Automated Reasoning 2012 (KB)
PROBLEMS 2 (Resolution and Clausal Form)

NOTE: *In general, in clausal data, terms beginning with letters from the beginning of the alphabet will be used for constants and those beginning with letters from the end of the alphabet will be used for variables. The first letter of a predicate, but not of a term, will be capitalised. Function symbols often begin with letters from the middle of the alphabet. Universal quantifiers will be implicit.*

QUICKIES

1. Resolve $Fx \vee Pxy$ with $\neg Pau \vee \neg Fu$ in two different ways

2. Find a factor of $Pxau \vee Pbyy$

3. Skolemise $\forall z\ (\exists u(Qu \vee Pz)\ )$ (See if you can manage to use a Skolem constant, not a Skolem function - ie distribute quantifiers first)

4. Show by resolution that $P(f(x)) \vee \neg P(x) \models P(f(f(x))) \vee \neg P(x)$

(Hint: You will need 2 copies of the given clause standardized apart)

5. Given the signature {F/1,G/2, H/1, a, b} (i.e. no function symbols) write down the Herbrand base HB. How many Herbrand Interpretations (HI) are there for this signature?

6. For the HB in Q5, find a HI that makes both the clauses below true:

$F(x) \vee \neg G(x,x)$, $\neg H(u) \vee \neg F(u)$.

LONGER QUESTIONS

1. a) *Derive the empty clause by resolution from*:

$\neg H(a)$, $\neg F(y) \vee \neg H(b)$, $Fx \vee Hx$, $\neg Gz \vee \neg Fb$, $Gu \vee \neg Fu$

where a and b are constants and x, y, u and z are variables. Note that, for simplicity, brackets surrounding the arguments in an atom are often left out; e.g. Fx means F(x), etc.

Use a saturation search:

(i) keeping all clauses except any variant of an earlier given or derived clause

(ii) with factoring and simple subsumption.

Recall that a factor of a clause is an instance of the clause in which two or more identical literals have been merged. eg $Fx \vee Fa$ factors to Fa, as $Fa \vee Fa$ is an instance and the two occurrences of Fa can be merged.

Simple subsumption occurs as follows. Let clause C be a subset of another clause D up to variable renaming, then D is less general than C and can be removed. We say C subsumes D or D is subsumed by C. eg $P(x) \vee P(a)$ is subsumed by both P(y) and P(a).

b) Think of an approach for organising the search for a derivation that is different from saturation search. How natural is it?

c) Repeat part (aii) for the clauses

$P(y,a) \vee P(f(y),y)$, $P(a,y) \vee P(y,f(y))$, $\neg P(x,y) \vee \neg P(y,x)$

(**Hint**: You can form a factor of one of the clauses – then the solution is very much simpler.)

2. *Convert to clausal form*

(a) $\forall x[\ \forall y\ P(x,y) \leftrightarrow \exists z\ Q(z,x)\ ]$

(b) $\exists z[\ Q(z) \wedge (\forall y[\neg (P(y,z) \rightarrow P(y,y))] \vee \neg \exists u \forall v[S(u,z) \rightarrow (T(v,z) \ \vee Q(v)\ )])]$

3a). *For each of the sets of clauses in i) and ii) enumerate the Herbrand universe and Herbrand base*. Assume the signatures are <{H, G, F}, ∅, <a,b}> and <{P}. {f}, {a}>.

(i) $\neg H(a)$, $\neg F(y) \vee \neg H(b)$, $Fx \vee Hx$, $\neg Gz \vee \neg Fb$, $Gu \vee \neg Fu$

(ii) $P(y,a) \vee P(f(y),y)$, $P(a,y) \vee P(y,f(y))$, $\neg P(x,y) \vee \neg P(y,x)$

b) *For the clauses in i) can you find a Herbrand model?*

4. Try to devise an algorithm to check for simple subsumption.

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