Logic-Based Learning: Brave Induction, Cautious Induction and ASPAL

Mark Law

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Question 1

For each of the following ILP_c tasks $\langle B, E^+, E^- \rangle$, find hypotheses H such that $H \in ILP_c\langle B, E^+, E^- \rangle$ and H contains no constant symbols (you may use $!=$, $>$ and $<$).

i) $B = \left\{ \begin{array}{l} cell(1..3, 1..3). \\ 1\{value(X,Y,1), value(X,Y,2), value(X,Y,3)\}1 \leftarrow cell(X,Y). \end{array} \right.$

$E^+ = \left\{ \begin{array}{l} value(1,1,1), \\ value(3,3,3) \end{array} \right.$

$E^- = \left\{ \begin{array}{l} value(1,2,1), \\ value(2,1,1) \end{array} \right.$

ii) $B = \left\{ \begin{array}{l} 1\{p(X,1), p(X,2)\}1 \leftarrow r(X). \\ q(b) \leftarrow p(b,1). \\ q(c) \leftarrow p(c,2). \\ r(a). \\ r(b). \\ r(c). \\ s \leftarrow q(X). \\ \leftarrow \text{not } s. \\ t(X) \leftarrow \text{not } q(X). \end{array} \right.$

$E^+ = \left\{ \begin{array}{l} t(a), \\ t(b) \end{array} \right.$

$E^- = \left\{ \; t(c) \right.$

Challenge: try finding a solution to part (i) such that $B \cup H$ has more than one Answer Set (or that only has one if your original solution had many).

Question 2

For each of the following ILP_b tasks $\langle B, E^+, E^-\rangle$, find hypotheses H such that $H \in ILP_b\langle B, E^+, E^-\rangle$ but $H \notin ILP_c\langle B, E^+, E^-\rangle$.

i) $$B = \left\{ \begin{array}{l} cell(1..3, 1..3). \\ 1\{value(X,Y,1), value(X,Y,2), value(X,Y,3)\}1 \leftarrow cell(X,Y). \end{array} \right.$$

$$E^+ = \left\{ \begin{array}{l} value(1,1,1), \\ value(3,3,3) \end{array} \right.$$

$$E^- = \left\{ \begin{array}{l} value(1,2,1), \\ value(2,1,1) \end{array} \right.$$

ii) $$B = \left\{ \begin{array}{l} 1\{p(X,1), p(X,2)\}1 \leftarrow r(X). \\ q(b) \leftarrow p(b,1). \\ q(c) \leftarrow p(c,2). \\ r(a). \\ r(b). \\ r(c). \end{array} \right.$$

$$E^+ = \left\{ \begin{array}{l} t(a), \\ t(b) \end{array} \right.$$

$$E^- = \left\{ \begin{array}{l} t(c) \end{array} \right.$$

Question 3

Consider the following ILP task:

$$M = \left\{ \begin{array}{l} modeh(1, murderer(+person)), \\ modeb(2, proven\_guilty(+person, \#crime)) \end{array} \right.$$

$$B = \left\{ \begin{array}{l} proven\_guilty(bob, murder). \\ proven\_guilty(fred, robbery). \\ person(bob). \\ person(fred). \\ crime(murder). \\ crime(robbery). \end{array} \right.$$

$$E^+ = \left\{ \begin{array}{l} murderer(bob). \end{array} \right.$$

$$E^- = \left\{ \begin{array}{l} murderer(bob). \end{array} \right.$$

1. Write down a maximal set of skeleton rules given the mode declarations ($L_{max} = 3$, $V_{max} = 3$).
2. Write down the ASP encoding which would be used by ASPAL to solve this task.
3. Solve the task using Clingo (give the argument –opt-mode=optN to generate all optimal solutions).
4. Solve the task using ASPAL and check whether the hypothesis found by ASPAL is in your set of optimal hypotheses.

Questions 4 and 5 have fairly large language biases. Don't worry, I wouldn't expect you to compute all the skeleton rules in the exam, but it is good practice.

Question 4

Consider the following ILP task:

$$M = \left\{ \begin{array}{l} modeh(1, holding(+object, +time))modeb(1, holding(+object, +time)), \\ modeb(1, not\ \ holding(+object, +time)), \\ modeb(1, next\_time(+time, -time)), \\ modeb(1, pick\_up(+time)), \\ modeb(1, drop(+time)) \end{array} \right.$$

$$B = \left\{ \begin{array}{l} time(0..10). \\ next\_time(X, X+1) \leftarrow time(X), time(X+1). \\ holding(ball, 0). \\ drop(ball, 1). \\ drop(ball, 7). \\ pick\_up(apple, 0). \\ pick\_up(apple, 1). \\ pick\_up(apple, 2). \\ object(apple). \\ object(ball). \end{array} \right.$$

$$E^{+} = \left\{ \begin{array}{l} holding(ball, 1). \\ holding(apple, 5). \\ holding(apple, 7). \end{array} \right.$$

$$E^{-} = \left\{ \begin{array}{l} holding(ball, 2). \\ holding(apple, 1). \\ holding(apple, 2). \\ holding(apple, 4). \\ holding(apple, 8). \end{array} \right.$$

1. Write down a maximal set of skeleton rules given the mode declarations ($L_{max} = 3$, $V_{max} = 3$).
2. Write down the ASP encoding which would be used by ASPAL to solve this task.
3. Solve the task using Clingo (give the argument –opt-mode=optN to generate all optimal solutions).
4. Solve the task using ASPAL and check whether the hypothesis found by ASPAL is in your set of optimal hypotheses.

Question 5

Consider the following ILP task:

$$M = \begin{cases} modeh(1, can\_play(+player, +card, +time)) modeb(1, \text{not } has\_suit(+player, +time)) \\ modeb(1, in\_hand(+player, +card, +time)) \\ modeb(1, leader(+player, +time)) \\ modeb(1, leader\_played\_suit(-suit, +time)) \\ modeb(1, is\_suit(+card, +suit)) \end{cases}$$

$$B = \begin{cases} player(p1).player(p2). \\ rank(1..10). \\ suit(clubs). \\ suit(diamonds). \\ suit(hearts). \\ suit(spades). \\ is\_suit(c(R,S),S) \leftarrow rank(R), suit(S). \\ time(1..5). \\ hand(p1,c(10,clubs)). \\ hand(p1,c(8,clubs)). \\ hand(p1,c(7,diamonds)). \\ hand(p1,c(5,hearts)). \\ hand(p1,c(6,hearts)). \\ hand(p2,c(2,clubs)). \\ hand(p2,c(5,clubs)). \\ hand(p2,c(9,clubs)). \\ hand(p2,c(2,diamonds)). \\ hand(p2,c(9,spades)). \\ play(p1,c(10,clubs),1). \\ play(p2,c(2,clubs),1). \\ play(p1,c(8,clubs),2). \\ play(p2,c(9,clubs),2). \\ play(p2,c(9,spades),3). \\ play(p1,c(5,hearts),3). \\ play(p2,c(5,clubs),4). \\ play(p1,c(7,diamonds),4). \\ play(p2,c(2,diamonds),5). \\ play(p1,c(6,hearts),5). \\ card(c(R,S)) \leftarrow rank(R), suit(S). \\ already\_played(c(R,S),T2) \leftarrow time(T2), play(P,c(R,S),T), T < T2. \\ in\_hand(P,c(R,S),T) \leftarrow hand(P,c(R,S)), not\ \ already\_played(c(R,S),T), time(T). \\ has\_suit(P,S,T) \leftarrow in\_hand(P,c(R,S),T). \\ leader(1,p1). \\ leader(T+1,P) \leftarrow play(P,c(R,S),T), play(OP,c(OR,S),T), R > OR. \\ leader(T+1,P) \leftarrow play(P,c(R,S),T), play(OP,c(OR,OS),T), S! = OS, leader(T,P). \\ leader\_played\_suit(S,T) \leftarrow play(P,c(R,S),T), leader(P). \end{cases}$$

$$E^+ = \left\{ \begin{array}{l} can\_play(p1, c(10, clubs), 1). \\ can\_play(p1, c(8, clubs), 1). \\ can\_play(p1, c(7, diamonds), 1). \\ can\_play(p1, c(5, hearts), 1). \\ can\_play(p1, c(6, hearts), 1). \\ can\_play(p2, c(2, clubs), 1). \\ can\_play(p1, c(7, diamonds), 3). \\ can\_play(p1, c(6, hearts), 3). \\ can\_play(p1, c(6, hearts), 4). \\ can\_play(p1, c(7, diamonds), 4). \\ can\_play(p1, c(5, hearts), 3). \end{array} \right.$$

$$E^- = \left\{ \begin{array}{l} can\_play(p2, c(2, diamonds), 1). \\ can\_play(p2, c(9, spades), 1). \\ can\_play(p2, c(8, clubs), 1). \\ can\_play(p2, c(10, clubs), 1). \\ can\_play(p1, c(2, diamonds), 1). \\ can\_play(p1, c(10, clubs), 3). \end{array} \right.$$

1. Write down a maximal set of skeleton rules given the mode declarations ($L_{max} = 5$, $V_{max} = 4$).
2. Write down the ASP encoding which would be used by ASPAL to solve this task.
3. Solve the task using Clingo (give the argument –opt-all to generate all optimal solutions).
4. Solve the task using ASPAL and check whether the hypothesis found by ASPAL is in your set of optimal hypotheses.

Question 6

The original definition for brave induction contained no concept of negative example. We will call this an ILP_{b*} task. This new task is denoted as a tuple $\langle B, E^+ \rangle$.
Write down how to map an ILP_b task to an ILP_{b*} task.
Hint: you may assume that you have a set of atoms $a_1, \ldots, a_n$ which do not occur in B, E^+ or E^-.

Advanced Questions

Question 7

An *Induction of Stable Models* (ILP_{sm}) task is a tuple $\langle B, S_M, E\rangle$ where B is an ASP program, S_M is a set of normal rules called the search space and E is a set of partial interpretations called the examples.
An hypothesis H is an inductive solution (written $H \in ILP_{sm}\langle B, S_M, E\rangle$) if and only if:

1) $H \subseteq S_M$

2) $\forall \langle E^{inc}, E^{exc}\rangle \in E : \exists A \in AS(B \cup H)$ such that $E^{inc} \subseteq A$ and $E^{exc} \cap A = \emptyset$

a) Write down the Answer Sets of the program P:

$$P = \begin{cases} heads \leftarrow \text{not } tails. \\ tails \leftarrow \text{not } heads. \end{cases}$$

b) Write down the Answer Sets of the program P':

$$P' = \begin{cases} e(heads, X) \leftarrow \text{not } e(tails, X), ex(X). \\ e(tails, X) \leftarrow \text{not } e(heads, X), ex(X). \\ ex(1). \\ ex(2). \end{cases}$$

c) Write down a mapping from an ILP_{sm} task to an ILP_{ASPAL} task.

Question 8

Prove that there is no Brave Induction task which has an optimal hypothesis H such that H contains a constraint.

Hint: the property that constraints only eliminate Answer Sets may come in handy (this means that for any constraint C and program P, $AS(P \cup C) \subseteq AS(P)$).