AUTOMATED REASONING

SLIDES 13:

EQUALITY IN TABLEAUX

Basic use of Equality in Tableaux

Use of Equality in ME

KB - AR - 12

EQUALITY IN TABLEAUX

13ai

- In developing a tableau two equality rules are used:
- Reflex (EQAX1) and Substitution (also called <u>paramodulation</u>) which uses EQAX2 and EQAX3 implicitly.:

$$\frac{a=b \quad P(\ldots,a,\ldots)}{P(\ldots,b,\ldots)} \qquad \frac{r=s \quad P(\ldots,t,\ldots)}{P(\ldots,s\theta,\ldots)\theta}$$

where $r\theta=t\theta$ and θ is mgu of r and t.

Example: (1) a<b ∨ a=b (2) ¬ a<c (3) b<c (4) ¬x<y ∨ ¬y<z ∨ x<z

¬a<c (**)
b<c (*)

a<b

- <u>¬x1<y1</u>
  x1==a
  y1==b
- ¬y1<z1⇒
  <u>¬b<z1</u>
  z1==c
- x1<z1⇒
  <u>a<c</u>

a=b

- <u>a<c</u> (Sub in *)
  or ¬b<c (Sub in (**))

NOTE:
a=b can be used from Left to Right or from Right to Left

Simulation of (Free variable) Tableau Equality Rules using Equality Axioms

13aii

r=s

P(...,f(t),...)

r, s may contain variables. Assume r and t unify with mgu θ

- ¬x1=y1
  <u>x1==r</u>
  y1==s
- f(x1)=f(y1)⇒ f(r)=f(s)
  - ¬x2=y2
    <u>x2==f(r)</u>
    y2==f(s)
  - ¬P(...,x2,...)
    ⇒
    <u>¬P(...,f(r),...)</u>
    unify r and t with mgu θ
  - P(...,y2,...)
    ⇒
    P(...,f(s),...)
    ⇒
    P(...,f(sθ),...)θ

This remaining open branch contains exactly the result of using the substitution rule

1. generate required equality by EQAX2

2. Apply appropriate EQAX3

13aiii

Models including the Equality Literal:

Recall from Slide 12dii that in a *normal* model the equality predicate is interpreted as identity and hence if p=q is true, then p and q must be interpreted as the same domain element. Alternatively, Herbrand models that satisfy the basic requirement of substitutivity can be used and as far as satisfiability is concerned the two approaches are equivalent.

The completeness proof for tableau involved constructing a saturated tableau from some consistent set and then constructing a model from the saturated tableau. A saturated tableau is one in which every rule is applied in every branch in every possible way. For the equality substitution rule, notice that it can be restricted to apply only to ground literals in the tableau. Substitution into sentences with quantifiers can be delayed until after the quantifier has been eliminated and the resulting sentence has been reduced to literals.

In order to show that the constructed model, which was derived from the literals in an open saturated branch, was indeed a model, a complexity ordering based on the length of formulas was used. When using equality substitution implicit is the use of the EQAX. The model that is found will have as domain the set of terms occurring in the branch and will definitely **not** be a normal model. Instead, it will be an E-model.

13bi

Controlling Equality Substitution in Tableaux

The most difficult aspect of dealing with equality is in *controlling* substitution as there are usually numerous ways to apply it in a tableau branch.

Here's one possible method:

- Form a tableau to some limit

(eg allow each universal rule to be expanded once and then allow a maximum number of "extra" applications.)

- Ignore substitution using equality literals, but allow those branches that can close in the usual way to do so.

- For each unclosed branch apply equality rules to equations in it in order to force a closure (see example on right).

P(a)
¬P(b)
Q(u1,f(b))
¬Q(c,g(b))
b=c
f(c)=g(c)

Branch will close if can show (a=b) or (u1=c & f(b)=g(b))

b=c
f(c)=g(c) (equalities from branch)

a≠b (from negated goal)
u1≠c ∨ f(b)≠g(b) (from negated goal)

Close? Yes, if u1==b

13bii

Controlling Equality Substitution in Tableaux:

There have been several methods proposed for controlling the use of equations in tableaux. Most involve the separation of the equational reasoning from standard rules in some way or other. A simple method is shown on 13bi. Here, a tableau is developed to a maximum depth, closing branches in the usual way if possible. If open branches remain, which also contain equations, an attempt is made to find a contradiction using the equations. Potential closure between 2 literals is made, subject to the constraint that the arguments can be made equal. e.g. P(a,f(X)) and ¬P(b, g(b)) would be complementary, if a=b and g(b)=f(X) (for some X) could be derived. (This is quite similar to the RUE refinement.) The contradiction can be derived in many ways; e.g. a refutation by resolution and substitution (paramodulation) using the equations E in the branch in which closure from E+{¬a=b}+ {¬g(b)=f(X)} is derived.

There are other approaches, but not considered here. Instead, using equations in tableaux can be simulated using the equality axioms, see 13aii, so an approach to their control would be to incorporate this simulation within the strategy used to develop the tableau. We'll look at just one such strategy, in which a RUE style of using the Alternative EQAX is introduced into clausal ME tableau.

13biii

EXAMPLE 1 First Approach

¬Q(g(b))
g(z1)=f(z1)
P(g(g(w1)))

Q(y1)
y1==g(b)
(unify)

¬P(f(y1)) ⇒ ¬P(f(g(b)))

(**)

Given:
¬Q(g(b))
Q(y) ∨ ¬P(f(y))
g(z)=f(z)
P(g(g(w)))

a) Use each non-unit clause a maximum of once in each branch
(assume this is the limit)

b) Can close first branch normally by unification

c) Can close at (**) if f(g(b))=g(g(w1)) is shown
To show f(g(b))=g(g(w1)), set z1=g(b) and w1=b.
That is, refute ¬(f(g(b))=g(g(w1)))
Substitute using f(z1)=g(z1) to give ¬(g(g(b))=g(g(w1)))
and then close with x=x

EXAMPLE 2 (see ppt)

First Approach

Given:
x=a ∨ x=b
g(x)≠g(y) ∨ x=y
g(a)=b
g(g(a))≠a

g(a)=b
g(g(a))≠a

x1=a
x1==g(g(a))
(unify)

x1=b ⇒ g(g(a))=b

g(x2)≠g(y2) (*)

x2=y2 (**)

a) Use each non-unit clause a maximum of once in each branch (the limit).
b) Can close first branch normally by unification (match with g(g(a))≠a).
c) Can close at (*) if g(y2)=b is shown **and**
either x2==g(a) (match with g(g(a))=b),
or x2==a (match with g(a)=b)

To show g(y2)=b, set y2==g(a) or y2==a; unify x2;

Together, the bindings for x2 and y2 give 4 possibilities for (**):

di) y2==g(a), x2==g(a): cannot refute g(a)=g(a).
dii) y2==g(a), x2==a: ==> refute a=g(a) (see (e) on slide 13bv)
diii) y2==a, x2==a: cannot refute a=a.
div) y2==a, x2==g(a): ==> refute g(a)=a (see (e) on slide 13bv)

13biv

EXAMPLE continued

13bv

g(a)=b
g(g(a))≠a

x1=b⇒ g(g(a))=b

x2=y2 =>g(a) = a (**)

e) From 13biv one remaining branch (**) to close: Can close if
(ei) g(g(a))=g(a) can be shown
(match g(a)=a with g(g(a))≠a), or if
(eii) a=b can be shown
(match g(g(a))=b with g(g(a))≠a)
See steps below – add to equations
either a≠b or g(g(a))≠g(a)

1. g(g(a))≠g(a)
2. g(a)=b
3. g(g(a))=b
4. g(a)=a

either:

g(a)≠g(a) (1+4)
close by reflex

b≠g(a) (1+3)
close by (2)

1. a≠b
2. g(a)=b
3. g(g(a))=b
4. g(a)=a

g(a)≠b (1+4)
close by (2)

Using Method of Slide 13bi in Tableaux:

The examples on 13biii - 13bv illustrate the method of forming a tableau to some limit (here using each clause a maximum of once in each branch) and then trying to close branches using equations. In the first example on 13biii, each clause is used once in a free variable tableau. One branch closes normally with the unifier y1==g(b). The second branch will close if f(g(b)) and g(g(z1)) can be shown to be equal.

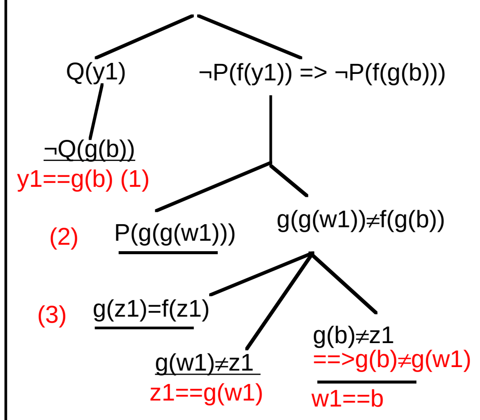
In the second example, there are two open branches and two possible closures for the first of these: between g(a)=b and g(x2)≠g(y2) or between g(g(a))=b and g(x2)≠g(y2). One can obtain closure either if x2==a and g(y2)=b can be derived, or if x2==g(a) and g(y2)=b can be derived. i.e. refute ¬g(y2)=b using the set of equations {g(a)=b, g(g(a))=b}, which is easy: y2==a or y2==g(a). There is another possibility, to close g(x2)=g(y2) by reflex, but this yields x2=x2 in the second open branch which cannot be refuted.

For the second open branch, two of the substitution pairs result in a=a or g(a)=g(a), which clearly cannot be refuted as they are instances of (Reflex). The other two substitution pairs both result in g(a)=a and the branch can be closed if g(g(a))≠a matches either g(g(a))=b or g(a)=a or g(a)=b. The first of these requires b=a to be shown using {g(a)=a, g(a)=b}, which is clearly possible; the second requires g(g(a))=g(a) to be shown, again using {g(a)=a, g(a)=b}. Again this is easy. The third requires to show g(g(a))=g(a) and a=b, which is done as before. (See 13biv.) Only one of these options is necessary.

Exercise: Show these things.

It is clear from these exemplars that there are many and various possibilities when using equations and that the search space can become very large.

EXAMPLE 3 ME tableau with equality - implicit EQAX RUE style



Given:
¬Q(g(b)), Q(y) ∨ ¬P(f(y))
g(z)=f(z), P(g(g(w)))

Remember can use symmetry

(1) here the match can be a normal closure

(2) try to match P(f(g(b))) with P(g(g(w1))); results in requiring to show g(g(w1))=f(g(b))

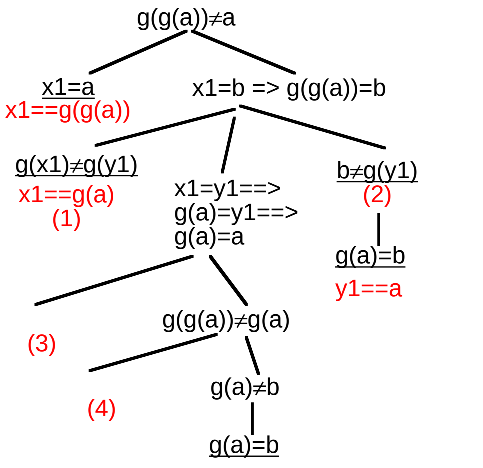
(3) try to match g(g(w1))=f(g(b)) with g(z1)=f(z1); need to show both g(w1)=z1, and g(b)=z1. The first binds z1==g(w1) and the second then binds w1==b.

Note: Resolution steps with EQAX1 and EQAX2 are made implicitly. eg at (3) RUE-style gives g(w1)≠z1 as well as g(b)≠z1.

There are still *many* different possibilities for closure:

What is the tableau that results if the literal order in the top clause is reversed?

EXAMPLE 4 ME tableau with equality - RUE style



Given:
x=a ∨ x=b g(g(a))≠a
g(a)=b g(x)≠g(y) ∨ x=y

(1) here try to match g(x1)≠g(y1) with g(g(a))=b; results in need to show b=g(y1) as in branch at (2)

(2) follows by a normal closure (remember can use symmetry)

(3) try to match g(a)=a with g(g(a))≠a; results in showing g(g(a))=g(a) (ie literal g(g(a))≠g(a)

(4) try to match g(g(a))≠g(a) with g(g(a))=b; results in showing g(a)=b, which results in literal g(a)≠b which closes normally.

Can you find a different tableau?

Using EQAX implicitly in Model Elimination Tableaux: 13ciii

Slides 13ci and 13cii show tableaux in which EQAX are used implicitly in Model Elimination to simulate a RUE style. The axioms used are particular for the predicate(s) involved, which may even be the equality predicate itself. In that case the axioms are

¬x=y ∨ g(x)=g(y), ¬x=w ∨ ¬y=z ∨ ¬x=y ∨ w=z

The first axiom is an instance of EQAX2 and the second is an instance of (Alt) EQAX3, where "P" of the axiom is the "=" predicate (c.f. ¬x=w ∨ ¬y=z ∨ ¬P(x,y) ∨ ¬P(w,z)).

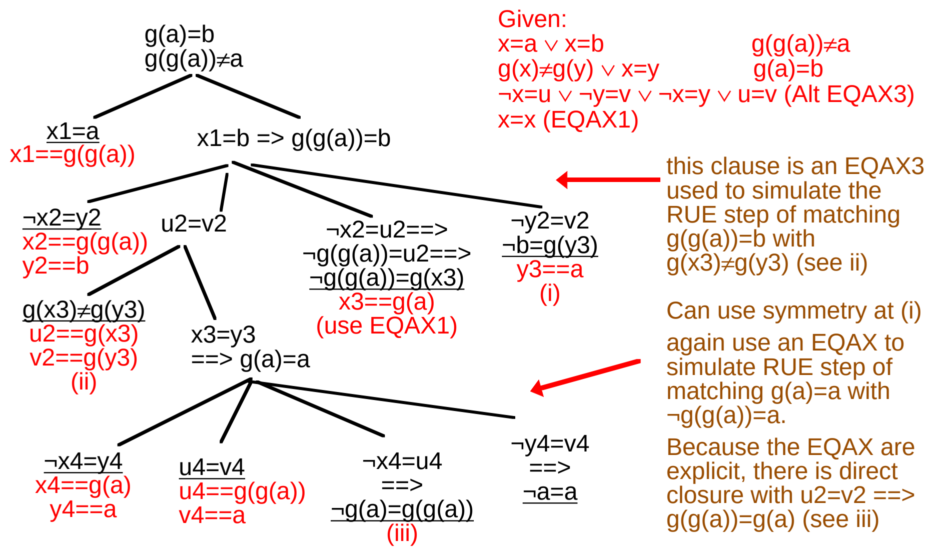
Use of the reflexive EQAX1 (x=x) is often needed. Symmetry can be built in; e.g. unifying (say) a=b with ¬x1=x2, results in two unifiers: {x1==a, x2==b} and {x1==b,x2==a}.

The implicit use of the Axioms are made by matching two literals with the same predicate. To match P(x,y) with ¬P(u,v), for example, the closure would be made but additional inequality branches of ¬x=y and ¬u=v would be added.

For example, this happens in several places on 13cii, namely at closures (2), (3) and (4). At (2) it arises due to matching g(g(a))=b in the tableau branch with g(x1)≠g(y1) from the clause x=y ∨ ¬g(x)=g(y). The additional inequality is b≠g(y1).

At (3) it arises due to matching g(a)=a with g(g(a))≠a.
At (4) it arises due to matching g(g(a))≠g(a) with g(g(a))=b.

EXAMPLE 5 (OPTIONAL) ME tableau with explicit EQAX - RUE style 13civ



Using EQAX explicitly in Model Elimination Tableaux (Optional): 13cv

Slide 13civ shows a tableau in which EQAX are used explicitly in Model Elimination to simulate a RUE style. As before, the axioms used are particular for the predicate(s) involved.

Various restrictions could be incorporated into the use of these axioms. For instance, the use of EQAX could be restricted to use in a branch B such that at most one literal remains unclosed. Closure could either be with a literal in B, or with a fact. This restriction could simulate both paramodulation and RUE, in different circumstances. For example, in the EQAX3, ¬x1=x2 ∨ ¬y1=y2 ∨ ¬P(x1,y1) ∨ P(x2,y2), if either ¬x1=x2 or ¬y1=y2 remained unclosed, this would be a RUE type step, whereas if ¬P(x1,y1) or P(x2,y2) remained unclosed it would be a paramodulation type step (**Check this**). Other restrictions were investigated by Rosa Gutierrez-Escudero in 2010 (available as a distinguished project on the doc website).

Unfortunately, the above restriction is not complete (can you find a counterexample?). Slide 13civ uses the restriction is used. Notice at most one literal in a clause is extended; all the others close immediately.

It is also possible to use the equality axioms explicitly to simulate paramodulation in a ME tableau. If the above restriction is incorporated, the effect is similar to using the standard equality substitution rule for tableau.

Summary of Slides 13 13ei

1. Equality reasoning can be incorporated into tableau, either standard tableau or free variable tableau or ME tableau.

2. In standard tableau the equality rule allows to derive new ground literals using equality substitution; in free variable tableau it allows to derive new literals, possibly applying a unifying substitution (also called paramodulation).

3. Usually, a tableau is developed to some depth, closing branches normally if possible, and then attempting to close remaining branches using any equalities in the branch.

4. Most methods using equality in tableau are quite difficult for humans and lead to large seach spaces.

5. Use of the equality axioms can be simulated within ME tableau, by using equality axioms implicitly (shown for the RUE approach) or explicitly.

6. Use of EQAX can also be used in ME style explicitly to simulate the RUE approach (shown here), or paramodulation (not shown here).

Question for next week

13eii

When simplifying an equation you are using equality reasoning.

How can the task of simplifying (1+ x) - 4 = 2 to give a binding for x, namely x==5 be recast as a paramodulation problem.

What reasoning steps do you use to solve the equation?
(Hint: there are more than you think, perhaps)