Automated Reasoning 2012 (KB)
PROBLEMS 5   Semantic Tableaux

QUICKIES:

1. *Use the standard tableau method to find a saturated tableau branch, and hence a model for*

$$F(x) \lor \neg G(x,y), \quad \neg H(v) \lor \neg F(v), \quad \neg F(u) \lor G(u,u), \quad H(a) \lor \neg G(a,z)$$

Assume the Signature ={a, b} and all variables {u,v,x,y,z} are universally quantified.

2. *Form a standard closed tableau of clauses of Q1 together with G(a,b) taking as top clause ¬F(u) ∨ G(u,u). Convert it to a free variable tableau and give the equations required for closure of each branch. Show the resulting unifier , when applied to the free variable tableau, gives the original standard tableau.*

LONGER:

1. Find a *standard* (not free variable) closed tableau for the sets of sentences (a) and (b):

a   (1) Fact(0,1)
     (2) $\forall$x,y,u,v [Fact*(x,y,u,v) $\rightarrow$ (Fact(x,y) $\rightarrow$ Fact(u,v))]
     (3) $\forall$x,y,u,v [(Fact(x,y) $\rightarrow$ Fact(u,v)) $\rightarrow$ Fact*(x,y,u,v)]
     (4) ¬ ($\forall$xy [Fact*(x,y,x,y)] $\land$ $\forall$xy[Fact*(0,1,x,y) $\rightarrow$ Fact (x,y)])

Hint: Start expanding the *last sentence* (4) first by the (¬$\land$) rule followed by the (¬$\forall$) rule and then use (3) in the left-hand branch and (1) and (2) in the right-hand branch.

b   (1) ¬([1, 2] $\subseteq$ [2,1])
     (2) $\forall$x,u,v [in(x, [u|v]) $\rightarrow$ is(x, u) $\lor$ in(x, v)]
     (3) $\forall$x,u,v [(is(x, u) $\lor$ in(x, v)) $\rightarrow$ in(x, [u|v])]
     (4) $\forall$x,y[$\forall$z [in(z, x) $\rightarrow$ in(z, y)] $\rightarrow$ x $\subseteq$ y]

Start with the *first sentence*. Uses Prolog list notation ie [1,2] is the list (1|(2|[])).

2. Find a *free variable* closed tableau for each of the sets of sentences (a) and (b):

a   (1) $\forall$x [¬blue(x) $\lor$ ¬green(x)],
     (2) $\forall$x[blue(x) $\lor$ green(x)],
     (3) On(A,B),   (4) On(B,C),   (5) green(C),   (6) blue(A),
     (7) $\forall$u$\forall$v[¬On(u,v) $\lor$ green(u) $\lor$ ¬green(v)]

Use the *first sentence at the top of the tableau*.
Hint: you will need to use sentence (7) twice in one of the branches.

b $\forall$x(($\exists$y.G(x,y)) $\rightarrow$ F(x)) $\forall$v(¬(H(v) $\land$ F(v))), $\exists$x$\forall$z(($\forall$y.G(x,y)) $\land$ (G(x,z) $\rightarrow$ H(x)))
Use as top sentence $\exists$x$\forall$z(($\forall$y.G(x,y)) $\land$ (G(x,z) $\rightarrow$ H(x)))

3. (Maybe for next week)

Find closed ME-tableaux for the sets of clauses in (a) - (c).

a   ¬Ha, ¬Fx $\lor$¬Hb, Fx $\lor$ Hx, ¬Gz $\lor$ ¬Fb, Gu $\lor$ ¬Fu

b   blue(A), On(A,B), green(C), On(B,C),
     ¬blue(x) $\lor$ ¬green(x), ¬On(u,v) $\lor$ green(u) $\lor$ ¬green(v)

c   ¬Pxa $\lor$¬Pax, Pxa $\lor$ Pxf(x), Pxa $\lor$ Pf(x)x