AUTOMATED REASONING (KB 2012) MAIN ASSESSED COURSEWORK Part 1

Issued: November 6th 2012

Due: by November 23rd

Marked scripts and solutions will be available at the latest by Dec. 07th

NOTE: variables begin with lower case u to z.

1. (17 marks) This question is similar to the "Quickies" on the problem sheets and covers weeks 2 - 6 of the course (slides 3 - 11) unless covered in a longer question.

(a) (slides 3) Give all binary resolvents of clause (i) with clause (ii):

(i) P(f(y) x) ∨ Q(x,y)   (ii) ¬Q(a,f(u)) ∨ ¬Q(z,z) ∨ ¬P(z,u)

(b) (slides 6) Find all factors of (iii) and state which are safe factors:

(iii) R(z,a,u) ∨ R(f(u),w,w) ∨ S(w,z) ∨ S(z,z) ∨ S(a,f(u))

(c) (slides 7) Use locking to derive the empty clause from clauses (16) - (19) in Q4 (repeated below). Try to assign locks so that using a saturation search there are as few steps as possible at each stage.

(16) S(z,y) ∨ S(h(h(y)),z)   (17) ¬S(a,x)

(18) R(u) ∨ S(a,u)   (19) ¬R(v) ∨ ¬S(h(v),v)

(d) (slides 8) Find a hyper-resolution refutation of the clauses (8) - (10).

(8) R(x,y) ∨ R(y,x)   (9) ¬R(b,w) ∨ ¬R(h(w),w)

(10) ¬R(z,h(z))

(e) (slides 9) What is the Herbrand Base for the following clauses (iv) - (vi)? Using the standard tableau method find a saturated branch from which you can read off a Herbrand model for the clauses. (Give the model).

(iv) P(x,y) ∨ ¬Q(x,y)   (v) ¬P(a ,b)   (vi) Q(x,y) ∨ Q(y,x)

2. (15 marks) (A variation of parts of 2 previous exam questions)

(a). Apply the Model Generation method of Slides 2 to the clauses (2) - (7). What is the outcome?

(2) ¬A ∨ C ∨ ¬B   (3) ¬B ∨ ¬A ∨ ¬C   (4) C ∨ ¬B ∨ A

(5) ¬B ∨ A ∨ ¬C   (6) ¬D ∨ B ∨ C   (7) B ∨ C ∨ D

(b) We'll call a set of clauses in which every clause has at most one positive literal an *acceptable* set of clauses. Let S be an acceptable set of clauses such that in addition every clause has at least one negative literal. Why does S have at least one model?

(c) Describe the form that a Model generation tree will take for an *unsatisfiable* acceptable set of clauses. Justify your answer.

Hint: what information does (b) give about an unsatisfiable acceptable set?

(d) *Assume for parts (e) and (f) that there are no function symbols in the signature.*

In Slides 11, Model Generation (MG) was compared with the tableau method. Using this analogy, suggest how MG might be extended to deal with first order clauses. Your answer should state the steps of the adapted procedure. (There are more than one possible answers.)

(e) Illustrate your answer to part (d) for the clauses

(11) P(y,x) ∨ ¬P(x,b)   (12) ¬P(x,a) ∨ ¬P(b,x)

(13) P(a,y) ∨ P(y,y)

3. **(9 marks)**. For the clauses (14) ¬Qud ∨ ¬Qvc   (15) Qbz ∨ Qaz

(a) Find a ME tableau refutation using clause (14) as the top clause. Show opportunities for application of merge or re-use (if there are any).

(Note that only one of these pruning techniques can be used in any particular tableau.)

(b) Use the ME extension of the Generalised Closure Rule with top clause (15).

4. (**5 marks**) Given clauses

(16) S(z,y) ∨ S(h(h(y)),z)   (17) ¬S(a,x)   (18) R(u) ∨ S(a,u)   (19) ¬R(v) ∨ ¬S(h(v),v)

For this question, use Model Elimination and assume clauses are tried in the order given in attempts to close branches. Show that non-essential back-tracking pruning leads to failure if the top clause is (16) but does not do so if the top clause is (17).

5. (14 marks) (Majority of a full question from a previous exam paper)

a) A new refinement called *Unit-resulting resolution with factoring* (FUR) has been suggested as a useful additional step in resolution refinements. Its single step consists of resolving none or more of the literals L_i in a "centre" clause $L_1 \vee L_2 \vee ... \vee L_n$, each with a complementary unit clause K_i such that the final resolvent is either the empty clause or is, or can be factored to, a unit clause.

i) Show that FUR on its own is not a complete resolution refinement by giving a propositional counter-example.

ii) Explain why factoring a clause to derive a unit clause is a FUR step.

iii) Find a refutation for the clauses (20) - (22) in which only FUR steps are used.

(20) ¬R(x, b) ∨ R(y, x)   (21) ¬R(b, z) ∨ ¬R(z, a)

(22) R(v, v) ∨ R(a, v)

b) Consider the following deduction rule, called Division, that may be applied to a set of clauses S:

Let C = D ∨ E be a clause in S such that D and E are disjunctions of one or more literals and the variables in D and E are distinct. Form the two sets of clauses S1 = (S − {C}) ∪ {D} and S2 = (S − {C}) ∪ {E}.

i) By considering derivations of the empty clause   from S1 and S2 and using the soundness and completeness properties of resolution, show that Division satisfies the property

if S1 |= [ ] and S2 |= [ ] **then** S |= [ ]

ii) Use the Division rule and resolution to derive the empty clause from clauses (23) - (27).

(23) ¬P(u) ∨ ¬Q(f(w), w)   (24) P(f(a)) ∨ P(f(b))   (25) S(a)

(26) ¬S(w) ∨ R(u,w)   (27) ¬P(x) ∨ ¬R(y, z) ∨ Q(x, y)