Tutorial Exercises 4 (mjs)
(ν-models for classical systems)

1. Show that each of the following can be falsified in ν-models:

   M. $\Box(A \wedge B) \longrightarrow (\Box A \wedge \Box B)$

   C. $(\Box A \wedge \Box B) \longrightarrow \Box(A \wedge B)$

   N. $\Box\top$

2. Consider the following conditions on a model $\mathcal{M} = \langle W, \nu, h \rangle$, for every world w and propositions (i.e. sets of worlds) X and Y:

   (m) if $X \cap Y \in \nu(w)$ then $X \in \nu(w)$ and $Y \in \nu(w)$
   (c) if $X \in \nu(w)$ and $Y \in \nu(w)$ then $X \cap Y \in \nu(w)$
   (n) $W \in \nu(w)$

   Prove that the schemas M, C, and N are valid in classes of ν-models satisfying conditions (m), (c), and (n), respectively.

3. Prove that condition (m) is equivalently expressed as follows:

   (rm) if $X \subseteq Y$, $X \in \nu(w) \Rightarrow Y \in \nu(w)$.

4. Prove that for any ν-model satisfying (m) or (rm)

$$\nu(w) \neq \emptyset \Leftrightarrow W \in \nu(w)$$

5. Re-express the model conditions (m), (c), (n), (rm) above in terms of the function $f : \wp(W) \rightarrow \wp(P)$ defined as $w \in f(X) \Leftrightarrow X \in \nu(w)$.

6. Every *normal* system contains D if and only if it contains P.

   P. $\neg\Box\bot$

   D. $\Box A \longrightarrow \Diamond A$

   Show that in a classical system P and D can be independent, in the sense that a classical system can contain P without containing D, and can contain D without containing P.

   Now show that every classical *EMD* system contains P but not every *EMP* system contains D.

   Finally, prove the assertion at the top of this question, that every *normal* system contains D if and only if it contains P.

7. Show that the schemas

   P. $\neg\Box\bot$

   D. $\Box A \longrightarrow \Diamond A$

   T. $\Box A \longrightarrow A$

   B. $A \longrightarrow \Box\Diamond A$

   4. $\Box A \longrightarrow \Box\Box A$

   5. $\Diamond A \longrightarrow \Box\Diamond A$

   are valid in classes of ν-models satisfying the following conditions (p), (d), (t), (b), (iv), and (v), respectively:

   (p) $\emptyset \notin \nu(w)$
   (d) $X \in \nu(w) \Rightarrow (W - X) \notin \nu(w)$
   (t) $X \in \nu(w) \Rightarrow w \in X$
   (b) $w \in X \Rightarrow \{w' \in W : (W - X) \notin \nu(w')\} \in \nu(w)$
   (iv) $X \in \nu(w) \Rightarrow \{w' \in W : X \in \nu(w')\} \in \nu(w)$
   (v) $X \notin \nu(w) \Rightarrow \{w' \in W : X \notin \nu(w')\} \in \nu(w)$

8. Re-express the model conditions (p), (d), (t), (b), (iv), and (v) above in terms of the function $f : \wp(W) \rightarrow \wp(P)$ defined as $w \in f(X) \Leftrightarrow X \in \nu(w)$.

   Notice anything?

9. Identify a model condition on ν-models that makes the following schema valid:

   G. $\Diamond\Box A \longrightarrow \Box\Diamond A$

   (Write out a guess based on the previous question, and then check it.)