1 Syntax

$$
\begin{array}{llll}
e ::= c \in \mathbb{Z} & e_e ::= \cdot +_1 e & s \in \mathit{stat} ::= \mathit{skip} & s_e ::= \mathsf{x} :=_1 \cdot \\
\quad \mid \mathsf{x} \in \mathit{Var} & \quad \mid \cdot +_2 \cdot & \quad \mid s_1 ; s_2 & \quad \mid \cdot ;_1 s_2 \\
\quad \mid e_1 + e_2 & \quad \mid @_1 \, (e_2) & \quad \mid \mathsf{x} := e & \quad \mid \mathit{if}_1 \; s_1 \; s_2 \\
\quad \mid \lambda \mathsf{x}.s & \quad \mid @_2 & \quad \mid \mathit{if} \; (e > 0) \; s_1 \; s_2 & \quad \mid \mathit{while}_1 \; (e > 0) \; s \\
\quad \mid e_1 \, (e_2) & \quad \mid @_3 & \quad \mid \mathit{while} \; (e > 0) \; s & \quad \mid \mathit{while}_2 \; (e > 0) \; s \\
 & & \quad \mid \mathit{return} \, e & \quad \mid \mathit{return}_1 \cdot
\end{array}
$$

2 Semantics

2.1 Expressions

$$
\textsc{red-const}(c) \quad \frac{}{H_e, \ell_e, \ell_c, c \Downarrow H_e, \ell_e, c}
\qquad
\textsc{red-var-local}(\mathsf{x}) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, \ell_c \, [\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom} \, (H_e \, [\ell_c])
$$

$$
\textsc{red-var-global}(x) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, E \, [\mathsf{x}]} \quad \mathsf{x} \in \mathit{dom} \, (H_e \, [\ell_e]) \wedge \mathsf{x} \notin \mathit{dom} \, (H_e \, [\ell_c])
$$

$$
\textsc{red-var-undef}(x) \quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow \mathit{err}} \quad \mathsf{x} \notin \mathit{dom} \, (H_e \, [\ell_e]) \wedge \mathsf{x} \notin \mathit{dom} \, (H_e \, [\ell_c])
$$

$$
\textsc{red-add}(e_1, e_2) \quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, \cdot +_1 e_2 \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 + e_2 \Downarrow r'}
\qquad
\textsc{red-add-1}(e_2) \quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad v_1, r, \cdot +_2 \cdot \Downarrow r'}{\ell_c, (H_e, \ell_e, v_1), \cdot +_1 e_2 \Downarrow r'}
$$

$$
\textsc{red-add-2} \quad \frac{}{v_1, (H_e, \ell_e, v_2), \cdot +_2 \cdot \Downarrow H_e, \ell_e, v_1 + v_2}
\qquad
\textsc{red-lambda}(\mathsf{x}, s) \quad \frac{}{H_e, \ell_e, \ell_c, \lambda \mathsf{x}.s \Downarrow H_e, \ell_e, (\ell_c, \lambda \mathsf{x}.s)}
$$

$$
\textsc{red-app}(e_1, e_2) \quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, @_1 \, (e_2) \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 \, (e_2) \Downarrow r'}
\qquad
\textsc{red-app-1}(e_2) \quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad \ell'_c, \mathsf{x}, s, r, @_2 \Downarrow r'}{\ell_c, (H_e, \ell_e, (\ell'_c, \lambda \mathsf{x}.s)), @_1 \, (e_2) \Downarrow r'}
$$

$$
\textsc{red-app-2}(s) \quad \frac{\ell'_c = \mathit{fresh} \, (H_e) \qquad C = H_e \, [\ell_c] \qquad H_e \, [\ell'_c \leftarrow C \, [\mathsf{x} \leftarrow v]], \ell_e, \ell'_c, s \Downarrow r \qquad r, @_3 \Downarrow r'}{\ell_c, \mathsf{x}, s, (H_e, \ell_e, v), @_2 \Downarrow r'}
\qquad
\textsc{red-app-3-ret} \quad \frac{}{\mathit{ret} \, (H_e, \ell_e, v), @_3 \Downarrow H_e, \ell_e, v}
$$

$$
\textsc{red-app-3-no-ret} \quad \frac{}{H_e, \ell_e, \ell_c, @_3 \Downarrow \mathit{err}}
$$

2.2 Statements

RED-SKIP

$$\frac{}{H_e, \ell_e, \ell_c, \mathit{skip} \Downarrow H_e, \ell_e, \ell_c}$$

RED-SEQ(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r \qquad r, \cdot ;_1 s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, s_1 ; s_2 \Downarrow r'}$$

RED-SEQ-1(s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{H_e, \ell_e, \ell_c, \cdot ;_1 s_2 \Downarrow r}$$

RED-ASN(x, e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathsf{x} :=_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathsf{x} := e \Downarrow r'}$$

RED-ASN-1(x)

$$\frac{\ell_e' = \mathit{fresh}\left(H_e\right) \qquad E = H_e\left[\ell_e\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell_e' \leftarrow E\left[\mathsf{x} \leftarrow v\right]\right], \ell_e', \ell_c} \quad \mathsf{x} \notin \mathit{dom}\left(H_e\left[\ell_c\right]\right)$$

RED-ASN-1-LOCAL(x)

$$\frac{\ell_c' = \mathit{fresh}\left(H_e\right) \qquad C = H_e\left[\ell_c\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell_c' \leftarrow C\left[\mathsf{x} \leftarrow v\right]\right], \ell_e, \ell_c'} \quad \mathsf{x} \in \mathit{dom}\left(C\right)$$

RED-IF(e, s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{if}_1\, s_1\, s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{if}\ \left(e > 0\right)\ s_1\, s_2 \Downarrow r'}$$

RED-IF-1-POS(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v > 0$$

RED-IF-1-NEG(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v \leqslant 0$$

RED-WHILE(e, s)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r'}$$

RED-WHILE-1-NEG(e, s)

$$\frac{}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow H_e, \ell_e, \ell_c} \quad v \leqslant 0$$

RED-WHILE-1-POS(e, s)

$$\frac{H_e, \ell_e, \ell_c, s \Downarrow r \qquad r, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r'}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'} \quad v > 0$$

RED-WHILE-2(e, s)

$$\frac{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r}{H_e, \ell_e, \ell_c, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r}$$

RED-RETURN(e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, \mathit{return}_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{return}\, e \Downarrow r'}$$

RED-RETURN-1

$$\frac{}{\left(H_e, \ell_e, v\right), \mathit{return}_1 \cdot \Downarrow \mathit{ret}\left(H_e, \ell_e, v\right)}$$

2.3 Aborting Rules

$$\frac{}{\sigma, e \Downarrow err}\ \textbf{abort}\,\sigma \wedge \neg\textbf{intercept}_e\,\sigma \quad \textsc{red-error-expr}(e)$$

$$\frac{}{\sigma, s \Downarrow err}\ \textbf{abort}\,\sigma \quad \textsc{red-error-stat}(s)$$

$$\frac{\sigma = C\,[\,err\,]}{\textbf{abort}\,\sigma}$$

$$\frac{}{\textbf{intercept}_{@_3}\,ret\,(H_e, \ell_e, v)}$$