

Automated Reasoning 2012 (KB) PROBLEMS 7
Equality in Resolution (Paramodulation) and Tableaux

QUICKIES (and not so quick!)

1. In how many ways can $f(x)=g(x)$ be paramodulated into $P(u,g(f(u)))$? Show a simulation by resolution using EQAX for the case of matching $f(x)$ with $f(u)$.

2. Derive $[]$ by (i) paramodulation and resolution and (ii) by RUE from $b(x) \neq x, \quad s(x) \neq x, \quad b(x) \neq s(y), \quad x=x, \quad x=a \vee y=a \vee x=y \vee x=u \vee y=u \vee u=a$, where $b(x) \neq x$ is $\neg(b(x)=x)$ etc.

You may assume symmetry of $=$; e.g. $a=b(a)$ resolves with $b(a) \neq a$.

3. Repeat Q2 (i) using tableau, and (ii) using Otter.

LONGER QUESTIONS.

1. (i) Paramodulate $a=b$ into $P(f(x), x) \vee Q(x)$ in all possible ways.
(ii) Paramodulate $f(u) = u \vee R(u)$ into $P(f(x), x) \vee Q(x)$ in all possible ways.

2. For each of the following sets of clauses, derive the empty clause using

(a) ordinary unrestricted paramodulation,

(b) hyper-paramodulation

(c) The RUE approach

(i) $x=x, \quad a=c, \quad b=c, \quad c=d, \quad c=e, \quad \neg P(d, x) \vee \neg P(e, x), \quad P(a, y) \vee P(b, y)$

(ii) $x=x, \quad x=a \vee x=b, \quad g(x) = g(x), \quad \neg(g(x) = x), \quad R(a, b), \quad R(g(a), g(b)), \quad \neg R(a, g(a)) \vee \neg R(b, g(b))$

(iii) $x=a \vee x=b, \quad \neg(g(g(a)) = a), \quad x=y \vee \neg(g(x) = g(y)), \quad x=x, \quad g(x) = g(x)$

Notes: To use paramodulation in Otter, use `set(para_into).` and `set(para_from).`

Resolving an equality $s=t$ with $t \neq s$ requires an implicit use of symmetry. Without that, you must first use $s=t$ in a paramodulation step on $t \neq s$ to obtain $t \neq t$, and then resolve with $x=x$, or alternatively paramodulate $s=t$ into $x=x$ to get $t=s$ and then resolve with $t \neq s$.

Question: What does Otter do? **Answer:** Otter does not seem to want to, or be able to, paramodulate into $x=x$ (or even into $x=y$ or $a=a$, etc.)

3. Use method on Slide 13bi to find a closed tableau for the following sets of clauses

(i) $x=x, a=c, b=c, c=d, c=e, \neg P(d, x) \vee \neg P(e, x), P(a, y) \vee P(b, y)$

(ii) $x=x, x=a \vee x=b, \neg(g(x) = x), R(a, b), R(g(a), g(b)), \neg R(a, g(a)) \vee \neg R(b, g(b))$

4. These clauses formalise the "Aunt Agatha" problem from Lecture 1.

$K(d, a), \quad d=a \vee d=b \vee d=c. \quad H(b, d), \quad x=b \vee H(b, x), \quad \neg a=b,$
 $\neg K(x, y) \vee H(x, y), \quad \neg H(x, f(x)), \quad \neg H(c, x) \vee x=b, \quad \neg K(a, a),$

Find a closed tableau using the given clauses by using the EQAX explicitly in the Model Elimination way. (**Hint:** start with clause $d=a \vee d=b \vee d=c$.)

(Using the RUE-tableau approach is much harder. You need a different top-clause. I managed by using $K(d,a)$ as top clause, but the tableau is not made exactly in the ME way.) Instead, why not try Otter on this example as well?