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Learning Weak Constraints in Answer Set Programming

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Preference Learning

- Learning to rank is an approach to preference learning where the goal is to learn to order objects given pairwise examples of a user's preferences.
- For example, learning academic's preferences about interview scheduling.

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

is preferred to

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

Although traditional machine learning techniques can be used to predict preferences, their reasoning is not easily human readable.

Inductive Logic Programming

- Given a set of positive examples E^+, negative examples E^- and a background knowledge B, the goal is to find a hypothesis H such that:
  - $\forall e \in E^+ : B \cup H \models e$
  - $\forall e \in E^- : B \cup H \not\models e$

Inductive Logic Programming

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  - $\forall e \in E^+ : B \cup H \models e$
  - $\forall e \in E^- : B \cup H \not\models e$
- The key advantages are that:
  - The hypotheses are human readable.
  - Can define useful concepts in the background knowledge.
  - Can give a very structured language bias to guide the search.

Learning from Answer Sets (ILP_{LAS})

- In ILP_{LAS} (Law et al. 2014), examples are *partial interpretations*.
- A partial interpretation e is a pair of sets of atoms $\langle e^{inc}, e^{exc} \rangle$.
- An answer set A extends e iff $e^{inc} \subseteq A$ and $e^{exc} \cap A = \emptyset$.

	1	2	3
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is extended by

	1	2	3
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W	c2	c1	c2

Learning from Answer Sets (ILP_{LAS})

Definition 1

An ILP_{LAS} task is a tuple $T = \langle B, S_M, E^+, E^- \rangle$. A hypothesis $H \subseteq S_M$ is in $ILP_{LAS}(T)$, the set of all inductive solutions of T, if and only if:

- $\forall e^+ \in E^+ \; \exists A \in AS(B \cup H)$ *such that* A *extends* e^+
- $\forall e^- \in E^- \; \nexists A \in AS(B \cup H)$ *such that* A *extends* e^-.

(Law et al. 2014)

Weak Constraints in ASP

:∼ assign(D, S),type(D, S, c1).[1@2, D, S]

:∼ assign(D, S1),assign(D, S2),neq(S1, S2).[1@1, D, S1, S2]

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Learning from Ordered Answer Sets

- As previous ILP frameworks could only give examples of what should (or shouldn't) be an answer set of the program, no existing framework could incentivise learning weak constraints.
- We now present an extension of ILP_{LAS} (Law et al. 2014), with a new type of example aimed at learning weak constraints.

Ordering Examples

Definition 3

An ordering example is a tuple $o = \langle e_1, e_2 \rangle$ where e_1 and e_2 are partial interpretations.

Brave Scheduling Example

```
:~ assign(D, S1),assign(D, S2),neq(S1, S2).[1@1, D, S1, S2]
:~ assign(D, S),type(D, S, c1).[1@2, D, S]
```

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

is sometimes preferred to

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

Definition 3

- *An ASP program P* bravely respects *o iff $\exists A_1, A_2 \in AS(P)$ such that A_1 extends e_1, A_2 extends e_2 and $A_1 \succ_P A_2$.*

Cautious Scheduling Example

```
:~ assign(D, S1),assign(D, S2),neq(S1, S2).[1@1, D, S1, S2]
:~ assign(D, S),type(D, S, c1).[1@2, D, S]
```

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

is always preferred to

	1	2	3
M	c1	c2	c2
T	c2	c2	c2
W	c2	c1	c2

Definition 3

- *P cautiously respects o iff $\forall A_1, A_2 \in AS(P)$ such that A_1 extends e_1 and A_2 extends e_2, it is the case that $A_1 \succ_P A_2$.*

Learning from Ordered Answer Sets (ILP_{LOAS})

Definition 4

An ILP_{LOAS} task is a tuple $T = \langle B, S_M, E^+, E^-, O^b, O^c \rangle$.
A hypothesis $H \subseteq S_M$ is in $ILP_{LOAS}(T)$, the inductive solutions of T, if and only if:

1. $\forall e^+ \in E^+ \; \exists A \in AS(B \cup H)$ *such that A extends* e^+
2. $\forall e^- \in E^- \; \nexists A \in AS(B \cup H)$ *such that A extends* e^-.
3. $\forall o \in O^b \; B \cup H$ *bravely respects o*
4. $\forall o \in O^c \; B \cup H$ *cautiously respects o*

Complexity

Theorem 3

Let T be any propositional ILP_{LAS} or ILP_{LOAS} task. Deciding whether T has at least one inductive solution is NP^{NP}-complete.

Algorithm

- Our new algorithm ILASP2 (Inductive Learning of Answer Set Programs) is sound and complete wrt the optimal (shortest) solutions of any ILP_{LOAS} task.
- It is available for download at `https://www.doc.ic.ac.uk/~ml1909/ILASP`.
- It extends our previous ILASP1 algorithm for solving ILP_{LAS} tasks (Law et al. 2014).

Positive and Violating Hypotheses

Definition 5/6

Let $T = \langle B, S_M, E^+, E^- \rangle$ be an ILP_{LAS} task. Any $H \subseteq S_M$ is a positive hypothesis *iff $\forall e \in E^+ \; \exists A \in AS(B \cup H)$ such that A extends e.*

A positive hypothesis H is violating iff $\exists e^- \in E^-, \exists A \in AS(B \cup H)$ such that A extends e^-.

(Law et al. 2014)

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Positive and Violating Hypotheses

$E^{+} = \{$

	1	2
M	c1	c2
T	c2	c2

$\}$

$E^{-} = \{$

	1	2
M	c1	c2
T	c2	c2

,

	1	2
M	c1	c2
T	c2	c2

$\}$

$B = \{$

```
slot(m, 1..2). slot(t, 1..2). type(m, 1, c1).
type(m, 2, c2). type(t, 1, c2). type(t, 2, c2).
0{assign(D, S)}1:-slot(D, S).
```

$\}$

$H = \emptyset$

This is a positive hypothesis, but is also violating.

Positive and Violating Hypotheses

$E^+ = \{$

	1	2
M	c1	c2
T	c2	c2

$\}$

$E^- = \{$

	1	2
M	c1	c2
T	c2	c2

,

	1	2
M	c1	c2
T	c2	c2

$\}$

$$B = \left\{ \begin{array}{l} \texttt{slot(m, 1..2). slot(t, 1..2). type(m, 1, c1).} \\ \texttt{type(m, 2, c2). type(t, 1, c2). type(t, 2, c2).} \\ \texttt{0\{assign(D, S)\}1:-slot(D, S).} \end{array} \right\}$$

$$H = \{ \ \texttt{:-assign(m, S).} \ \}$$

This is a positive hypothesis which is not violating. Hence, it is an inductive solution.

Positive and Violating Hypotheses

Definition 5/6

Let $T = \langle B, S_M, E^+, E^-, O^b, O^c \rangle$ be an ILP_{LOAS} task. Any $H \subseteq S_M$ is a positive hypothesis *iff $\forall e \in E^+\ \exists A \in AS(B \cup H)$ such that A extends e, and $\forall o \in O^b$ $B \cup H$ bravely respects o.*

A positive hypothesis H is violating iff at least one of the following cases is true:

- $\exists e^- \in E^-, \exists A \in AS(B \cup H)$ *such that A extends e^-.*
- $\exists A_1, A_2 \in AS(B \cup H)$ *and* $\exists \langle e_1, e_2 \rangle \in O^c$ *such that A_1 extends e_1, A_2 extends e_2 and $A_1 \not\prec_P A_2$ with respect to $B \cup H$.*

ILASP1

procedure ILASP1(T)
  solutions = []
  $n = 0$
  while *solutions* == [] **do**
    $V = violating\_hypotheses(T, n)$
    $solutions = remaining\_positive\_hypothses(T, n, V)$
    $n = n + 1$
  end while
  return *solutions*
end procedure

(Law et al. 2014)

Positive Hypotheses and Violating Reasons

Definition 5/6

Let $T = \langle B, S_M, E^+, E^-, O^b, O^c \rangle$ *be an* ILP_{LOAS} *task. Any* $H \subseteq S_M$ *is a* positive hypothesis *iff* $\forall e \in E^+ \; \exists A \in AS(B \cup H)$ *such that* A *extends* e*, and* $\forall o \in O^b$ $H \cup B$ *bravely respects* o*.*

A positive hypothesis H *is violating iff at least one of the following cases is true:*

- $\exists e^- \in E^-, \exists A \in AS(B \cup H)$ *such that* A *extends* e^-*.*
  In this case we call A *a violating interpretation of* T*.*
- $\exists A_1, A_2 \in AS(B \cup H)$ *and* $\exists \langle e_1, e_2 \rangle \in O^c$ *such that* A_1 *extends* e_1*,* A_2 *extends* e_2 *and* $A_1 \not\succ_P A_2$ *with respect to* $B \cup H$*.*
  In this case, we call $\langle A_1, A_2 \rangle$ *a violating pair of* T*.*

Meta Representation: Properties

Given a task T and a set of violating reasons VR, we defined a meta program $\mathcal{M}(T, VR)$.

- The A's $\in AS(\mathcal{M}(T, VR))$ map to the positive hypotheses $\mathcal{M}^{-1}_{hyp}(A)$ which do not violate any $vr \in VR$.
- If H is violating, there is an $A \in AS(\mathcal{M}(T, VR))$ st $H = \mathcal{M}^{-1}_{hyp}(A)$ and $\texttt{violating} \in A$.
- The optimality of each answer set A is $2|\mathcal{M}^{-1}_{hyp}(A)|$ if $\texttt{violating} \in A$ and $2|\mathcal{M}^{-1}_{hyp}(A)| + 1$ otherwise.

ILASP2

procedure $\mathrm{ILASP2}(T)$
  $VR = []$
  $A = solve(\mathcal{M}(T, VR))$
  while $A \neq \texttt{nil}$ && $\texttt{violating} \in A$ **do**
    $VR \mathrel{+}= \mathcal{M}_{vr}^{-1}(A)$
    $A = solve(\mathcal{M}(T, VR))$
  end while
  return $\{\mathcal{M}_{hyp}^{-1}(A) \mid A \in AS^*(\mathcal{M}(T, VR))\}$
end procedure

- $solve(P)$ returns an optimal answer set of P (it returns `nil` if $AS(P) = \emptyset$).
- $\mathcal{M}_{hyp}^{-1}$ maps an answer set of $\mathcal{M}(T, VR)$ to the corresponding hypothesis.
- $\mathcal{M}_{vr}^{-1}$ maps an answer set of $\mathcal{M}(T, VR)$ to the corresponding violating reason.

Soundness and Completeness

Theorem 5

Let T be an ILP_{LOAS} task. If $ILASP2(T)$ terminates, then $ILASP2(T)$ returns the set of optimal inductive solutions of $ILP_{LOAS}(T)$.

Sudoku Experiment

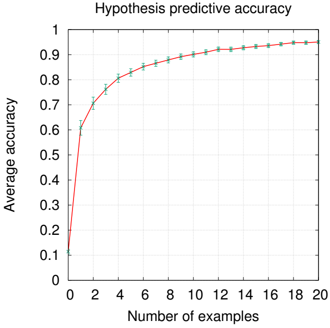
- In (Law et al. 2014) we showed that ILASP1 could be used to learn the rules of a 4x4 version of sudoku.
- This took 486.2s (over 8 minutes) to solve with ILASP1 as there were 332437 violating hypotheses found before the first inductive solution.
- ILASP2 only needs 9 violating reasons and so solves the same task in 0.69s (less than 1 second).

Experiments

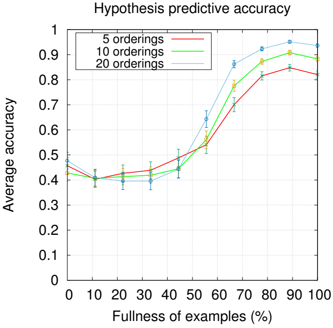
Our main experiments were on the interview scheduling example.

- For each experiment, we randomly generated hypotheses H with up to 3 weak constraints from S_M.
- We used each H to randomly generate orderings.
- We then used ILASP2 to learn a hypothesis H' which covered these examples.
- We checked the accuracy of H' at predicting the orderings of the timetables given by H.

Experiments



(a)



(b)

Figure : Accuracy with varying (a) numbers of examples; (b) fullness of examples

Experiments

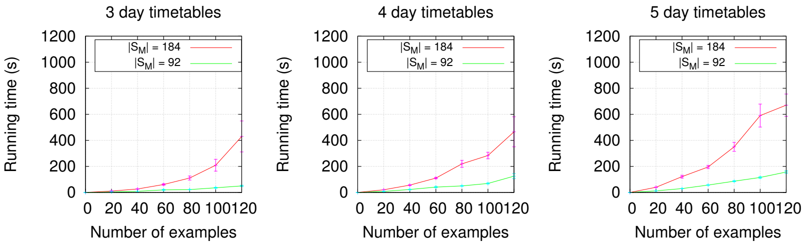


Figure : Average running time of ILASP2 with varying numbers of examples

Comparison of Non-monotonic ILP systems

Learning Task	Normal Rules	Choice Rules	Constraints	Brave	Cautious	Weak Constraints	Algorithm for optimal solutions
Brave Induction [Sakama, Inoue 2009]	✔	✔	✘	✔	✘	✘	✘
Cautious Induction [Sakama, Inoue 2009]	✔	✔	✘	✘	✔	✘	✘
XHAIL [Ray 2009] & ASPAL [Corapi et al 2011]	✔	✘	✘	✔	✘	✘	✔
Induction of Stable Models [Otero 2001]	✔	✘	✘	✔	✘	✘	✘
Induction from Answer Sets [Sakama 2005]	✔	✘	✔	✔	✔	✘	✘
LAS [Law et al 2014]	✔	✔	✔	✔	✔	✘	✔
LOAS	✔	✔	✔	✔	✔	✔	✔

Other Related Work in ILP

- ILP systems have previously been used for preference learning (Dastani et al. 2001, Horváth 2012). This addressed learning ratings, such as *good*, *poor* and *bad*, rather than rankings over the examples.

Summary

- Presented ILP_{LOAS}, which is the first framework capable of learning weak constraints.
- Presented ILASP2, which is sound and complete wrt ILP_{LOAS}.
- Proved the complexity of deciding the existence of solutions for ILP_{LAS} and ILP_{LOAS}.
- Showed that ILASP2 is more efficient than ILASP1 for the previous ILP_{LAS} task.
- Gave experimental results of ILASP2 in the setting of learning accademic's preferences for interview scheduling.

Future Work

- Support noisy examples.
- Improve the performance with larger numbers of examples.

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