

1 Syntax

$$\begin{array}{l}
 e ::= c \in \mathbb{Z} \\
 \quad | x \in \text{Var} \\
 \quad | e_1 + e_2
 \end{array}
 \quad
 \begin{array}{l}
 e_e ::= \cdot +_1 e \\
 \quad | \cdot +_2 \cdot
 \end{array}
 \quad
 \begin{array}{l}
 s ::= \text{skip} \\
 \quad | s_1; s_2 \\
 \quad | x := e \\
 \quad | \text{if } (e > 0) s_1 s_2 \\
 \quad | \text{while } (e > 0) s
 \end{array}
 \quad
 \begin{array}{l}
 s_e ::= x :=_1 \cdot \\
 \quad | \cdot;_1 s_2 \\
 \quad | \text{if}_1 s_1 s_2 \\
 \quad | \text{while}_1 (e > 0) s \\
 \quad | \text{while}_2 (e > 0) s
 \end{array}$$

2 Semantics

2.1 Expressions

$$\begin{array}{c}
 \text{RED-CONST}(c) \\
 \hline
 E, c \Downarrow c
 \end{array}
 \quad
 \begin{array}{c}
 \text{RED-VAR}(x) \\
 \hline
 E, x \Downarrow E[x]
 \end{array}
 \quad
 x \in \text{dom}(E)
 \quad
 \begin{array}{c}
 \text{RED-VAR-UNDEF}(x) \\
 \hline
 E, x \Downarrow \text{err}
 \end{array}
 \quad
 x \notin \text{dom}(E)$$

$$\begin{array}{c}
 \text{RED-ADD}(e_1, e_2) \\
 \hline
 \frac{E, e_1 \Downarrow r \quad E, r, \cdot +_1 e_2 \Downarrow r'}{E, e_1 + e_2 \Downarrow r'}
 \end{array}
 \quad
 \begin{array}{c}
 \text{RED-ADD-1}(e_2) \\
 \hline
 \frac{E, e_2 \Downarrow r \quad E, v_1, r, \cdot +_2 \cdot \Downarrow r'}{E, v_1, \cdot +_1 e_2 \Downarrow r'}
 \end{array}$$

$$\begin{array}{c}
 \text{RED-ADD-2} \\
 \hline
 E, v_1, v_2, \cdot +_2 \cdot \Downarrow v_1 + v_2
 \end{array}$$

2.2 Statements

$$\begin{array}{c}
\text{RED-SKIP} \\
\frac{}{E, \text{skip} \Downarrow E}
\end{array}
\qquad
\frac{\text{RED-SEQ}(s_1, s_2)}{E, s_1 \Downarrow r \quad r, \cdot ;_1 s_2 \Downarrow r'}{E, s_1 ; s_2 \Downarrow r'}
\qquad
\frac{\text{RED-SEQ-1}(s_2)}{E, s_2 \Downarrow r}{E, \cdot ;_1 s_2 \Downarrow r}$$

$$\frac{\text{RED-ASN}(x, e)}{E, e \Downarrow r \quad E, r, x :=_1 \cdot \Downarrow r'}{E, x := e \Downarrow r'}
\qquad
\frac{\text{RED-ASN-1}(x)}{E, v, x :=_1 \cdot \Downarrow E[x \leftarrow v]}$$

$$\frac{\text{RED-IF}(e, s_1, s_2)}{E, e \Downarrow r \quad E, r, \text{if}_1 s_1 s_2 \Downarrow r'}{E, \text{if } (e > 0) s_1 s_2 \Downarrow r'}
\qquad
\frac{\text{RED-IF-1-POS}(s_1, s_2)}{E, s_1 \Downarrow r}{E, v, \text{if}_1 s_1 s_2 \Downarrow r} \quad v > 0$$

$$\frac{\text{RED-IF-1-NEG}(s_1, s_2)}{E, s_2 \Downarrow r}{E, v, \text{if}_1 s_1 s_2 \Downarrow r} \quad v \leq 0
\qquad
\frac{\text{RED-WHILE}(e, s)}{E, e \Downarrow r \quad E, r, \text{while}_1 (e > 0) s \Downarrow r'}{E, \text{while } (e > 0) s \Downarrow r'}$$

$$\frac{\text{RED-WHILE-1-NEG}(e, s)}{E, v, \text{while}_1 (e > 0) s \Downarrow E} \quad v \leq 0
\qquad
\frac{\text{RED-WHILE-1-POS}(e, s)}{E, s \Downarrow r \quad r, \text{while}_2 (e > 0) s \Downarrow r'}{E, v, \text{while}_1 (e > 0) s \Downarrow r'} \quad v > 0$$

$$\frac{\text{RED-WHILE-2}(e, s)}{E, \text{while } (e > 0) s \Downarrow r}{E, \text{while}_2 (e > 0) s \Downarrow r}$$

2.3 Aborting Rules

$$\frac{\text{RED-ERROR-EXPR}(e)}{\sigma, e \Downarrow \text{err}} \quad \mathbf{abort} \sigma
\qquad
\frac{\text{RED-ERROR-STAT}(s)}{\sigma, s \Downarrow \text{err}} \quad \mathbf{abort} \sigma$$

$$\frac{\sigma = C[\text{err}]}{\mathbf{abort} \sigma}$$