.

Describe the (abstract) property lattice and the concrete and abstract domain (incl. ordering and least upper bound operation). Furthermore, define the abstraction, \( \alpha \), and concretisation, \( \gamma \), functions.

Construct formally the abstraction (in the sense of Abstract Interpretation) of the doubling and square function, i.e. \( f^\# \) and \( g^\# \) for

\[ f(n) = 2 \times n \; \text{and} \; g(n) = n^2 \]
Exercise 4 Consider a Sign Analysis for the imperative WHILE language. That is: We are interested in the sign of variables, i.e. whether we can guarantee that for a given program point and a variable \( x \) (at least) one of the following properties holds: \( x = 0 \), \( x < 0 \), \( x > 0 \), \( x \leq 0 \) and \( x \geq 0 \).

Define a representation function \( \beta \) for this Sign Analysis. How can one define the corresponding correctness relation \( R_\beta \)? State formally what it means that the transfer functions \( f_L \) for all labels are fulfilling the correctness condition.