

# Exercises

## Program Analysis (CO70020)

### Sheet 6

**Exercise 1** Consider the following PWHILE program with  $x \in \{-1, 0, +1\}$ :

```
x ?= {-1,1};
while (x > 0) || (x < 0) do
  choose 1: x := ((x+1)+1)%3-1 or 1: x := ((x+1)-1)%3-1 ro
od;
stop
```

By “%” we denote the remainder operation (i.e.  $x \bmod 3$ ). Describe what this program is doing. Does this program always terminate?

Label the program and give the probabilistic program flow. Construct the transfer functions/matrices for every block. Give the symbolic construction of the LOS semantics. What is the dimension of the DTMC operator representing the semantics of this program and explicitly write it down and explain what each row/column corresponds to.

For these exercises it is helpful to use tools like `octave`, `Julia` or `Matlab`.

**Exercise 2** Change the above PWHILE program such that we have  $x \in \{-3, 0, +3\}$ . Label the program and give the probabilistic program flow.

Give the symbolic construction of the LOS semantics. What is the dimension of the DTMC operator representing the semantics of this program.

Consider the sign abstraction defined by the extraction function:

$$\eta(n) = \begin{cases} - & \text{if } n < 0 \\ 0 & \text{if } n = 0 \\ + & \text{if } n > 0 \end{cases}$$

Give the (probabilistic) abstraction and concretisation matrices  $\mathbf{A}$  and  $\mathbf{G}$  for the variable  $x$ . Describe the corresponding classical abstract interpretation setting (domains,  $\alpha$ ,  $\gamma$ , etc.).

Give the symbolic construction of the abstract semantics of the program using the sign abstraction for variable  $x$ . Give the **abstract** transfer functions/matrices for all blocks. What is the dimension of the abstract DTMC generator and construct it explicitly.

Optionally: Construct the transfer functions/matrices for the **concrete** semantics for every block.