Automated Reasoning 2012 (KB)
PROBLEMS 6 Clausal Tableaux

QUICKIES.

1. Why is the Generalised Closure Rule (GCR) complete?
Why does soundness of GCR not follow directly from tableau soundness?

2a. Why does disallowing non-essential backtracking not affect completeness of ME for propositional sets of clauses?

2b. Can you think of a circumstance for first order sentences, such that disallowing non-essential backtracking does not affect completeness?

3. A ME tableau T from clauses S using top clause C (in S) is only guaranteed to close if C is in a minimally unsatisfiable subset of S. (That is, removing C from S would give a satisfiable set.) Find an unsatisfiable set S and top clause C such that no ME tableau starting from C will close.

LONGER QUESTIONS.

1 (Parts (a) and (b) are the same as Sheet 5 Q3!)

Find closed ME-tableaux for the sets of clauses in (a) - (d).

a ¬Ha, ¬Fx ∨ ¬Hb, Fx ∨ Hx, ¬Gz ∨ ¬Fb, Gu ∨ ¬Fu

b blue(a), On(a,b), green(c), On(b,c),
¬blue(x) ∨ ¬green(x), ¬On(u,v) ∨ green(u) ∨ ¬green(v)

c Q(x,y) ∨ Q(y,x), ¬P(a,b), ¬P(x,y) ∨ P(y,z) ∨ ¬P(x,z), P(x,y) ∨ ¬Q(x,y)

d S(x,y) ∨ ¬ in(el(x,y), y)), S(x,y) ∨ in(el(x,y), x),
in(x, [u|v]) ∨ ¬is(x, u), ¬S([1, 2], [2,1]),
in(x,[u|v]) ∨ ¬in(x,v), ¬in(x,[u|v]) ∨ in(x,v) ∨ is(x,u)

Here, the [u|v] is Prolog list notation, and the predicates *S, in* and *is* can be read, respectively, as "subset", "belongs to" and "same as".

2a Make use of the ReUse rule and refute the following using top clause ¬B ∨ ¬A ∨ ¬C:

¬A ∨ C ∨ ¬B, ¬C ∨ B, ¬B ∨ ¬A ∨ ¬C, B ∨ C ∨ D,
¬D ∨ B ∨ C, ¬B ∨ A ∨ ¬C, C ∨ ¬B ∨ A

2b Repeat part a) for the Merge rule.

3 How does the use of universal literals in the generalised closure rule reduce the size of the refutation for the following clauses if the top clause is ¬Fx ∨ ¬Hb.?

¬Ha ∨ ¬Hc, ¬Fx ∨ ¬Hb, Fx ∨ Hx, ¬Gz ∨ ¬Fb, Gu ∨ ¬Fu

4 Consider the standard Model Elimination search space for the clauses in part (1b) using top clause green(c). Which parts of it are removed (if any) when non-essential backtracking is disallowed?