AUTOMATED REASONING

SLIDES 7:

SOME SYNTACTIC STRATEGY REFINEMENTS
Predicate Ordering
Locking

The Predicate Ordering Syntactic Refinement (ppt)

Predicate Ordering Strategy:
- Give each predicate a value (called an index) from some (partial) order (usually use positive integers as indices)
- Resolve only on a predicate in a clause that is *minimal in the order*
- Within the strategy perform a saturation search.

Example: S= (1) ¬Ha, (2) Fx ∨Hx, (3) ¬Gz ∨¬Fb, (4) ¬Fx ∨¬Hb, (5) Gx ∨¬Fx

Stage 0

Order predicates as G <H <F. i.e. resolve on an F literal only if no H or G literals and resolve on an H literal only if no G literals.

6 (1,2) Fa — 7 (2,4) Fb ∨¬Fx

8 (3,5) ¬Fb ∨¬Fx (factors to ¬Fb - safe factoring as ¬Fb subsumes ¬Fb ∨¬Fx which can be removed. Also subsumes clause 3.) Stage 1

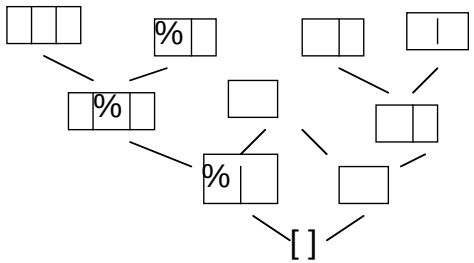
9 (6,7) Fb (subsumes clause 7) — 10 (8,9) [] Stage 2

Exercise:
(1) Identify forwards and backwards subsumption steps in an alternative Stage 2
(2) Try a different ordering.

Is Predicate Ordering a good strategy? (ppt)

If some predicates are regarded as being "more important" than others and the problem involves several different predicates, predicate ordering can be useful.

eg In a resolution derivation every literal from the *used* instances of given clauses must eventually be resolved upon and eliminated. In the schematic example below assume factoring occurs within a resolution step (when required).



If the literal "%" is one which it is considered might **not** easily be resolved away, it may be better to know this early in a derivation attempt; so give it an index near minimal in the order. If it is one which can **very likely** be resolved away then can give it a low priority) i.e. its index should come near maximal in the order).

Generally, make higher priority predicates come early in the order (i.e. high priority predicates are "<" low priority predicates); think of this as the strategy "preferentially choose high priority predicates".

Properties of the Predicate Ordering Strategy

7aiii

Is the strategy *SOUND* ? (When it yields a refutation, is that "correct".)
YES - WHY?

The reason is typical of refinements that restrict the resolvents that may be made in a resolution derivation, but always perform correct steps.

Is the strategy *COMPLETE* ?

(Does it yield a refutation whenever one is possible?) YES

To show completeness can modify the semantic tree argument.

Consider atoms in the tree according to their order: those highest in the order (or least preferred) go first (i.e. at the top) and those lowest (or most preferred) go at the end, with the rest in order in between.

i.e. if G<F then F atoms are put above G atoms.

Question: Why will this show completeness?
(Hint: think about how a semantic tree is reduced to produce a refutation)

Proving Completeness of Resolution Refinements

7aiv

Most completeness proofs for resolution strategies have two parts:

(i) show that a ground refutation of the right kind exists for some finite subset of the ground instances of the given clauses (eg see 5ai), and then

(ii) transform such a ground refutation to a general refutation (also of the right kind) (called *Lifting*).

To show (i) there are 3 possible methods:

(a) show that some refutation exists and transform it to one of the right kind; (useful for restrictions of resolution to control the search space), or

(b) show a refutation of the right structure exists by induction on a suitable metric (e.g. number of atoms, size of clauses), or

(c) give a prescription for directly constructing a resolution refutation of the right structure (eg using a semantic tree)

For Predicate ordering we used (c) adapting the method of semantic trees.
For Locking (next) we'll use (b).

7av

Strategies for Controlling resolution:

The control strategies in Slides 7 make use of a syntactic criterion on atoms to order literals in clauses. Literals are selected in order (<), lowest in order first. The strategies are quite simple, but in some circumstances can have a dramatic effect on the search space.

In all the strategies we assume that subsumption occurs;
however, as the examples on 7biii show, for the locking refinement subsumption may be a problem; furthermore, although factoring can probably be restricted to safe factoring, and be applied at any time, I am not aware that this has been proved formally. Here, it is assumed that when factoring (whatever kind) occurs in locking, the factored literals that are retained are the ones least in the order. e.g. if some factoring substitution unifies three literals with indices 1, 5, 7 and two other literals with indices 4 and 10, then the indices on the retained literals are 1 and 4.

Exercise:
In which situations would *predicate ordering* be useless? When might it be useful?
When might *locking* not be very effective? When could it prove useful?

The *atom ordering strategy* (in the optional material) is the most fine-grained (and complex) This strategy orders atoms by taking into account their arguments as well as their predicate.

NOTE for resolution applications: It is usually required to find just one refutation of []
(cf logic programming where all refutations of the goal clause may be required – this is usually applicable only if the "goal clause" is identified. Unless otherwise stated, backwards subsumption will be applied, even though some “proofs” may be lost – see slide 6biv.)

The Locking Refinement (ppt)

A different kind of literal ordering is *Locking*, invented by BOYER (1973)
Each literal in a clause is assigned a numeric index (not necessarily unique)
Literals in a clause are ordered by index, lowest first

Locking Strategy:

1. Assign arbitrary indices (called locks) to literals in given clauses.
2. Only resolve on a literal with lowest lock in its clause.
Note:There is no need for indices on unit clauses.
3. Resolvent literals inherit indices from the parents

Example: (We'll omit the "∨" symbol in clauses in what follows)
(locks are indicated in brackets after each literal and the lowest is underlined)

1. ¬Ha
2. { Fx (3), Hx(8)}
3. {¬Gz(5), ¬Fb (2)}
4. {¬ Fx(6),¬ Hb (4)}
5. {Gx(1), ¬ Fx(7)}

6. (2,3) {Hb (8), ¬Gz(5)}
7. (5,6) {¬Fz(7), Hb(8)}
8. (2,7) {Hz(8), Hb(8)} factors to Hb which subsumes 6 and 7
9. (8,4) ¬Fx subsumes 3,4 and 5
10. (9,2) Hx subsumes 2 and 8
11. (10,1) []

Use of Locking

Locking can be used in applications to restrict the use of clauses. eg.

- some clauses are more suited to 'top-down' use; (Horn clauses ?)

eg a clause A ∧B → C might only be useful in the context of the goal C

- some clauses are better suited to 'bottom-up' use (security access ?)

eg a clause A∧B → C may have originated from A →(B → C)
and the B → C part should only be made available when A is known to be true; ie perhaps A might be a more important condition than B.

- Locking can simulate predicate ordering - HOW?

Locking is *COMPLETE*.
(An inductive proof of this is given on Slide 7bvi)

Properties of the Locking Refinement

Some Unexpected Problems:

(i) *Problem with Tautologies*:
In most resolution refinements a tautology is redundant, so is never generated
In the locking refinement it is sometimes necessary to generate tautologies

Example
{P(1),Q(2)}, {P(13),¬Q(5)}, {¬P(9), Q(6)}, {¬P(3),¬Q(8)}

The only resolvents are tautologies; e.g. {P(13), ¬P(9)} (match ¬Q(5) and Q(6))

(ii) *Problem with Subsumption*:
Question: Does {A(8), B(6)} subsume {A(3), B(5), C(7)}?
Although it does so *without* the locks, could it be that removing the longer clause will prevent a locking refutation, since the first clause allows only to resolve on B, whereas the second allows only to resolve on A?

7biv gives an example of forward subsumption causing a problem of this sort.

Example of the locking subsumption problem

7biv

1. {P(1),Q(2)}, 2. {P(13),¬Q(5)}, 3. {¬P(9), Q(6)}, 4. {¬P(3),¬Q(8)}

5. (1+4) {Q(2), ¬Q(8)} a tautology (**can** safely be deleted)
6. (2+3) {¬P(9), P(13)} another tautology (**cannot** safely be deleted)
7. (5+2) {¬Q(8),P(13)} ``subsumed'' by 2
8. (1+6) {Q(2), P(13)} ``subsumed'' by {P(1), Q(2)} **MUST KEEP**

If the subsumed 8 is removed there is no way to derive the empty clause,
even if all of 1-7 are kept.

If {Q(2),P(13)} (i.e. clause 8) is retained, the refutation continues:

9. (8+2) P(13), 10. (4+9) ¬Q(8), 11. (3+10) ¬P(9), 12. (9+11) [ ]

(Indices on unit clauses are retained here to show the origin of the literals.)

Continued on 7bv .....

What went wrong .... on 7biv?

7bv

In the example the two clauses that subsume each other were

8. {Q(2), P(13)} and 1. {P(1), Q(2)}

A backward subsumption step that kept 8. and removed 1. admitted a solution

But a forward subsumption step that removed 8. and kept 1. did not

Moreover, in case of backward subsumption, if the subsumed clause 1. had not been used in all ways, deleting it too soon might also prove problematic.

Although locking is a good refinement, it shows how careful one must be regarding completeness. However, in general, locking does work well.

Factoring in refinements using literal ordering

In all literal ordering refinements non-safe factoring should involve one literal lowest in the ordering. Of the factored literals the lowest is retained. Other factoring steps can be delayed as they won't affect the refutations that can be found.

On the other hand safe factoring can occur at any time **(*)**

In Locking (*) is not immediately obvious due to the subsumption problem
Exercise: Is the statement (*) true for locking? (I believe it is!)

Proof that Locking for ground clauses is a Complete Refinement

7bvi

The proof uses induction on the number of *excess literals* in a set of clauses
excess literals = total number of literals - total number of clauses

The IDEA: (See diagram on 7bvii)

Assume S is unsatisfiable set of ground clauses with k≥0 excess literals.

Assume as induction hypothesis (IH) that for unsatisfiable set of ground clauses with <k excess literals there is a locking refutation from S

Case 1: (k = 0) All clauses unit clauses, so as unsatisfiable must exist A and ¬A

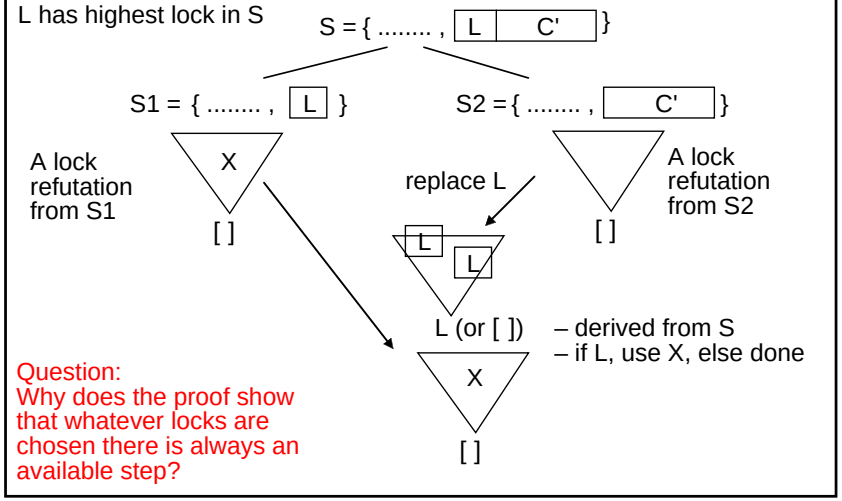
Case 2: (k>0)

- Select literal L with highest lock, from a non-unit clause C = L∨C', and temporarily remove it from C.
- Divide S into two sets S1=S-{C}+{L} and S2=S-{C}+{C'}; by IH refutations exist
- Add L back into S2 and combine the resulting refutation of L with that of S1.
- (The details are on Slide 7bviii)

This proof is an example of approach (b) on Slide 7aiv. It still has to be lifted to the general case, but that is not difficult.

Structure of Locking derivation used in completeness

7bvii



More details of Proof that Locking for ground clauses is complete: 7bviii

Let set of ground clauses S be unsatisfiable. We show by induction on the excess literal count (k) that there is a locking refutation from S.

Assume as induction hypothesis (IH) that for any unsatisfiable clause set S' with excess literal count <k there is a lock refutation.

Case 1 (k=0). Every clause in S must be a unit clause and as S is unsatisfiable there must be A and ¬A for some A which can be resolved to give [ ].

Case 2 (k>0) Select the literal L in S with highest lock (it must be in a non-unit clause C=L∨C') and form S1=S-{C}+{L} and S2=S-{C}+{C'}. Both are still unsatisfiable: for if either had a model that model would satisfy L or C', as well as S-{C}, and hence also S. Both have an excess literal count <k as C' and L are both smaller than C. Therefore, IH can be used to find a lock refutation of both. In the case of S2, adding back L gives either a derivation of L from S or of [ ] from S, in case L merges with a literal of lower lock index.

In the second of these we are done. In the first case, use the derived clause L wherever L was used in the refutation from S1 to derive [ ].

It is instructive to follow the construction for an example, say for the one on Slide 7bi. If you do so, you should find it constructs exactly the proof given.

Notice also that since locking is complete, for any ground clauses S and any set of locks there must always exist a first step.

Summary of Slides 7

7ci

1. Resolution derivations need to be controlled to avoid the search space exploding, even when tautology deletion, subsumption deletion and factoring are applied.

2. Syntactic methods that restrict particular literals in a clause can be used. Two methods were considered: Predicate Ordering and Locking.

3. Locking orders literal occurrences in a given set of clauses. Literals in resolvents inherit their locks from the corresponding literals in the parent clauses. (For recursive clauses this might be a problem - can you see why? - and an extension is to use dynamic locking, whereby locks are globally reset during a proof.)

4. There are problems with subsumption and tautolgy deletion in the Locking strategy - completeness may be lost if unrestricted tautology deletion and subsumption is allowed.

5. Locking can simulate Predicate Ordering. That is, suitable locks can be found, or a suitable atom ordering can be found, such that applying the strategy of Locking generates the same search space as using the strategy of predicte ordering.
(**Exercise**: Suggest suitable locks/atom ordering for the simulations.

START of OPTIONAL MATERIAL (SLIDES 7)

Atom Ordering

The Atom Ordering Strategy (continued on 7dii): 7di

In the *atom ordering strategy*, shown in Slides 7e as an optional Case Study, atoms are ordered by taking into account their arguments as well as their predicate.

For example, a *ground atom* might be given a weight according to the symbols (predicate, function and constant) that occur in it. Each symbol is given a weight ≥0 and the atomic weight of atom A is the sum of the weights of the symbols occurring in A. Such an ordering is called a *weight ordering* (a particular kind of atom ordering). Assuming the signature is finite then a set of ground atoms may be partially-ordered by their weight. (See slide 7dvi for details.) If no two symbols have the same weight, then the ordering on 7dvi will be total. (Find an example of a non-total ordering if this condition is false.)

Example:
Sig = <{H,G, F}, {}, {a, b}> and Clauses (from slide 7ai)
(1) ¬Ha, (2) Fx ∨Hx, (3) ¬Gz ∨¬Fb, (4) ¬Fx ∨¬Hb, (5) Gx ∨¬Fx

Assign symbols a weight according to their order in the signature (H=1, b=5);
the weights of ground literals are then: Ha:5, Hb:6, Ga:6, Gb:7, Fa:7, Fb:8. Then Hb and Ga would be ordered Hb<Ga, for example, since the weights are equal, but wt(H)<wt(G).

The weights of literals with variables will depend on the bindings to the variables. In each clause the literal with lowest weight is (1): ¬ Ha, (2): Hx, (3): ¬Gz, (4): ¬Hb, (5): Gx. In clause (2), whatever the binding to x, Hx always has lower weight than Fx. In clause (3), Gz is always smaller than Fb, whatever the binding to z.

Atom Orderings continued: 7dii

A derivation using the ordering on 7ei is:
(6=1+2): Fa, (7=2+4): Fb ∨¬ Fx, (8=6+7): Fb, (note literals in 7 are not comparable)
(9=3+5): ¬Fx ∨ ¬Fb, which safe factors to ¬Fb, which subsumes (3),
(10=8+9):[ ]

The weights could have been differently assigned; e.g. a=1, b=2, F=3, G=4, H=5;
In this case Fb has weight 5 and Gz has weight either 5 or 6 depending on the binding to z. Therefore, in clause (3) Fb and Gz are not comparable, since in one case ¬Fb must be selected and in the other case either ¬Fb or ¬Gz could be selected. Thus both are deemed (potentially) minimal in clause (3) and either can be selected.

A refutation using this weight ordering is:
(6=2+3): Hb ∨¬Gz, (7=2+4): Hx ∨ ¬Hb, (8=2+5): Hx ∨Gx,
(9=6+8): Hb∨ Hx, which safe factors to Hb and then subsumes (6),
(10=1+7): Hx, (11=1+10): []

Other kinds of orderings have been investigated;
e.g. we might ignore variables, so Fx is assigned a weight depending only on F, etc.
In this case, literals that are not ground are preferred over ground literals with a similar structure (e.g. Fx <Fa).
The derivation according to this ordering is: (6=1+2): Fa, (7=4+6): ¬ Hb,
(8=3+5): ¬ Fb ∨¬ Fx, which safely factors to ¬Fb, (9=2+7): Fb, (10=8+9): [ ]

The atom ordering strategy can simulate predicate ordering but not locking.
Locking can simulate predicate ordering and sometimes atom ordering.

Atom ordering (1) (A case study) 7diii

An extension of predicate ordering is to order ground literals according to all of their symbols, not just the predicate.

Example of atom order 1:
P(a) < P(b) < P(f(a)) < … < Q(a,a)<Q(a,b)< ...< R(a) etc.
(i.e. A<B if A is lexicographically before B when A and B are written as lists)
This ordering is used in the example on slide 7cvi.

Atom Ordering Strategy:
Literal L is *minimal* in clause C if not(∃K in C s.t. K<L) .
Select for resolution only literals that are minimal in a clause.
Notice that a non-empty clause always has at least one such literal.

Can you think of a problem when all symbols are considered?

An ordering on ground atoms can be partially extended to non-ground atoms as long as it ***respects instantiation:***
i.e. A ≤ B iff Aθ ≤ Bθ for every θ.
(A ≤B if A < B or A=B)
This is called a ***stable ordering***.

Atom ordering (2) 7div

Unfortunately for general clauses an atom order is not a total order
(given two atoms X and Y, it may not hold that X<Y or that Y<X

Examples: These all use the lexicographic ordering

In the clause P(a) ∨ P(b), P(a) < P(b)

In P(a) ∨ P(x) ∨ R(x), P(a) ≤ P(x)θ for all substitutions θ hence P(a) ≤ P(x) < R(x)

In P(b) ∨ P(x) ∨ R(x), P(b) and P(x) are incomparable
(it depends whether x≥b or not), so can select either

In P(x) ∨ P(y) ∨ Q(x,y)
if x==a, y==b, then P(x) is the minimal literal
if x==b and y==a then P(y) is the minimal literal

In Q(x, y) ∨ ¬Q(y, x), is Q(x,y) < Q(y,x) ?
It depends on the bindings to x and y, so Q(x,y) and ¬Q(y,x) are not comparable in the clause and *both* are minimal.

Example of a refutation using Atom ordering

(1) Px ∨ Ry ∨ ¬Qxy (2) ¬Sz ∨ ¬Rz (3) Pu ∨ Qf(v)v
(4) Sa (5) Sb (6) ¬Pf(a) ∨ ¬P(f(b)

(7) (1+6) Ry ∨ ¬Qf(a)y ∨ ¬ Pf(b)
(8) (3+6) Qf(v)v ∨ ¬Pf(b)

(9) {1+7) Ry ∨ ¬Qf(a)y ∨ ¬Qf(b)y1 ∨ Ry1
(10) (3+7) Ry ∨ ¬Qf(a)y ∨ Qf(v)v
(11) (1+8) Ry ∨ ¬Qf(b)y ∨ Qf(v)v
(12) (3+8) Qf(v)v (after factoring)
(12) subsumes (3), (8), (10), (11)

} Note 2 minimal literals

(13) (9+12) Ra ∨ ¬Qf(b)y1 ∨ Ry1

(14) (12+13) Ra ∨ Rb

(15) (14+2) ¬Sa ∨ Rb

(16) (15+2) ¬Sa ∨ ¬Sb

(17) (16+4) ¬Sb
(18) (17+5) []

Use Lexicographic ordering as in Example 1 on 7div

The Weight Atom Ordering

(Sig = ({P,Q,R,S}, {f}, {a,b});
give each symbol a weight (e.g. P:1, a: 1, Q:2, b: 2, R:3, f: 3, S:4);
define wt(atom / term) = sum of wts of symbols in atom / term; wt(¬A)=wt(A)

For ground atoms or terms A and B, we say A < B if
either wt(A) < wt(B),
or wt(A) = wt(B) & wt(arg_j(A)) < wt(arg_j(B))
& wt(arg_i(A)) = wt(arg_i(B)) , for all i<j

(**Note**: we count the predicate symbol of an atom, or functor of a term, as arg0.)

For the above weights:
Pa<Pb<P(f(a))<P(f(b))<P(f(f(a)))…;
P(x) < Q(x,a) < R(x) < S(x) for any x;
Qaa<Qab<Qba<Qbb<Qaf(a) <Qf(a)a<Qbf(a)<…;
P(f(a))<Q(a,b)<...

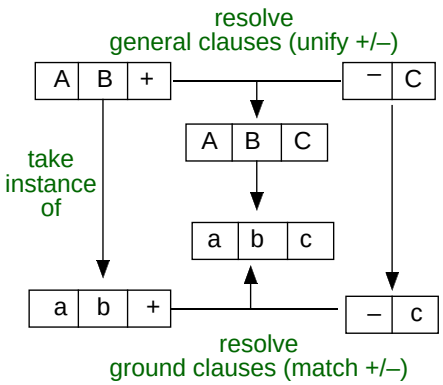
If no two predicate symbols or functors (including constants) have the same weight, then the order on ground terms is total. i.e. given 2 ground atoms A and B then either A<B or B<A. (**Easy exercise**: Show this.)

Features of Atom Ordering (1)

Is Atom Ordering Complete?

Can employ the semantic tree method as for predicate ordering to show that an atom ordered refutation exists for any set of *ground* unsatisfiable clauses.

Lifting to general case (Proof, see 7eix)



If A>+ and B>+ there is no problem as we will be able to find the required step corresponding to the step taken at ground level.

Could +>A?
No - because of stability; it would force +>a, contradicting assumption.

It could be that both A and + were minimal, so the step resolving between + and – would still be allowed.

Analogously for B,C and –.

(Assume: a>+, b>+, c>–

Features of Atom Ordering (2)

A possible improvement?
Do *not* resolve on an atom if the mgu would make it non-minimal in its clause.

e.g. In Px ∨ Pb, don't resolve on Px if x becomes bound by the mgu to c>b.

Do you see any problems with this suggestion?

Questions:
Suggest a simple ordering of atoms that enables atom ordering to simulate predicate ordering.

Explain why locking can simulate atom ordering for ground clauses.

Explain why locking cannot usually simulate atom ordering for general clauses? (Hint: consider the example on slide 7dv)

Can atom ordering simulate locking for ground clauses?
(Hint: locks can be arbitrary)

Lifting an Atom Ordering Derivation - proof of completeness 7dix

Assume the atom ordering is stable and that a ground refutation exists

Consider lifting a step (at ground level) involving a minimal literal L and its complement to the general level; then either:

- all instances Lθ of the general literal L used for resolving respect the ordering in the clause - i.e. L is minimal in the clause so no problem, or
- some instances do not respect the ordering; then L would be incomparable with other literals in the clause and so still be available by the strategy; thus no proofs are lost.

Notice that no other literal M in the clause would satisfy M <L;

Proof: Assume for contradiction there is such a literal M, M<L.
Then by stability Mθ ≤ Lθ for all ground substitutions θ;
this includes the ground instance of L used in the given ground refutation contradicting its minimality.

Hence L is minimal (possibly by incomparability).

Summary of Optional Material in Slides 7

1. Atom Ordering orders atoms according to some syntactic criteria, such as weight ordering, or lexicographic weight ordering.

2. An order is *stable* iff,
A<B in a clause C implies Aθ ≤ Bθ in Cθ, for any instance Cθ of C.

3. In Locking, the order of a literal in a clause depends solely on its lock relative to other literals and does not vary with instance. Thus the locking order is stable: if A<B in a clause C then the lock on A is < the lock on B and similarly Aθ < Bθ in Cθ, for any instance Cθ of C.

4. Atom Orderings are not necessarily stable. (**Exercise**: make up an example of a non-stable atom ordering). If C is a clause in which literals A and B may be ordered A<B or B<A, depending on the particular instance of C, then A and B are not comparable. If there is no literal definitely < A, then A is minimal in the clause and may be selected for resolution.

5. Atom Ordering cannot simulate Locking, nor can Locking simulate Atom ordering (in general).

7ei