Automated Reasoning 2012 (KB)
PROBLEMS 8 Rewriting – Basic Ideas

QUICK QUESTIONS:

1. Why must f(x,y) = g(y) be ordered as a rewrite rule from L to R?

2. Find critical terms and corresponding critical pairs by superposing the following sets of rules:

(a) s(j(x,y)) => j(y,x) and j(x,n) => x (n is a constant)

(b) f(x, x) => e, h(g(x, y)) => g(y, x), g(x, y) => f(y, x)

(c) f(x, x) => e, h(g(x, y)) => g(y, x), f(x, y) => g(y, x)

3. Show each set of rules in Q2 is ordered for termination.
Also order each critical pair obtained.
(Use rpo or kbo as appropriate, but use the same ordering for each set.)

LONGER QUESTIONS:

1. Find all critical pairs among the following sets of rewrite rules.

a (1) f(x,e) => x, (2) f(x,x) => e, (3) f(g(x),y) => g(f(x,y))

b (1) f(x,e) => x, (2) f(x,x) => e, (3) g(f(x,y)) => f(g(x),y)

c (1) g(f(x)) => f(f(g(x))), (2) f(h(x)) => h(g(x))

d (1) g(f(x)) => f(f(g(x))), (2) h(g(x))=> f(h(x))

e (1) min(x, x+y) => x, (2) min(y+x,x) => x, (3) x+0 => x, (4) 0+x => x

f (1) g(a) =>a, (2) g(b) =>b, (3) h(a,x) => f(a,x), (4) h(b,x) => f(b,x),
(5) f(h(x,y),z) => f(y,f(x,z)), (6) g(h(x,y)) => h(g(y),g(x))

g (1) f(x, x) => e, (2) h(g(x, y)) => g(y, x), (3) g(x, y) => f(y, x)
(4) g(g(x, y), z) => g(z, f(y, x))

2. Show that the rule n(y,n(L,x)) => n(n(y,y),x) is not terminating by finding a term that leads to a non-terminating sequence of rewrites. Show that the rule can be ordered in the opposite direction for termination by kbo.

3. Check that each of the following sets of equations can be turned into a set of terminating rewrite rules.

a g(f(x)) = f(f(g(x))) (*try rpo or kbo or a simple measure*)

b f(h(x)) = h(g(x)) (*try a simple measure*)

c (a) and (b) together (*try rpo*)

d f(x,e) = x, f(x,x) = e, f(g(x),y) = g(f(x,y)) (*try rpo*)

e g(a) =a, g(b) =b, h(a,x) = f(a,x), h(b,x) = f(b,x),
f(h(x,y),z) = f(y,f(x,z)), g(h(x,y)) = h(g(y),g(x))

f x+0 = x, 0+x = x, min(y+x,x) = x, min(x, x+y) = x

g f(x, x) = e, f(x, y) = g(y, x), h(g(x, y)) = g(y, x),
g(g(x, y), z) = g(z, f(y, x))