Dialectical Formalisations of Non-monotonic Reasoning: Rationality under Resource Bounds

Sanjay Modgil
Department of Informatics, King’s College London

Outline of Talk

- Introduction to ASPIC+ and argumentative formalisations of non-monotonic logics

- Limitations of ASPIC+
  - not all rationality postulates are satisfied
  - rationally postulates that are satisfied assume unbounded reasoners

- A dialectical account of ASPIC+ that is *fully* rational under assumption that agents have bounded resources

The ASPIC+ Framework [1]

- A framework for structured argumentation
- Establishes guidelines guaranteeing satisfaction of rationality postulates when defining non-monotonic inferences via argumentation

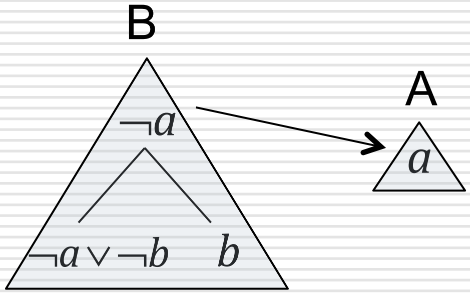
- Define for some arbitrary language $\mathcal{L}$:

    1) KB of infallible and/or fallible premises that are wff in $\mathcal{L}$
    2) Strict and/or defeasible rules *inference* rules respectively encoding inference in some deductive logic and domain specific defeasible/ default inferences
    3) Contrary function declaring when one formula conflicts with another e.g., α and $\neg\alpha$ are contraries of each other

1. S. Modgil, H. Prakken. A General Account of Argumentation and Preferences. In: **Artificial Intelligence (AIJ)** . 195(0), 361 - 397, 2013.

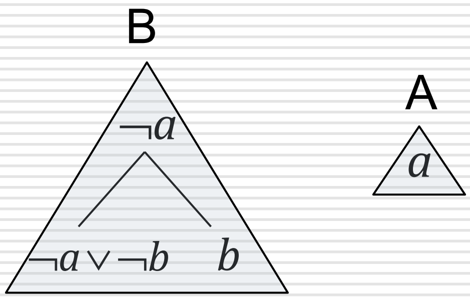
ASPIC+ Arguments, Attacks and Defeats (Example 1)

- Totally ordered set of (inconsistent) formulae Δ = fallible premises
- Strict inference rules encoding classical logic inference



B *attacks* A

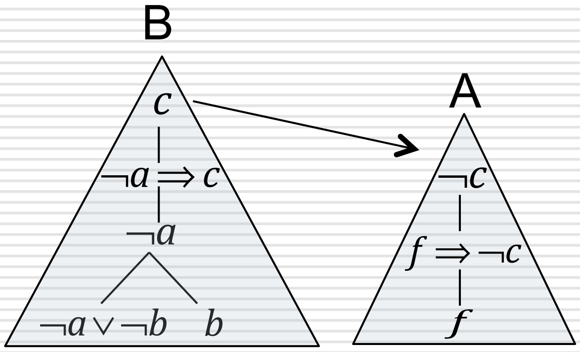
ASPIC+ Arguments, Attacks and Defeats (Example 1)



B does not *defeat* A given b < a (and so B < A)

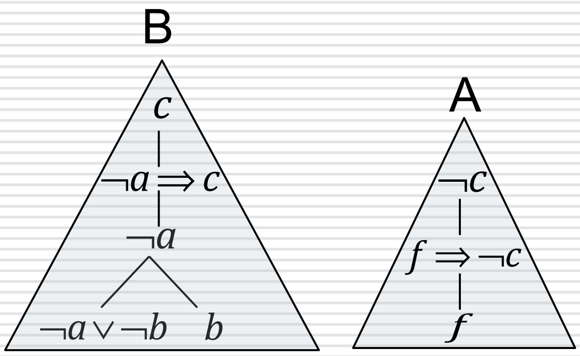
ASPIC+ Arguments, Attacks and Defeats (Example 1)

- Unordered set of (consistent) formulae W = infallible (axiom) premises
- Strict inference rules (classical logic) and (ordered) defeasible inference rules (defaults)



B *attacks* A

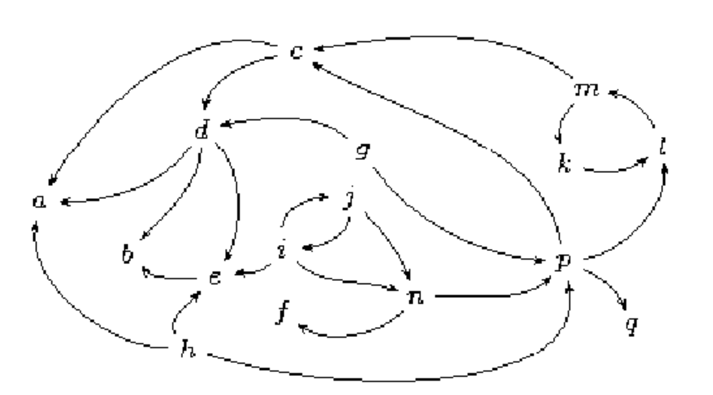
ASPIC+ Arguments, Attacks and Defeats (Example 1)



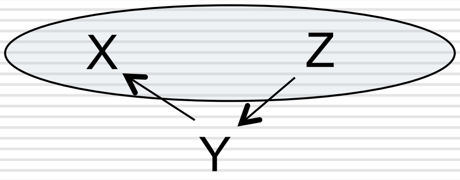
B does not *defeat* A given $\neg a \Rightarrow c < f \Rightarrow \neg c$ and so B < A

Evaluating Dung Framework[1] of Arguments and Defeats

- ($\mathcal{Args}$,$\mathcal{Defeats}$) defined by ASPIC+ theory = (KB, Inference rules, and strict preference ordering over $\mathcal{Args}$)



- Intuitive, principle of defense establishes membership of arguments in sets of winning/justified *extensions*

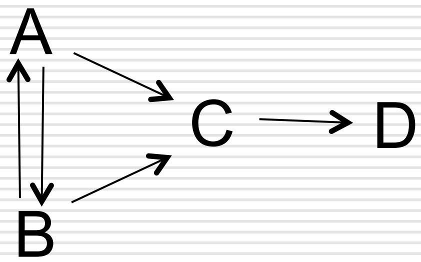


- Sceptical / credulously justified arguments under different semantics (arguments in all/at least one extension)

1. P. M. Dung. On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. ***Artificial Intelligence***, 77(2):321–358, 1995

Argument Evaluation

- (*Args*,*Def*) =



- 2 *preferred* ***extensions***
  - **{A,D}** (A defends itself against B, and A defends D against C)
  - **{B,D}** (B defends itself against A, and B defends D against C)

- Single *grounded* extension = ∅ (arguments cannot defend themselves)
- Many other semantics extensively studied in research literature

Argumentation-based characterisations of non-monotonic inference relations in ASPIC+

$(\mathcal{A}rgs, \mathcal{D}ef)_\Delta \mathrel{|\!\sim} \alpha$ (the claim of an argument in grounded extension)

iff

$\Delta \mathrel{|\!\sim}_{LP} \alpha$ (under well founded semantics of logic programming)

$(\mathcal{A}rgs, \mathcal{D}ef)_\Delta \mathrel{|\!\sim} \alpha$ iff $\Delta \mathrel{|\!\sim}_{PS} \alpha$

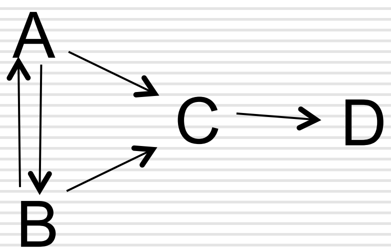
PS – *Brewka's Preferred Subtheories* inferences from totally ordered set of classical wff Δ (Δ = fallible premises + $R_{S(CL)}$)

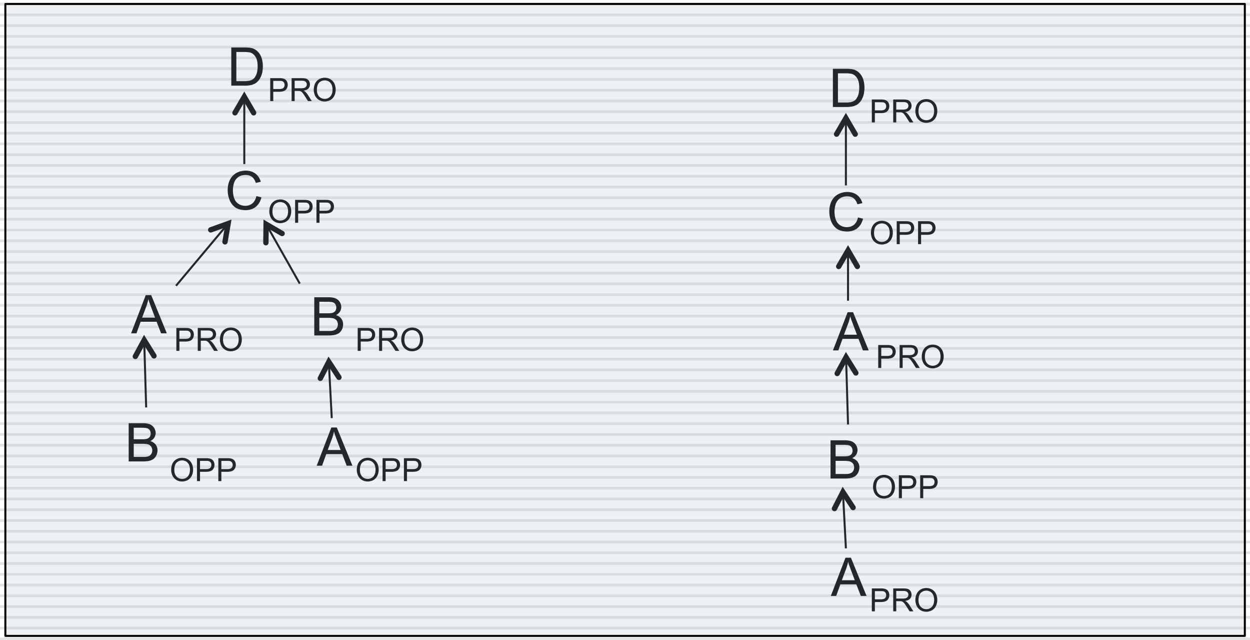
$(\mathcal{A}rgs, \mathcal{D}ef)_\Delta \mathrel{|\!\sim} \alpha$ iff $\Delta \mathrel{|\!\sim}_{PDL} \alpha$

PDL – *Prioritised Default Logic* (Reiter's normal default logic + priorities (W,D,<))

(W = axiom premises, D = defeasible inference rules + $R_{S(CL)}$)

Semantic Specific Argument Game Proof Theories for Deciding Membership of Arguments in Extensions *





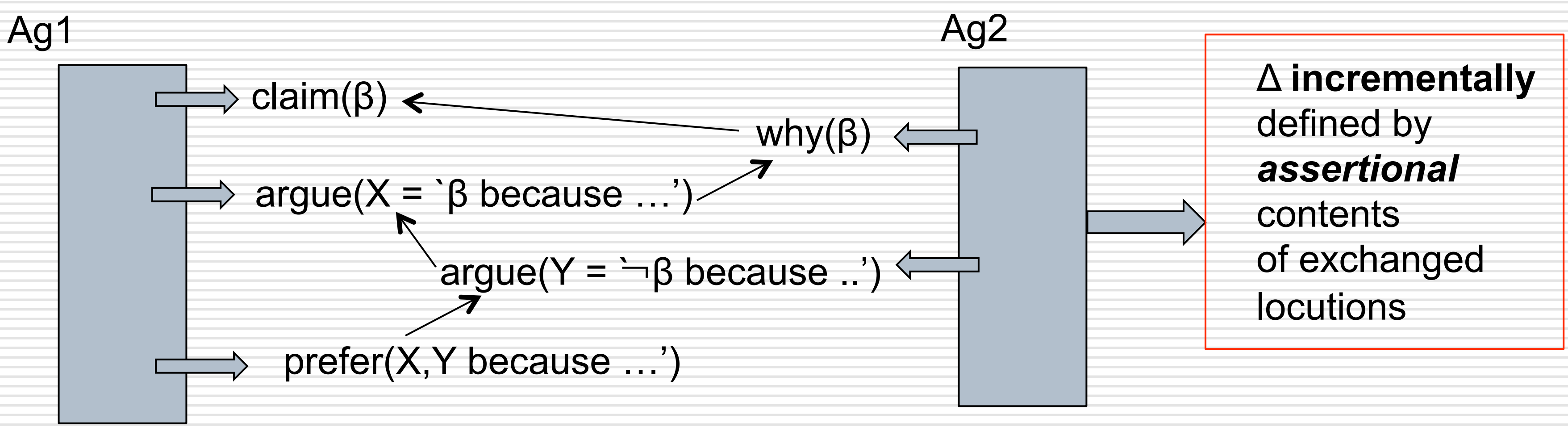
In *grounded* game (PRO loses)

In *preferred* game PRO wins and is said to have a *winning strategy*

* S. Modgil and M. Caminada. Proof Theories and Algorithms for Abstract Argumentation Frameworks. In: *Argumentation in AI,* I. Rahwan and G. Simari (eds), 105-132, 2009.

From single agent reasoning to distributed (non-monotonic) reasoning via dialogue

"The lonesome thinker in an armchair is as marginal as he looks: most of our logical skills are displayed in interaction" – *J. Van Bentham*



Ag1 wins dialogue ***iff*** $\Delta \mid\!\sim \beta$

ASPIC+ and Rationality (Consistency)

- *Consistency*: premises, intermediate conclusions and claims of arguments in an extension are mutually consistent

- Shown under two assumptions:

  1) **Logical Omniscience**: ($\mathcal{Args}, \mathcal{Defeats}$) includes ***all*** arguments that can be constructed from premises and inference rules

  e.g., $\mathcal{Args} = \{(\Gamma, \alpha) \mid \Gamma \in P(\Delta), \Gamma \vdash \alpha\}$ where Δ = set of classical wff

  2) **'Reasonable' Preference Relations**: Preference relation over arguments must satisfy certain properties

ASPIC+ and Rationality (Non-contamination)

- ***Non-contamination***:

  Argumentation defined inferences from KB and inference rules R are not invalidated when adding premises and rules that are syntactically disjoint from KB and R

- Satisfied *only by classical logic argumentation*, under the assumption that arguments' premises are checked for *consistency and subset minimality*

  e.g.

  $\mathcal{A}rgs = \{(\Gamma,\alpha) \mid \Gamma \in P(\Delta), \Gamma \mid\!- \alpha, \Gamma$ *is consistent and minimally entails* $\alpha\}$

ASPIC+ and Rationality (Non-contamination)

- Suppose consistency check not implemented

KB = {s} and so ({s},s) is in single grounded extension

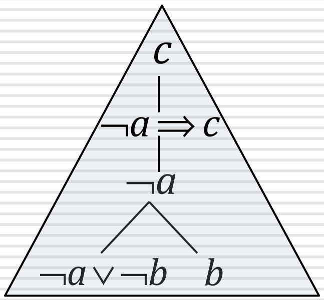
KB' = {s,p,¬p} and now ({p,¬p}, ¬s) defeats ({s},s) which is now no longer in grounded extension !

- As we will see later, if subset minimality check not implemented this may also result in contamination

ASPIC+ and Rationality (Non-contamination)

- Logical Omniscience and subset minimality/consistency checks on arguments' premises are clearly not feasible for real world resource bounded agents

- As of yet no solution to contamination problem for ASPIC+ arguments incorporating defeasible inference rules (e.g. ASPIC+ formalisations of (prioritised) Default Logic)



A *Dialectical* formulation of ASPIC+ that is fully rationality under resource bounds

- We want a framework for dialectical formalisations of non-monotonic reasoning for use by resource bounded agents reasoning individually and via dialogue, that:

  1) Drops computationally expensive consistency and subset minimality checks on arguments

  2) Drops assumption of logical omniscience

  3) Is fully rational (non-contaminating and consistent)

A *Dialectical* formulation of ASPIC+ that is fully rationality under resource bounds

Joint Work with M. D'Agostino, Dept. of Philosophy Milan

M. D'Agostino and S.Modgil
Classical Logic, Argument and Dialectic.
In Artificial Intelligence (AIJ). 262, 15 - 51, 2018.

M. D'Agostino and S.Modgil.
A Study of Argumentative Characterisations of Preferred Subtheories
In: 27th Int. Joint Conference on Artificial Intelligence (IJCAI-ECAI-18), 1788-1794, 2018

M. D'Agostino and S.Modgil.
Dialectical Formalisations of Non-monotonic Reasoning: Rationality under Resource Bounds. In preparation. 2019.

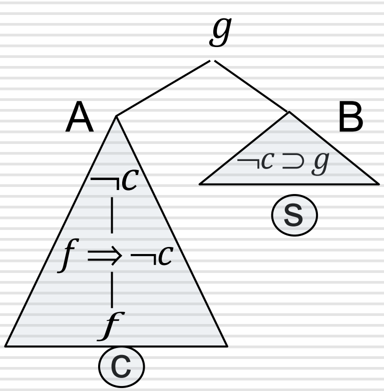
A Dialectical Ontology for Arguments

- The solution is to define an ontology for arguments (*qua* proofs) and evaluation of arguments that accounts for their *dialectical* use
- In practice, arguments are of the following form :

Given that I am committed to the claims Δ and supposing for the sake of argument your commitment to the claims Γ, it then necessarily (deductively) follows that α

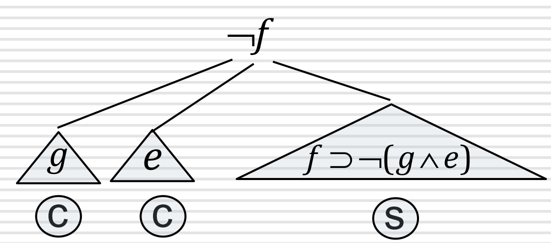
So an argument is now a *triple* (*Δ, Γ, α*) – **no subset minimality or consistency checks**

Δ are the *commitments* and *Γ* the *suppositions*



({*A*}, {*B*}, *g*)

$(\{g, e\}, \{f \supset \neg(g \wedge e)\}, \neg f)$



Dialectical Defeat and Defense

- Recall that an ‘extension’ E is a set of arguments that defend themselves against all defeats

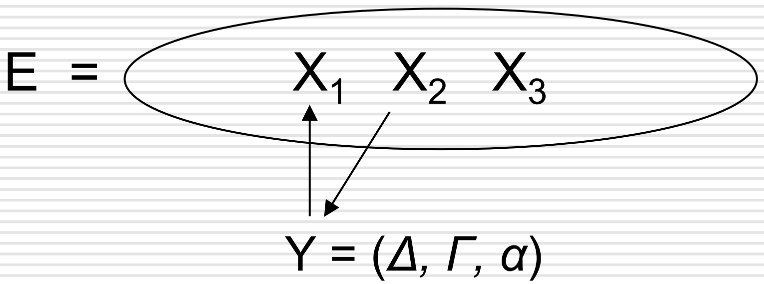
$$E = \{ X_1 = (\Phi_1 , \Sigma_1 , \beta_1) \quad X_2 = (\Phi_2 , \Sigma_2 , \beta_2) \quad X_3 = (\Phi_3 , \Sigma_3 , \beta_3) \}$$

Y

- $Y = (\Delta, \Gamma, \alpha)$ *dialectically defeats* $X_1 = (\Phi_1 , \Sigma_1 , \beta_1)$ if $-\alpha$ is a fallible premise π, or conclusion of a defeasible rule of some argument π in Φ_1, and $Y \nless \pi$ and suppositions Γ of Y are a subset of the commitments of $X_{1\text{-}3} = \Phi_1 \cup \Phi_2 \cup \Phi_3$

Intuitively, given that I commit to Δ and supposing for the sake of argument your commitments in E, then Y is a counter-argument to X_1

Dialectical Defeat and Defense

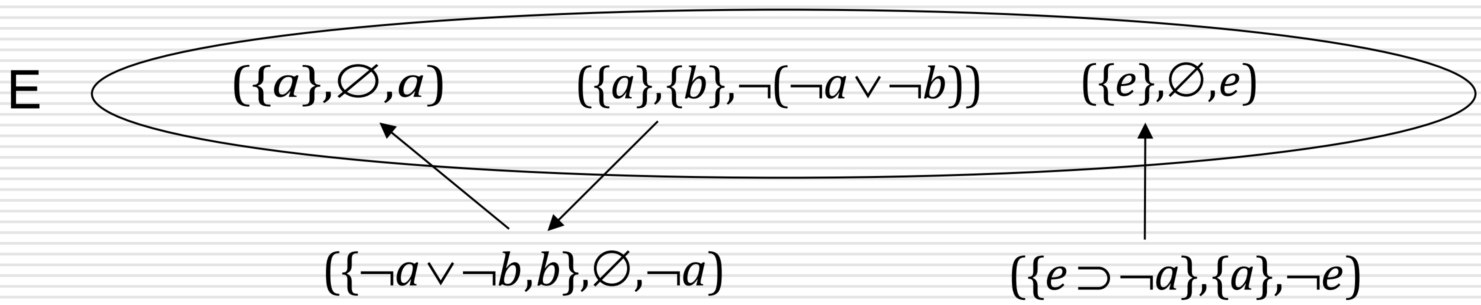


- $X_2 = (\Phi_2\ , \Sigma_2\ , \beta_2)$ counter-argues Y (and so defends X_1) if $-\beta_2$ is a fallible premise δ, or conclusion of a defeasible rule of some argument δ in *Δ*, and $X_2 \nless \delta$

  and the suppositions Σ_2 of X_2 are a subset of the commitments *Δ* of Y

Intuitively, given my premises Φ_2 and supposing for the sake of argument Σ_2 that you've committed to (in Y), then X_2 is a counter-argument to Y

Classical Logic Example



Dialectical Demonstrations of Inconsistency

- Preferences over dialectical arguments are used in the usual way to define defeats, except that

attacks from falsum arguments $(\emptyset,\Delta,\bot)$ ***always succeed as defeats (independently of preferences)***

- Arguments of the form $(\emptyset,\Delta,\bot)$ cannot be defeated since they have empty commitments – they are said to be unassailable

- Eg Galileo's famous refutation of Aristotle's theory of falling bodies, in the form of a dialogue demonstrating that the premises of arguments justifying that heavier bodies fall faster than lighter bodies, lead to a contradiction

Consistency under standard ASPIC+ formalisation of Classical Logic Argumentation

Logical omniscience and *conditions on preference relations* assumed as sufficient conditions to guarantee consistency

E.g., to ensure that A, E, F cannot coexist in an extension ...

A= $\{p \rightarrow \neg q\}: p \rightarrow \neg q$ E = $\{p\}$:p F=$\{q\}:q$

$B = \{p,q\}: \neg(p \rightarrow \neg q)$ $C = \{q, p \rightarrow \neg q\}: \neg p$ $D = \{p, p \rightarrow \neg q\}: \neg q$

need to assume $B, C, D \in Args$ **and** that either $B \nprec A$ or $C \nprec E$ or $D \nprec E$

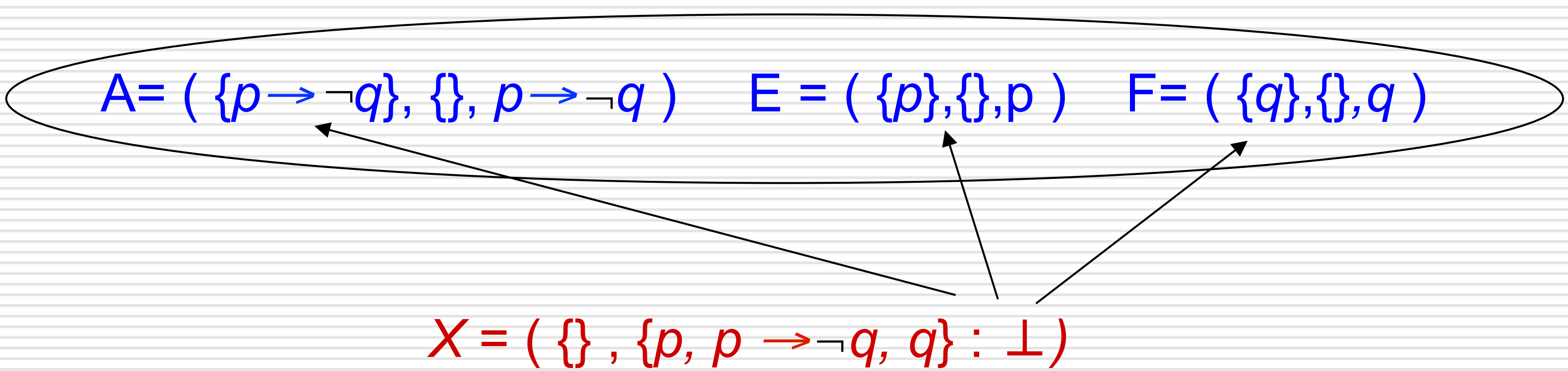
Satisfying Consistency in Dialectical Formalisation of ASPIC+ (Classical Logic Example)

If resources suffices to recognise inconsistency through construction of arguments that make contradictory claims e.g. $F = (\{q\},\{\},q)$ and $D = \{p, p \rightarrow \neg q\} : \neg q$

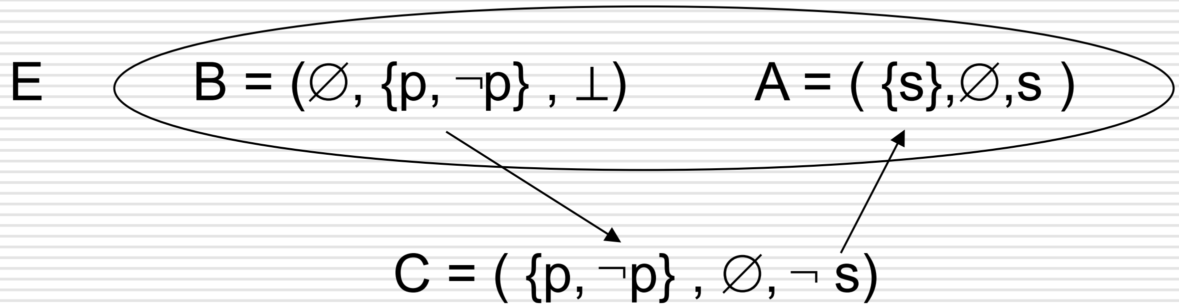
and

resources suffice to combine premises of arguments with conflicting conclusions, so obtaining unassailable X which (independently of preferences) defeats each of the arguments with the culpable premises, and cannot itself be defeated

Hence consistency postulates satisfied ***independently of preferences***



Satisfying Non-contamination in Dialectical Formalisation of ASPIC+ (Classical Logic Example)



Despite dropping consistency checks on arguments' premises, *explosivity* does not result in contamination:

A = ({s},∅,s) is in the grounded extension since B defeats C (independently of preferences) and so defends A, and B itself cannot be defeated

Satisfying Non-contamination in ASPIC+
The problem of relevance

$A = (\{s\}, \{\}, s)$

$B = (\{\neg s\}, \emptyset, \neg s)$

$B \prec A$ and so B does not defeat A and A is in the grounded extension

Satisfying Non-contamination in ASPIC+ : The problem of relevance

A = ({s}, {} , s)

B = ({¬s} , ∅, ¬s)　　　C= ({p, ¬s} , ∅, ¬s)

B ≺ A but C ⊀ A and so C defeats A and A is not in the grounded extension

- Subset mimimality is an unfeasible means of enforcing relevance
- We require a notion of relevance that can be enforced *proof theoretically*

Relevance defined in terms of syntactic disjointedness

Proposition: If the deductive inference encoded in the strict rules is such that

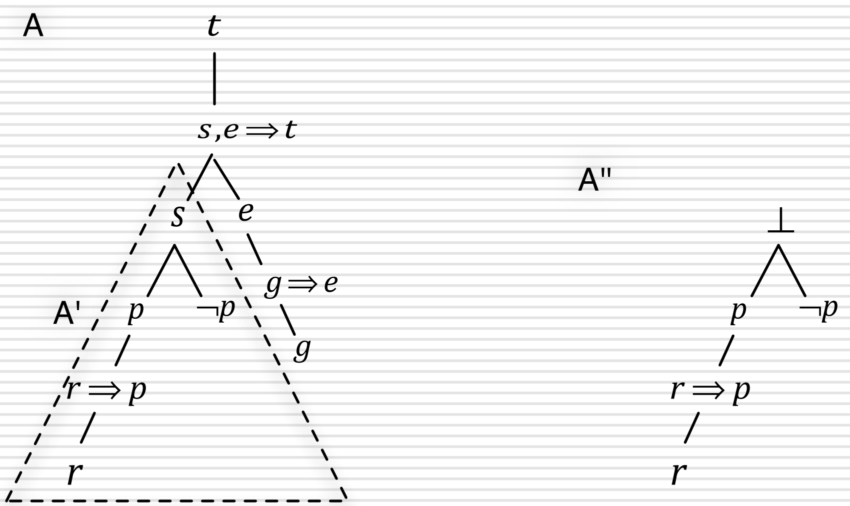
if $\Gamma \vdash \alpha$ *and* $\exists \Delta \subseteq \Gamma$ *such that* $\Delta \,/\!/\, (\Gamma \setminus \Delta) \cup \{\alpha\}$, *then: either* $\Delta \vdash \bot$ *or* $\Gamma \setminus \Delta \vdash \alpha$ *

then given a *contaminated* ASPIC+ argument X that includes a set of syntactically disjoint premises and defeasible inference rules Δ, there exists either:

- a non-redundant counterpart to X constructed from $\Gamma \setminus \Delta$ and that concludes the same claim as X, or;
- an inconsistent component of X constructed from Δ that can be defeated by an unassailable falsum argument

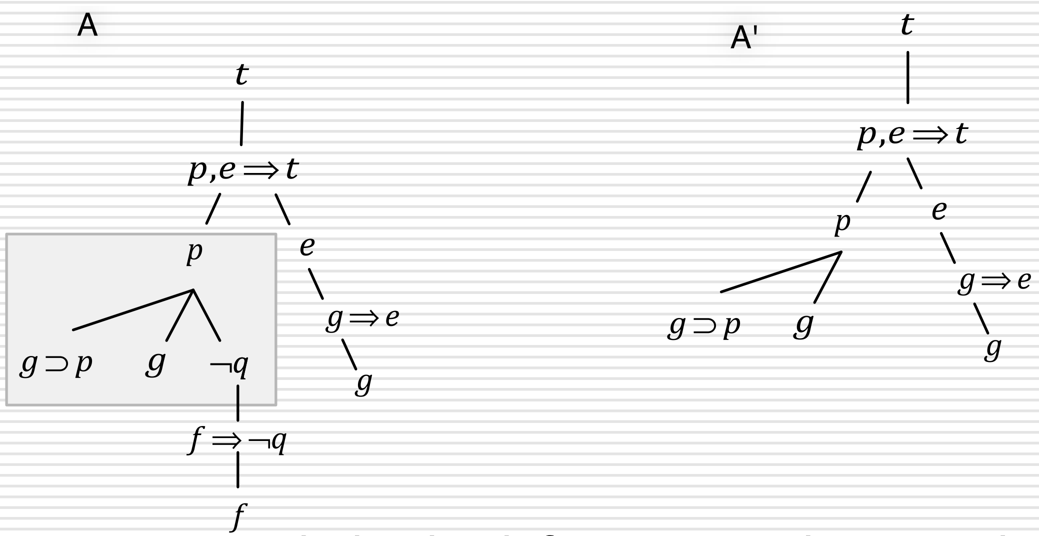
Note that * is satisfied by classical logic

Excluding arguments that are contaminated due to explosivity



the unassailable $(\emptyset, A'', t)$ defeats A on A' independently of preferences and so precludes the contaminating effect of A

Proof theoretic exclusion of arguments that are contaminated dues to non-explosive redundant components

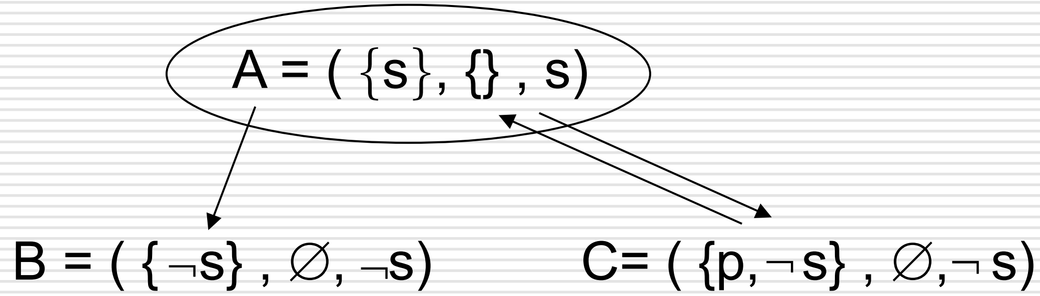


Redundancy due to non-relevant deductive inference can be excluded proof theoretically e.g., use of *Intelim* classical natural deduction system in

M. D'Agostino, D. Gabbay and S.Modgil *Normality, non-contamination and logical depth in classical natural deduction. In: Studia Logica,* pp 1–67 *Feb, 2019.*

will not license redundant inference of *p* from from *g*, $g \supset p, \neg q$. Hence only non redundant argument (A') can be constructed

Satisfying Non-contamination in ASPIC+ (Classical Logic Example)



$B \prec A$ but $C \not\prec A$ and so C defeats A and A is not in the grounded extension

- Either C is excluded proof theoretically (e.g. through use of *Intelim* natural deduction system)

or

- If proof system licences construction of arguments such as C then preference relation must be such that arguments are not strengthened when adding syntactically disjoint premises/rules. Hence $C \prec A$ *and so C does not defeat A*

A Fully Rational ASPIC+ For Resource Bounded Agents

Let ($\mathcal{A}rgs$,$\mathcal{D}efeats$) be defined by ASPIC+ theory Δ = (*KB,R*), where $\mathcal{A}rgs$ is ***any*** subset of the dialectical arguments defined by Δ such that

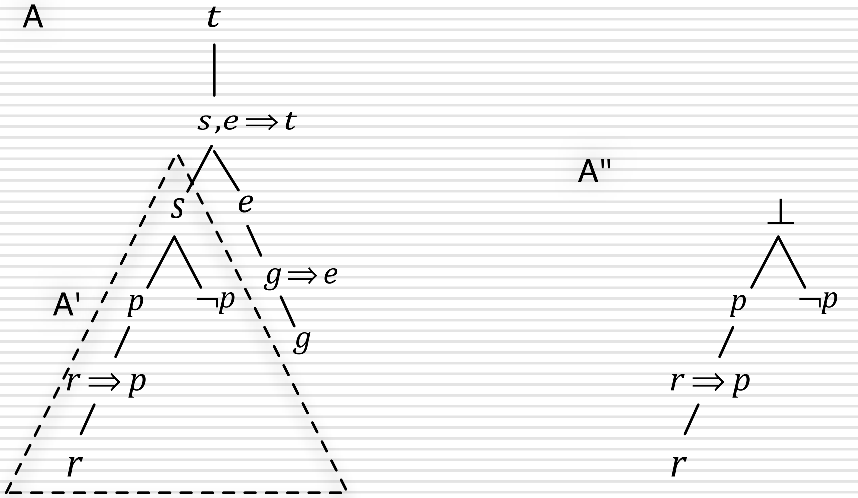
1) If $\boldsymbol{\alpha}$ is a premise in KB then $(\{\boldsymbol{\alpha}\},\{\},\boldsymbol{\alpha}) \in \mathcal{A}rgs$

2) If $(\Delta,\{\},\boldsymbol{\alpha})$ and $(\Gamma,\{\},\text{-}\boldsymbol{\alpha}) \in \mathcal{A}rgs$ then $(\Delta \cup \Gamma,\{\},\perp)$ and so $(\{\},\Delta \cup \Gamma,\perp) \in \mathcal{A}rgs$

3) If $(\Delta \cup \Gamma,\emptyset,\boldsymbol{\alpha}) \in$ *Args* and Δ syntactically disjoint from $\Gamma \cup \{\boldsymbol{\alpha}\}$ then

i) if redundant arguments are proof theoretically excluded (i.e., $\Gamma = \{\}$) then $(\Delta,\{\},\perp) \in \mathcal{A}rgs$

Excluding arguments that are contaminated due to explosivity



If A $\in \mathcal{A}rgs$ then A`` $\in \mathcal{A}rgs$

A Fully Rational ASPIC+ For Resource Bounded Agents

Let ($\mathcal{A}rgs$,$\mathcal{D}efeats$) be defined by ASPIC+ theory $\Delta = (KB,R)$, where $\mathcal{A}rgs$ is ***any*** subset of the dialectical arguments defined by Δ such that

1) If $\boldsymbol{\alpha}$ is a premise in KB then $(\{\boldsymbol{\alpha}\},\{\},\boldsymbol{\alpha}) \in \mathcal{A}rgs$

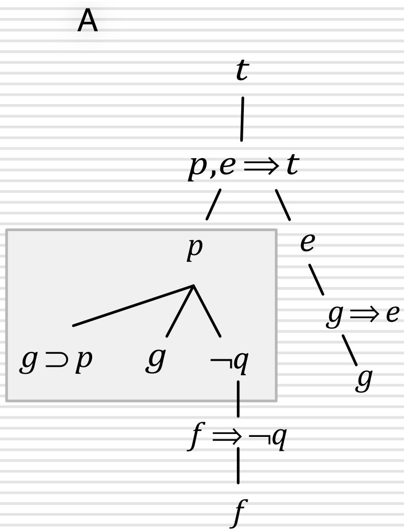
2) If $(\Delta,\{\},\boldsymbol{\alpha})$ and$(\Delta',\{\},-\boldsymbol{\alpha}) \in \mathcal{A}rgs$ then $(\Delta \cup \Delta',\{\},\bot)$ and so $(\{\},\Delta \cup \Delta', \bot) \in \mathcal{A}rgs$

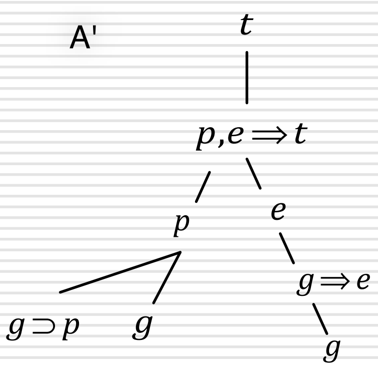
3) If $(\Delta \cup \Gamma,\emptyset,\boldsymbol{\alpha}) \in Args$ and Δ syntactically disjoint from $\Gamma \cup \{\boldsymbol{\alpha}\}$ then

i) if redundant arguments are proof theoretically excluded (i.e., $\Gamma = \{\}$) then $(\Delta,\{\},\bot) \in \mathcal{A}rgs$

ii) else $(\Delta,\{\},\bot) \in \mathcal{A}rgs$ or $(\Gamma,\{\},\boldsymbol{\alpha}) \in \mathcal{A}rgs$ and $(\Delta \cup \Gamma,\emptyset,\boldsymbol{\alpha})$ and $(\Gamma,\{\},\boldsymbol{\alpha})$ are of the same strength

Proof theoretic exclusion of arguments that are contaminated dues to non-explosive redundant components





If A $\in \mathcal{A}rgs$ then A\` $\in \mathcal{A}rgs$ and A is neither stronger or weaker than A\`

A Fully Rational ASPIC+ For Resource Bounded Agents

Let ($\mathcal{Args}$,$\mathcal{Defeats}$) be defined by ASPIC+ theory Δ = (KB,R), where $\mathcal{Args}$ is ***any*** subset of the dialectical arguments defined by Δ such that

1) If $\boldsymbol{\alpha}$ is a premise in KB then $(\{\boldsymbol{\alpha}\},\{\},\boldsymbol{\alpha}) \in \mathcal{Args}$

2) If $(\Delta,\{\},\boldsymbol{\alpha})$ and$(\Delta',\{\},-\boldsymbol{\alpha}) \in \mathcal{Args}$ then $(\Delta \cup \Delta',\{\},\bot)$ and so $(\{\},\Delta \cup \Delta', \bot) \in \mathcal{Args}$

3) If $(\Delta \cup \Gamma,\emptyset,\boldsymbol{\alpha}) \in Args$ and Δ syntactically disjoint from $\Gamma \cup \{\boldsymbol{\alpha}\}$ then

i) if redundant arguments are proof theoretically excluded (i.e., $\Gamma = \{\}$) then $(\Delta,\{\},\bot) \in \mathcal{Args}$

ii) else $(\Delta,\{\},\bot) \in \mathcal{Args}$ or $(\Gamma,\{\},\boldsymbol{\alpha}) \in \mathcal{Args}$ and $(\Delta \cup \Gamma,\emptyset,\boldsymbol{\alpha})$ and $(\Gamma,\{\},\boldsymbol{\alpha})$ are of the same strength

Then all rationality postulates are satisfied

Example dialectical formalisation of a non-monotonic logic [1]

- Brewka's Preferred Subtheories (PS) defines non-monotonic inferences from maximal consistent subsets (*mcs*) of a totally ordered set of classical wff Δ, obtaining preferred *mcs* and credulous/sceptical non-monotonic inferences

- Suppose PS defined based on a resource bounded $\vdash_r \subseteq \vdash_c$ such that 1) $\forall \alpha \in \Delta,\ \Delta \vdash_r \alpha$ 2) $\Delta \vdash_r \alpha, \neg\alpha$ implies $\Delta \vdash_r \bot$

Then $(\mathcal{A}rgs, \mathcal{D}ef)_\Delta \mathrel{\mid\!\sim}_{\substack{credulous\\preferred}} \alpha$ iff $\Delta \mathrel{\mid\!\sim}^{PS}_{credulous} \alpha$

Also $(\mathcal{A}rgs, \mathcal{D}ef)_\Delta \mathrel{\mid\!\sim}_{\substack{sceptical\\grounded}} \alpha$ implies $\Delta \mathrel{\mid\!\sim}^{PS}_{sceptical} \alpha$

but less sceptical than standard ASPIC+ formalisation of preferred subtheories !

1 M. D'Agostino and S, Modgil. *A Study of Argumentative Characterisations of Preferred Subtheories .* In: **27th Int. Joint Conference on Artificial Intelligence (IJCAI-ECAI-18)**, 1788-1794, July 2018

Propositional Classical Logic Argumentation Using *Intelim* Natural Deduction [1]

- Arguments are intelim natural deduction proofs that do not use virtual information (assumptions) e.g.,$\rightarrow$ I and $\vee$ E
- Instead just one rule of bivalence (RB)

$$\begin{array}{cc} \alpha & \neg\alpha \\ \vdots & \vdots \\ \hline \beta & \beta \end{array} \qquad \dfrac{\begin{array}{cc} \not\alpha & \neg\not\alpha \\ \vdots & \vdots \\ \hline \beta & \beta \end{array}}{\beta}$$

- Degree k of nested use of RB in ND proof – k-depth arguments
  - Increments in depth equate with nested use of *virtual information*
  - Equates with stepwise increments in computational complexity/cognitive effort for decision problem

Whether or not $\Delta \vdash_k \alpha$ can be decided in polynomial O(n $^{2k+2}$) time, where n is the total number of symbols occurring in $\Delta \cup \{\alpha\}$ ($\vdash_\infty = \vdash_{CL}$)

1 M. D'Agostino, D. Gabbay and S.Modgil *Normality, non-contamination and logical depth in classical natural deduction. In: Studia Logica,* pp 1–67 *Feb, 2019*

Propositional Classical Logic Argumentation Using *Intelim* Natural Deduction

- Depth bounded argumentation allows us to accommodate agents with bounded resources ($\mathcal{A}rgs_k$, $\mathcal{D}ef_k$)

- We show [1] that each ($\mathcal{A}rgs_k$, $\mathcal{D}ef_k$) satisfies rationality postulates

Thank you for your attention

Questions ?