Synthesis of distributed mobile programs using monadic types in Coq

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The problem

The extraction of certified *functional* and effect-free programs is a well-know practice in the field of Type Theory, however:

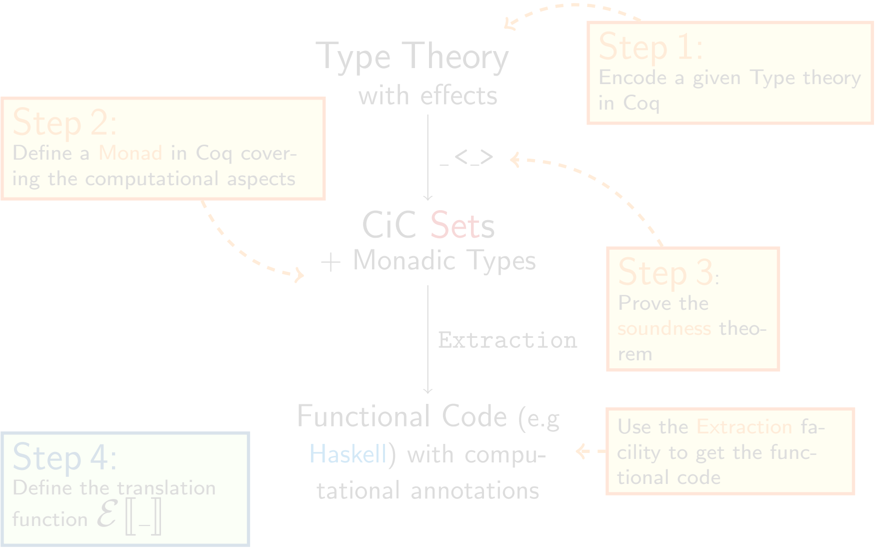
- ✔ There are many other computational effects (and corresponding Type Theories, possibly)
- ✔ These scenarios would greatly benefit from a mechanisms for extraction
- ✘ Languages implementing these aspects usually do not support the Curry-Howard isomorphism
- ✘ Implementing a specific proof-assistant would be a daunting task anyway.

Our contribution

We propose:

- a general methodology for circumventing this problem using the existing technology (Coq)
  - encapsulate non-functional aspects in monadic types
  - implement a post-extraction compiler for realizing monadic constructors in the target language
- example: distributed programs with effects in Erlang.

A general methodology



Type Theory
with effects

Step 1:
Encode a given Type theory in Coq

_<_>

Step 2:
Define a Monad in Coq covering the computational aspects

CiC Sets
+ Monadic Types

Step 3:
Prove the soundness theorem

`Extraction`

Functional Code (e.g Haskell) with computational annotations

Use the Extraction facility to get the functional code

Step 4:
Define the translation function $\mathcal{E}[\![\_]\!]$

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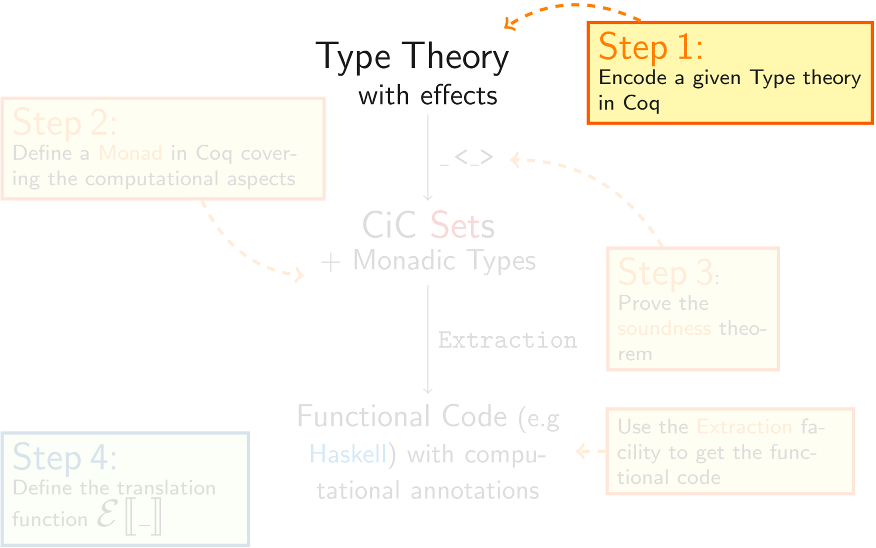

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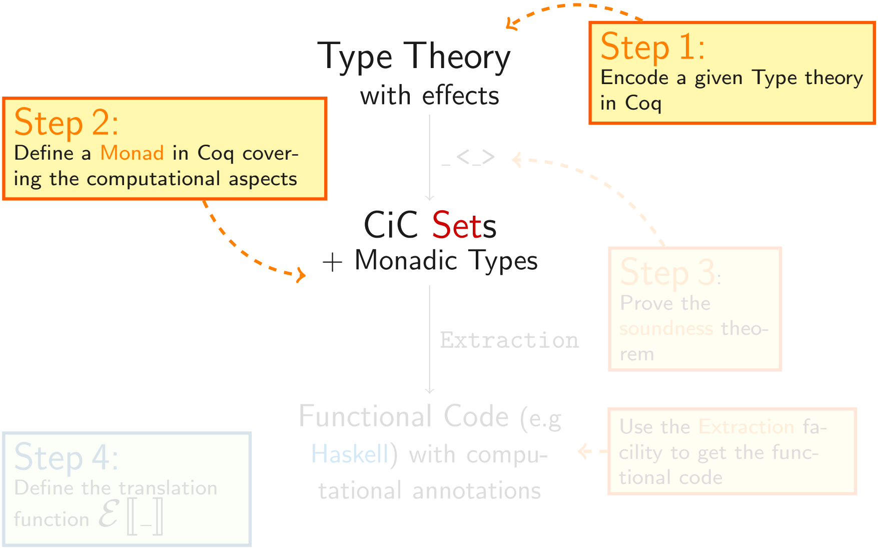

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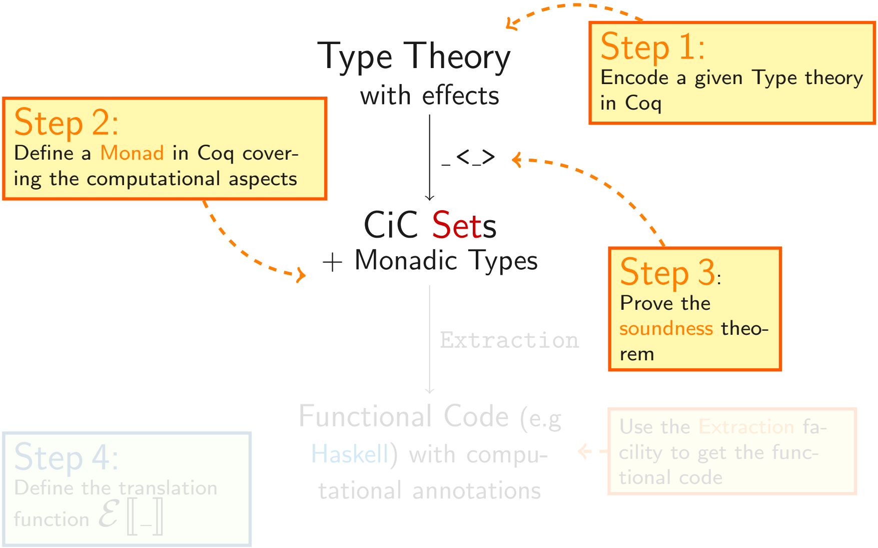

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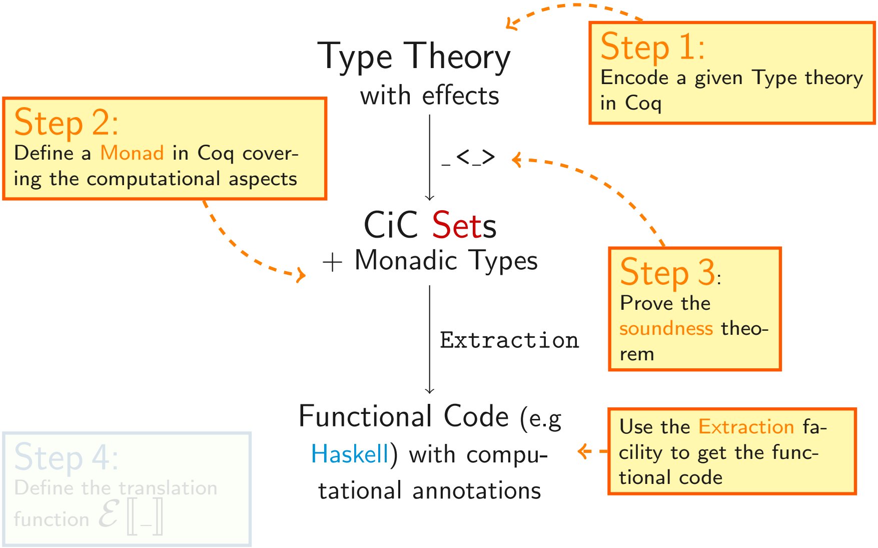

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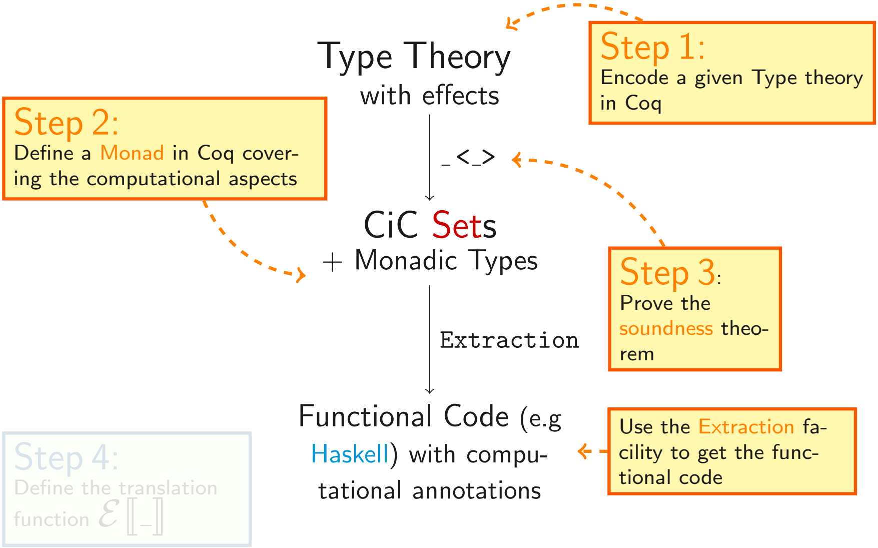

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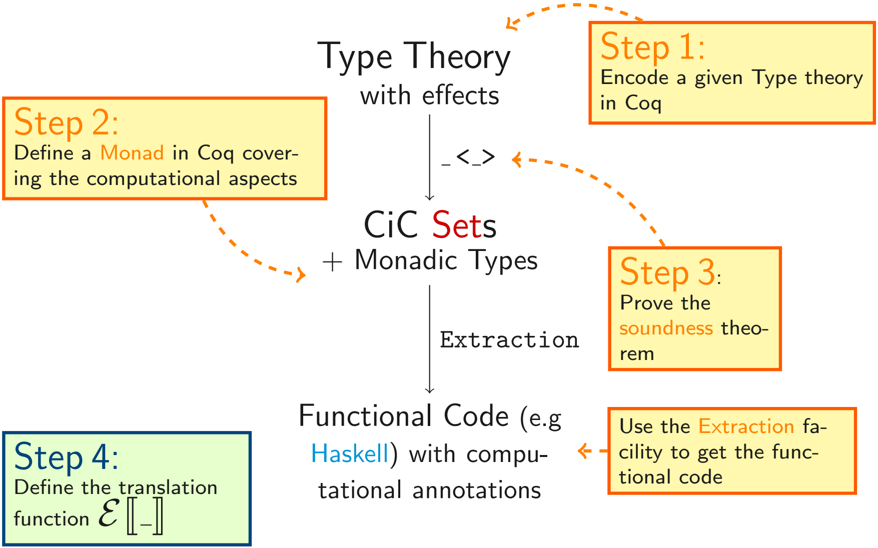

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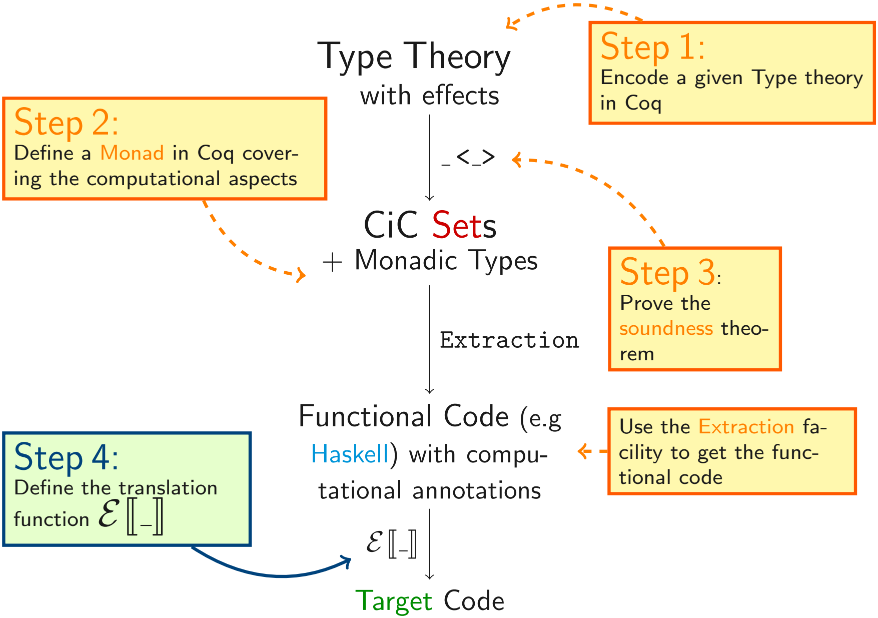

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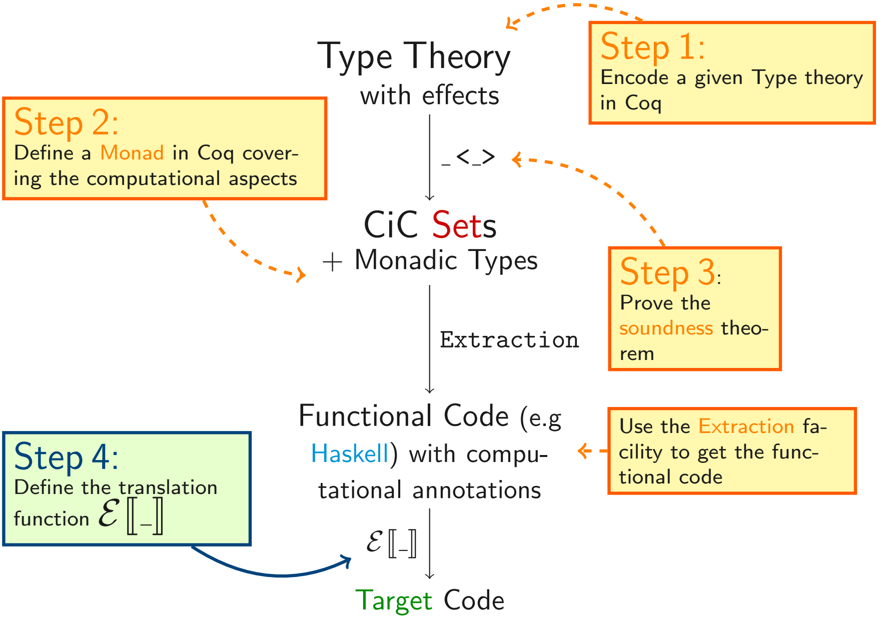

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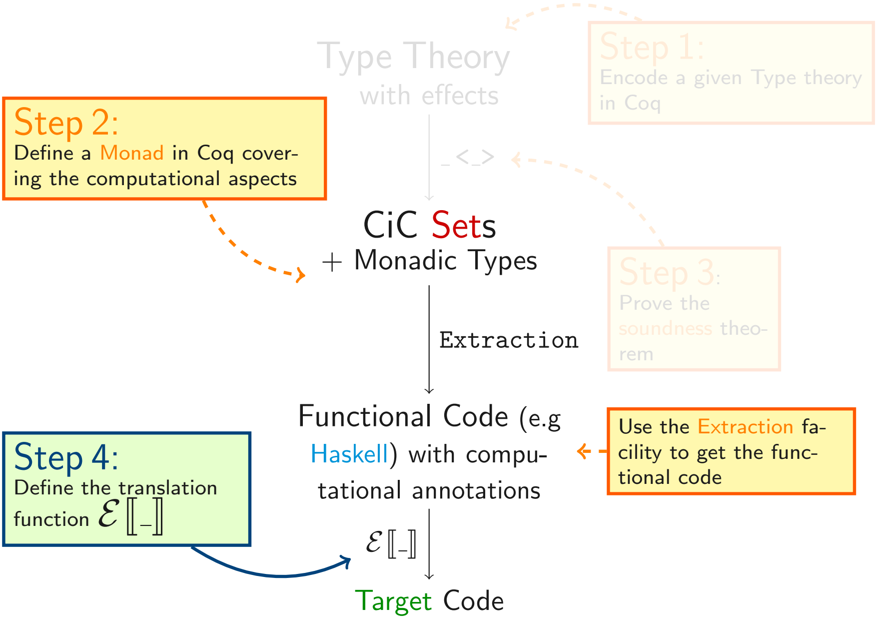

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Extraction of distributed code

We define the distributed monad in the **Calculus of Inductive Constructions**

`forall w: World` and `A: Set`

`IO w A :Set`

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A computation localized on the specified host

Extraction of distributed code

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$$\overset{\texttt{forall w: World} \text{ and } \texttt{A: Set}}{\texttt{IO w A :Set}}$$

By Curry-Howard Isomorphism, the (constructive) proofs of these specifications are turned into **decorated Haskell** code

$$\texttt{IO w A} \overset{\textit{Extraction}}{\Longrightarrow} \mathbb{H}$$

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These decorations are exploited by the Haskell-**Erlang** Compiler

$$\mathcal{E}[\![\_]\!] : \mathbb{H} \rightarrow \mathbb{E}$$

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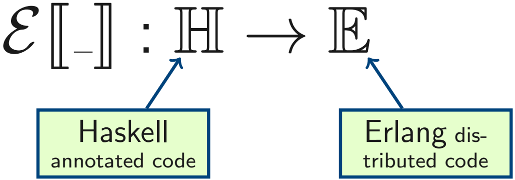

$$\mathcal{E}[\![\_]\!] : \mathbb{H} \longrightarrow \mathbb{E}$$

Haskell
annotated code

Erlang distributed code

Monads in Coq

We define a *family* of monads indexed by worlds from `Set` to `Set`. Given a world `w` a monad is a functor defined as

$$\mathtt{IO\ w\ A} = S \ \rightarrow ((\mathtt{R\ w\ A}) + \mathtt{Error})$$

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A **Localized** computation

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Function Space
from the global store to the results plus the error state

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Monadic Operators

$$\texttt{IOget}_{\texttt{w}}\ \texttt{A} : \texttt{IO remote A} \rightarrow \texttt{IO w A}$$

$\lambda\ \kappa\ \sigma.\ \kappa(\sigma)$ (Operator's implementation)

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Other monadic operators

$\mathtt{IOreturn_w\ A} : A \rightarrow \mathtt{IO\ w\ A}$

$\mathtt{IObind_w\ A\ B} : \mathtt{IO\ w\ A} \rightarrow (A \rightarrow \mathtt{IO\ w\ B}) \rightarrow \mathtt{IO\ w\ B}$

$\mathtt{IOlookup_w\ A} : \mathtt{Ref_w} \rightarrow (\mathbb{N} \rightarrow \mathtt{IO\ w\ A}) \rightarrow \mathtt{IO\ w\ A}$

$\mathtt{IOupdate_w\ A} : \mathtt{Ref_w} \rightarrow \mathbb{N} \rightarrow \mathtt{IO\ w\ A} \rightarrow \mathtt{IO\ w\ A}$

$\mathtt{IOnew_w\ A} : \mathbb{N} \rightarrow (\mathtt{Ref_w} \rightarrow \mathtt{IO\ w\ A}) \rightarrow \mathtt{IO\ w\ A}$

Extraction

Lemma (Remote Procedure Call)

$\forall \mathtt{w}\ \mathtt{w'}, (\mathbb{N} \to \mathtt{IO}\ \mathtt{w'}\ \mathtt{bool}) \to (\mathbb{N} \to \mathtt{IO}\ \mathtt{w}\ \mathtt{bool})$

Proof. simpl; introv f. intro n. apply* IOget.
Qed.

Haskell

`rpc w w' f n =` iOget w w' (f n)

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Given two worlds w w'

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Given a function f

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Given a value, say *n*

Haskell

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$\forall w\ w', (\mathbb{N} \to IO\ w'\ bool) \to (\mathbb{N} \to IO\ w\ bool)$

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Apply `IOget` to $(f\ n)$

(World parameters are inferred)

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Notice: The annotated Haskell code is not runnable yet!

The HEC Compiler: the mobility fragment

$$\begin{aligned}\mathcal{M}[\![\texttt{iOget}\ A_1\ A_2\ (F\ A_3)]\!]_\rho = \ & \textbf{spawn}\texttt{(element(2, } \mathcal{E}[\![A_1]\!]_\rho\texttt{), } \rho(\eta)\texttt{,}\\ & \quad \texttt{dispatcher,[fun () -> } \mathcal{E}[\![F]\!]_\rho \mathcal{E}[\![A_3]\!]_\rho \textbf{end},\\ & \quad \mathcal{E}[\![A_2]\!]_\rho, \{\textbf{self}\texttt{(), } \textbf{node}\texttt{()}\}\texttt{]),}\\ & \textbf{receive}\{\texttt{result, Z}\}\texttt{ -> Z}\end{aligned}$$

Erlang Primitives remind:

`Pid` = **spawn** `(Host, Module, Function, Parameters)` (Code Mobilty)

receive {`Pattern`} `->` `Expression` (Receive)

`Pid !` Expression (Send)

The HEC Compiler: the location fragment

New

$$\mathcal{M}[\![\texttt{iOnew}\ A_1\ A_2\ A_3]\!]_\rho = \\ (\mathcal{E}[\![A_3]\!]_\rho)(\textbf{spawn}\texttt{(}\textbf{element}\texttt{(2, }\mathcal{E}[\![A_1]\!]_\rho\texttt{)},\rho\texttt{("module name"),} \\ \texttt{location, [}\mathcal{E}[\![A_2]\!]_\rho\texttt{]))}$$

Update

$$\mathcal{M}[\![\texttt{iOupdate}\ w\ A_1\ A_2\ A_3]\!]_\rho = \mathcal{E}[\![A_1]\!]_\rho!\{\textbf{update},\mathcal{E}[\![A_2]\!]_\rho\},\mathcal{E}[\![A_3]\!]_\rho$$

Lookup

$$\mathcal{M}[\![\texttt{iOlookup}\ w\ A_1\ A_2]\!]_\rho = \mathcal{E}[\![A_1]\!]_\rho\texttt{!}\{\mathrm{get},\{\textbf{self}(),\textbf{node}()\}\}, \\ \textbf{receive}\{\texttt{result, Z}\} \rightarrow (\mathcal{E}[\![A_2]\!]_\rho)(Z)$$

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The `rpc` example

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rpc w w' f n = iOget w w' (f n)
```

Erlang

```
spawn(element(2, w), ρ(η),
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[fun () -> f(n) end,w',{self(), node()}]),
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$\mathcal{M}[\![\_]\!] : \mathbb{H} \to \mathbb{E}$

Haskell annotations are exploited by the HEC compiler

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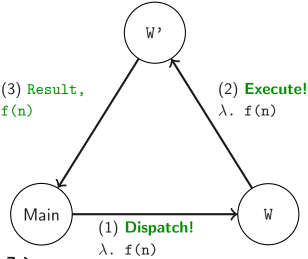
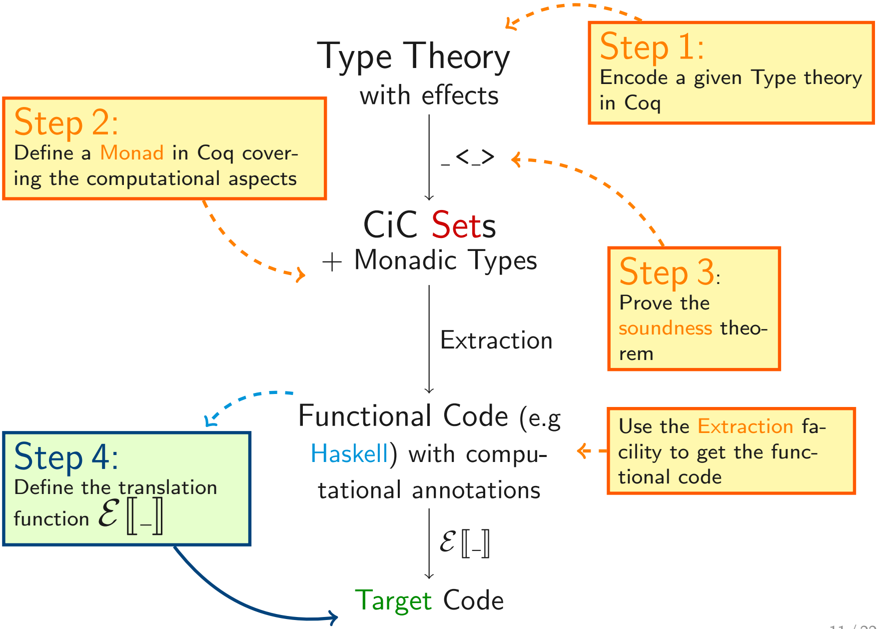


Adding a Type Theory



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λ_{XD} : A Type Theory for distributed computations

We give a Type theory similar to the Intuitionistic Hybrid Modal Logic of Licata and Harper[1]

Syntax (Hybrid Modal Logic IS5)

$$
\begin{array}{lll}
\text{Types} & A ::= & \text{Unit} \mid \text{Bool} \mid \text{Nat} \mid A \times B \mid A \rightarrow B \\
 & & \mid \text{Ref} \qquad\qquad (\text{Locations}) \\
 & & \mid \forall w.A \qquad\qquad (\text{Necessarily} \quad \Box A) \\
 & & \mid \exists w.A \qquad\qquad (\text{Possibly} \quad \Diamond A) \\
 & & \mid A @ w \qquad\qquad (\text{Nominal}) \\
 & & \mid \bigcirc A \qquad\qquad (\text{Lax Modality})
\end{array}
$$

---

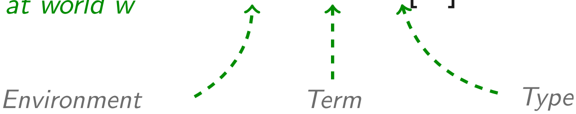
[1] "A monadic formalization of ML5". In Proc. LFMTP, EPTCS 34, 2010.

λ_{XD}: Type System

“*A* is inhabited with *M at world w*”

$$\Gamma \vdash M : A[w]$$

(Judgment)



Environment

Term

Type

▸ World Fragment

▸ Monadic Fragment

λ_{XD}: Type System

"A is inhabited with M at world w"

$$\Gamma \vdash M : A[w]$$

(Judgment)

Environment *Term* *Type*

World Fragment

(A small subset)

$$\frac{\Gamma \vdash M : A[w]}{\Gamma \vdash \ \text{some } w \ M : \exists w.A[w]} \ (\text{some})$$

Intuitionistic $\exists$

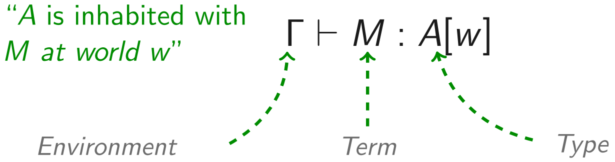
- a world w as Witness
- a Proof of $A(w)$

$$\frac{\Gamma \vdash M : \exists u.A[w] \quad \Gamma, x : A\{z/u\}[w] \vdash N : C[w'] \quad z \text{ fresh in } \Gamma}{\Gamma \vdash \ \text{letd } (z, x) = M \ \text{ in } N : C[w']} \ (\text{letd})$$

‣ World Fragment

‣ Monadic Fragment

λ_{XD}: Type System



“*A* is inhabited with *M* at world *w*”

$$\Gamma \vdash M : A[w]$$

(Judgment)

Environment

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Type

Monadic Fragment

(A small subset)

$$\frac{\text{Mobile } A \quad \Gamma \vdash M : \bigcirc A[w']}{\Gamma \vdash \text{ get } w' \; M : \bigcirc A[w]} \;(\text{Get})$$

World Fragment

Monadic Fragment

Soundness

We restrict the mobile types which are movable:

$$\frac{b \in \{\text{Unit, Nat, Bool}\}}{\text{Mobile } b}\ (\text{Basic}) \qquad \frac{\text{Mobile } A \quad \text{Mobile } B}{\text{Mobile } A \times B}\ (\text{Prod})$$

$$\frac{}{\text{Mobile } A@w}\ (\text{At}) \qquad \frac{\text{Mobile } A}{\text{Mobile } \forall w.A}\ (\text{Forall}) \qquad \frac{\text{Mobile } A}{\text{Mobile } \exists w.A}\ (\text{Exists})$$

Lemma (Mobility)

$\forall A \in$ Type, if Mobile A, then $\forall w,\ w' \in \mathtt{W}$, A <w> $=$ A <w'>.

`[Coq proof]`

λ_{XD}: Translation into CIC `Sets`

$$\_\,\texttt{<}w\texttt{>} : \text{Type}_{\lambda_{XD}} \to \texttt{Type}$$

$\text{Unit}\,\texttt{<w>} = \texttt{unit}$

$\text{Nat}\,\texttt{<w>} = \texttt{nat}$

$A \times B\,\texttt{<w>} = A\,\texttt{<w>}\ \texttt{*}\ B\,\texttt{<w>}$

$\text{Bool}\,\texttt{<w>} = \texttt{bool}$

$\text{Ref}\,\texttt{<w>} = \texttt{ref}\ w$

$A@w'\,\texttt{<w>} = A\,\texttt{<w'>}$

$A \to B\,\texttt{<w>} = A\,\texttt{<w>}\ \texttt{->}\ B\,\texttt{<w>}$

$\forall w'.A\,\texttt{<w>} = \texttt{forall w'}, (A\ w')\,\texttt{<w>}$

$\exists w'.A\,\texttt{<w>} = \{\ \texttt{w' : world}\ \&\ (A\ w')\,\texttt{<w>}\ \}$

$\bigcirc\, A\,\texttt{<w>} = \texttt{IO w}\ (A\,\texttt{<w>})$

Theorem (Soundness)

Let $\Gamma \in \mathit{Ctxt}$, $t \in \mathit{Term}$, $A \in \mathit{Type}$, $w \in \mathit{World}$, let $\{w_1, \ldots, w_n\}$ be all free worlds in Γ, A, t, w.

$$\Gamma \vdash_{XD} t : A[w] \quad \to \quad w_1 : W, \ldots, w_n : W \vdash \_ : [\![\Gamma]\!] \to A\,\texttt{<}w\texttt{>}.$$

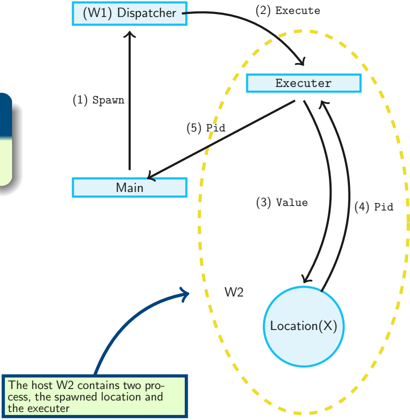
`[Coq proof]`

Example: Remote Write

Lemma (Remote Write)

$\forall w' \;\; w'', \mathbb{N}@w' \rightarrow_{XD} \bigcirc(\mathit{Ref}@w'')\ \text{<}w'\text{>}$

Proof. simpl; introv value. apply IOget with (remote := w2). apply IOnew. exact value. intros address. apply IOret. exact address. **Defined**.



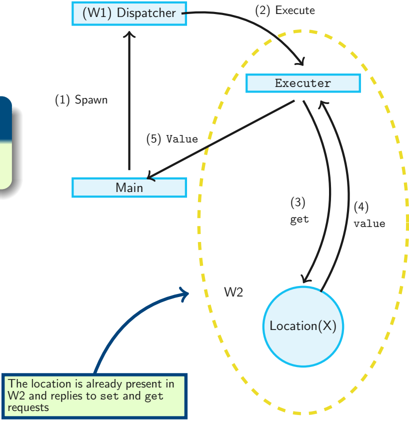
The host W2 contains two process, the spawned location and the executer

Example: Remote Read

Lemma (Remote Read)

$\forall w' \ w'', Ref @ w'' \rightarrow_{XD} \bigcirc(\mathbb{N}) <w'>$

Proof. simpl; introv address. apply IOget with (remote := w2). apply* IOlookup. intros. apply* IOret.
Defined.



The location is already present in W2 and replies to set and get requests

Update Lemma

A suitable lemma can be stated and proved in order to guarantee the system behaves correctly

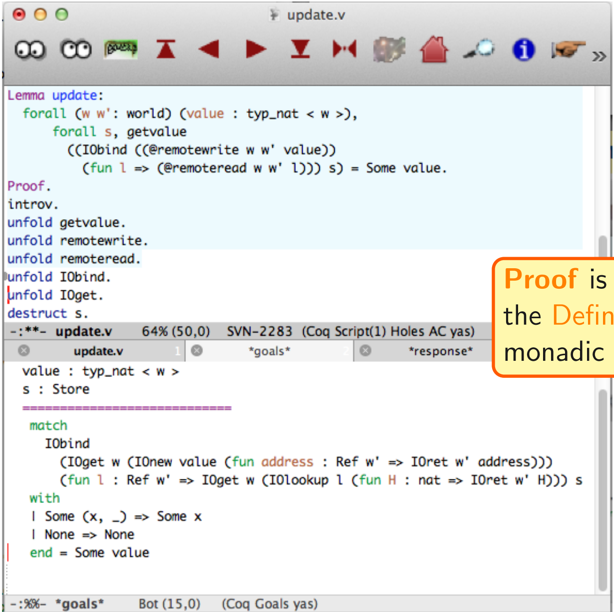
Lemma (Update operation is correct)

$\forall$*w w'*,$\forall\mathbb{N}$, $\forall s$: *Store*,

```
getvalue ((IObind ((@remotewrite w w' value ))
(λ l.  (@remoteread w w' l))) s) = Some value.
```

[Coq proof]

Update lemma: proof deals with monadic constructors



```
Lemma update:
  forall (w w': world) (value : typ_nat < w >),
      forall s, getvalue
        ((IObind ((@remotewrite w w' value))
          (fun l => (@remoteread w w' l))) s) = Some value.
Proof.
introv.
unfold getvalue.
unfold remotewrite.
unfold remoteread.
unfold IObind.
unfold IOget.
destruct s.
-:**-  update.v      64% (50,0)    SVN-2283  (Coq Script(1) Holes AC yas)
```

```
  value : typ_nat < w >
  s : Store
  ============================
   match
     IObind
       (IOget w (IOnew value (fun address : Ref w' => IOret w' address)))
       (fun l : Ref w' => IOget w (IOlookup l (fun H : nat => IOret w' H))) s
   with
   | Some (x, _) => Some x
   | None => None
   end = Some value

-:%%-  *goals*      Bot (15,0)    (Coq Goals yas)
```

Proof is by unfolding the Definitions of the monadic operators

Conclusions

Summary

- We presented a generic methodology for code extraction which works in principle with every computational aspect
  - + Define a front-end type theory on top (optional)
  - + Define a `IO` monad over `Set`
  - + Implement the back-end compiler
- We shown how to extract distributed code by defining a Distributed Monad `IO w A`
- We defined the λ_{XD} on top

Conclusions

Future work

- Other computational aspects / target languages
- is the compiler correct? ⇒ Compiler Certification could be achieved by using the implementation of monadic operators as the specification of how the target code must behave
- Alternatively, an Axiomatization for the operators is needed
  - + which takes into account the mobility
  - + not available yet (AFAIK)
  - + but could be achived extending similar work (see e.g., Power and Plotkin ▸ Axioms)

Thanks for your attention.
Questions?

Appendix: λ_{XD} Typesystem - World Fragment

$$\frac{\Gamma \vdash M : \ A[w'] \quad w \text{ fresh in } \Gamma}{\Gamma \vdash \Lambda w.M : \forall w.A[w']} \text{(Box)} \qquad \frac{\Gamma \vdash M : \forall w.A[w]}{\Gamma \vdash \ \text{unbox } w' M : A[w]} \text{(Unbox)}$$

$$\frac{\vdash \Gamma \quad x : A[w] \in \Gamma}{\Gamma \vdash x : A[w]} \text{(Var)} \qquad \frac{\Gamma \vdash M : A[w]}{\Gamma \vdash \ \text{some } w \ M : \exists w.A[w]} \text{(Some)}$$

$$\frac{\Gamma \vdash M : \exists u.A[w] \quad \Gamma, x : A\{z/u\}[w] \vdash N : C[w'] \quad z \text{ fresh in } \Gamma}{\Gamma \vdash \ \text{letd } (z, x) = M \ \text{ in } N : C[w']} \text{(LetD)}$$

$$\frac{\Gamma \vdash M : A[w]}{\Gamma \vdash \text{hold } M : A@w[w']} \text{(Hold)}$$

$$\frac{\Gamma \vdash M : A@z[w] \quad \Gamma, x : A[z] \vdash N : C[w]}{\Gamma \vdash \ \text{leta } x = M \text{ in } N : C[w]} \text{(LetA)}$$

▸ Return

Appendix: λ_{XD} Typesystem - Monadic fragment

$$\frac{\Gamma \vdash M : A[w]}{\Gamma \vdash \text{ return } M : \bigcirc A[w]} \text{(Ret)} \qquad \frac{\Gamma \vdash M : \bigcirc A[w] \quad \Gamma \vdash N : A \to \bigcirc B[w]}{\Gamma \vdash \text{ bind } M\ N : \bigcirc B[w]}$$

$$\frac{\text{Mobile } A \quad \Gamma \vdash M : \bigcirc A[w']}{\Gamma \vdash \text{ get } w'\ M : \bigcirc A[w]} \text{(Get)}$$

$$\frac{\Gamma \vdash M : \text{Nat}[w] \quad \Gamma \vdash N : \text{Ref} \to \bigcirc A[w]}{\Gamma \vdash \text{ new } M\ N : \bigcirc A[w]} \text{(New)}$$

$$\frac{\Gamma \vdash M : \text{ Ref } [w] \quad \Gamma \vdash N : \text{Nat} \to \bigcirc A}{\Gamma \vdash \text{ lookup } M\ N : \bigcirc A[w]} \text{(Lookup)}$$

$$\frac{\Gamma \vdash T1 : \text{ Ref } [w] \quad \Gamma \vdash T2 : \text{Nat}[w] \quad \Gamma \vdash T3 : \bigcirc A[w]}{\Gamma \vdash \text{ update } T1\ T2\ T3 : \bigcirc A[w]} \text{(Update)}$$

▸ Return

Appendix: On the axiomatization

Rules for the `lookup` and `update` are borrowed from Plotkin and Power [1]:

1.$l_{loc}(u_{loc,v}(x))v = x$

2.$l_{loc}(l_{loc}(t_{vv'})_v)_{v'} = l_{loc}(t_{vv})_v$

3.$u_{loc,v}(u_{loc,v'}(x)) = u_{loc,v'}(x)$

4.$u_{loc,v}(l_{loc}(t_{v'})_{v'}) = u_{loc,v}(t_v)$

5.$l_{loc}(l'_{loc}(t_{vv'})_{v'})_v = l'_{loc}(l_{loc}(t_{vv'})_v)_{v'}$where $loc = loc'$

6.$u_{loc,v}(u_{loc',v'}(x)) = u_{loc',v'}(u_{loc,v}(x))$where $loc = loc''$

▸ Return

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[1] G. D. Plotkin and J. Power. Notions of computation determine monads. In Proc. FoSSaCS, LNCS 2303, 2002.