Exercises

Program Analysis

Sheet 5

Exercise 1 Consider a Sign Analysis for the imperative While language. That is: We are interested in the sign of variables, i.e. whether we can guarantee that for a given program point and a variable $x$ (at least) one of the following properties holds: $x = 0$, $x < 0$, $x > 0$, $x \leq 0$ and $x \geq 0$.

Define a representation function $\beta$ for this Sign Analysis. How can one define the corresponding correctness relation $R_\beta$? State formally what it means that the transfer functions $f_\ell$ for all labels are fulfilling the correctness condition.

Exercise 2 Consider the following pWhile program with $x \in \{-1, 0, +1\}$:

```plaintext
x ?= \{-1, 1\};
while (x > 0) || (x < 0) do
  choose 1: x := (((x+1)+1)%3)-1 or 1: x := (((x+1)-1)%3)-1 ro
  od;
stop
```

By "\%" we denote the remainder operation (i.e. $x \mod 3$). Describe what this program is doing. Does this program always terminate?

Label the program and give the probabilistic program flow. Construct the transfer functions/matrices for every block. Give the symbolic construction of the LOS semantics. What is the dimension of the DTMC operator representing the semantics of this program and explicitly write it down and explain what each row/column corresponds to.

For these exercises it is helpful to use tools like octave, Julia or Matlab.

Exercise 3 Change the above pWhile program such that we have $x \in \{-3, 0, +3\}$.

Label the program and give the probabilistic program flow.

Give the symbolic construction of the LOS semantics. What is the dimension of the DTMC operator representing the semantics of this program.

Consider the sign abstraction defined by the extraction function:

$$\eta(n) = \begin{cases} 
- & \text{if } n < 0 \\
0 & \text{if } n = 0 \\
+ & \text{if } n > 0 
\end{cases}$$

Give the (probabilistic) abstraction and concretisation matrices $A$ and $G$ for the variable $x$. Describe the corresponding classical abstract interpretation setting (domains, $\alpha$, $\gamma$, etc.).
Give the symbolic construction of the abstract semantics of the program using the sign abstraction for variable \( x \). Give the abstract transfer functions/matrices for all blocks. What is the dimension of the abstract DTMC generator and construct it explicitly.

Optionally: Construct the transfer functions/matrices for the concrete semantics for every block.