Compilers - Chapter 6:

Optimisation and data-flow analysis

Part 2: Data flow analysis via live variables

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Textbooks:

- Cooper and Torczon (EaC):
  - Chapter 9
- Aho, Lam, Sethi, Ullman (Dragon book 2nd ed):
  - Chapter 9, 9.2
- Appel (Modern Compiler Construction in Java 2nd ed):
  - Chapter 10

Dataflow analysis (DFA)

- Optimisation consists of **analysis** and **transformation**
- **Analysis**: deduce program properties from IR
  - Analyse effect of each instruction
  - Compose these effects to derive information about the entire procedure
- Consider: Add (Reg T0) (Reg T1)
  - Uses temporaries T0 and T1
  - Kills old definition of T1
  - Generates new definition of T1
- We will see how to do “dataflow analysis” in order to use this local information to derive global properties

Example dataflow analysis: **live ranges**

- Recall graph colouring:

1. Generate code using temporaries T0… instead of registers
2. For each temporary T*i*, find T*i*'s "**live range**" – the set of instructions for which T*i* must reside in a register
3. If LiveRange(T*i*) **intersects** LiveRange(T*j*) they have to be allocated to different registers – they *interfere*
4. Assemble the register interference graph (RIG)
5. Colour the RIG by assigning real registers to temporaries avoiding interference
6. If successful, replace temporaries with registers and generate code
7. If graph cannot be coloured, find a temporary to *spill* to memory, then retry

Preliminary: build the control flow graph

- data CFG = ControlFlowGraph [CFGNode]
- data CFGNode = Node Id Instruction [Register] [Register] [Id]
  uses *defs* *succs*
- type Id = Int
- data Register = D Int | T Int *(temporaries before, real after)*
- buildCFG :: [Instruction] -> CFG

- Each node of the control flow graph contains an instruction, together with:
  - nodeDefs cfgnode = list of temporaries which this instruction updates
  - nodeUses cfgnode = list of temporaries which this instruction reads
  - nodeSuccs cfgnode = list of nodes which might be executed next

Source code

```
while (b<10)
{
  if (b<a)
    a = a*7;
    b = a+1;
  else
    a = b-1;
}
```

Intermediate code

```
    Bra L2
L1:
    cmp b a
    bge L3
    mul #7 a
    mov a b
    add #1 b
    bra L4
L3:
    mov b a
    sub #1 a
L4:
L2:
    Cmp b #10
    Blt L1
```

Control flow graph

		uses	*defs*	*succs*
1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

Finding live ranges… example

Live variable analysis - definition

- **Point**: any location *between* adjacent nodes
- **Path**: a sequence of points $p_1..p_i\ p_{i+1}..p_n$ such that p_{i+1} is the immediate successor of p_i in the CFG

- "**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Live variable analysis - definition

- **Point**: any location *between* adjacent nodes
- **Path**: a sequence of points $p_1..p_i\ p_{i+1}..p_n$ such that p_{i+1} is the immediate successor of p_i in the CFG

- "**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

We *could* work this out with a depth-first search – for every variable, and for every point. We are looking for a more efficient algorithm, that computes the *set* of *all* live variables at *every* point.

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

"**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Is b live-out from node 1?

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

"**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Consider variable b after node 1

b is live-out from node 1 because it is used at node 10

Is b live-out from node 2?

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

“**x is live at p**”: for some variable x and point p, the value of x could be used along some path starting at p.

Consider variable b after node 2

Is b live-out from node 2?

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

"**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Consider variable b after node 2

b ***is*** live-out from node 2 because it ***might*** be used at node 8 (it might not be used but it ***might***)

Is b live-out from node 4?

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

"**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Consider variable b after node 4

1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #10	b		11
11	Blt L1			12,2

Is b live-out from node 4?

"**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.

Consider variable b after node 4

b is not live-out from node 4: node 5 overwrites b's value – the value from node 4 can't reach further

Dataflow equations for live variable analysis

Define:

- LiveIn(n): the set of temporaries live immediately **before** node n
- LiveOut(n): the set of temporaries live immediately **after** node n

- A variable is live immediately ***after*** node n if it is live before any of n's successors

- A variable is live immediately ***before*** node n if:
  - It is live after node n (ie some later instruction reads it)
  - Unless it is overwritten by node n

  OR

  - It is used by node n (ie the instruction reads it)

Dataflow equations for live variable analysis

- LiveIn(n): set of temporaries live immediately **before** node n
- LiveOut(n): set of temporaries live immediately **after** node n

- A variable is live immediately after node n if it is live before any of n's successors:
  - $\text{LiveOut}(n) = \bigcup_{s \in \text{succ}(n)} \text{LiveIn}(s)$

- A variable is live immediately before node n if:
  - It is live after node n (ie some later instruction reads it)
  - Unless it is overwritten by node n

  OR
  - It is used by node n (ie the instruction reads it)
  - $\text{LiveIn}(n) = \text{uses}(n) \cup (\text{LiveOut}(n) - \text{defs}(n))$

Dataflow equations for live variable analysis

- LiveIn(n): set of temporaries live immediately **before** node n
- LiveOut(n): set of temporaries live immediately **after** node n
- A variable is live immediately after node n if it is live before any of n's successors:

The union of the LiveIns of all this node's successors

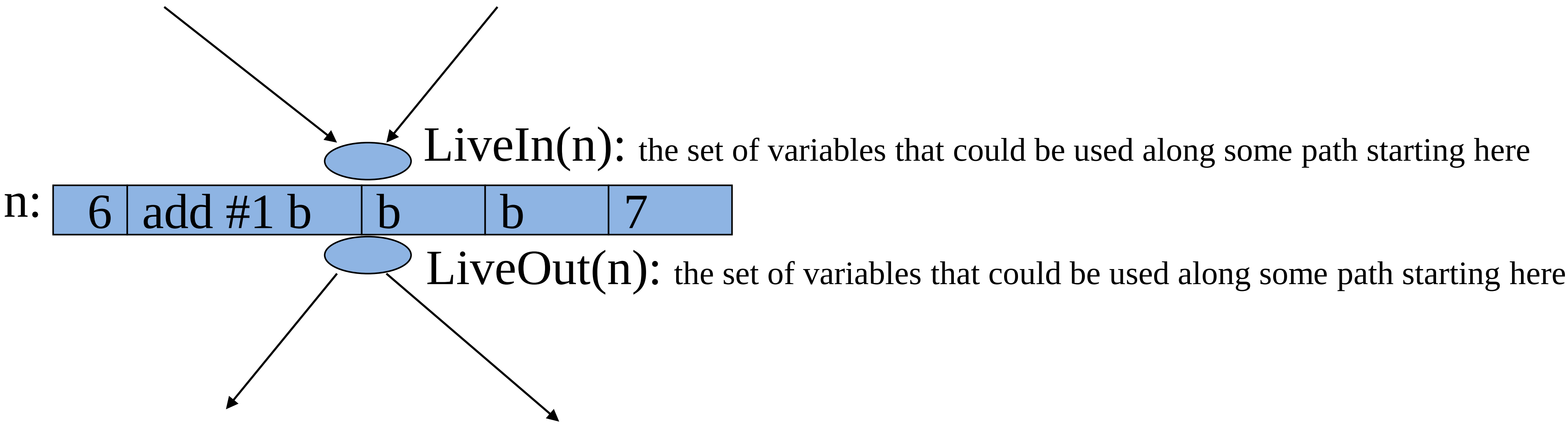
  - $\text{LiveOut}(n) = \bigcup_{s \in \text{succ}(n)} \text{LiveIn}(s)$

- A variable is live immediately before node n if:
  - It is live after node n (ie some later instruction reads it)
  - Unless it is overwritten by node n

  OR
  - It is used by node n (ie the instruction reads it)

  - $\text{LiveIn}(n) = \text{uses}(n) \cup (\text{LiveOut}(n) - \text{defs}(n))$

- What's the difference between LiveIn and LiveOut?



$LiveIn(1) = uses(1) \cup (LiveOut(1) - defs(1))$

$LiveOut(1) = \bigcup_{s \in succ(1)} LiveIn(s)$

$LiveIn(2) = uses(2) \cup (LiveOut(2) - defs(2))$

$LiveOut(2) = \bigcup_{s \in succ(2)} LiveIn(s)$

$LiveIn(3) = uses(3) \cup (LiveOut(3) - defs(3))$

$LiveOut(3) = \bigcup_{s \in succ(3)} LiveIn(s)$

$LiveIn(4) = uses(4) \cup (LiveOut(4) - defs(4))$

$LiveOut(4) = \bigcup_{s \in succ(4)} LiveIn(s)$

$LiveIn(5) = uses(5) \cup (LiveOut(5) - defs(5))$

$LiveOut(5) = \bigcup_{s \in succ(5)} LiveIn(s)$

$LiveIn(6) = uses(6) \cup (LiveOut(6) - defs(6))$

$LiveOut(6) = \bigcup_{s \in succ(6)} LiveIn(s)$

$LiveIn(7) = uses(7) \cup (LiveOut(7) - defs(7))$

$LiveOut(7) = \bigcup_{s \in succ(7)} LiveIn(s)$

$LiveIn(8) = uses(8) \cup (LiveOut(8) - defs(8))$

$LiveOut(8) = \bigcup_{s \in succ(8)} LiveIn(s)$

$LiveIn(9) = uses(9) \cup (LiveOut(9) - defs(9))$

$LiveOut(9) = \bigcup_{s \in succ(9)} LiveIn(s)$

$LiveIn(10) = uses(10) \cup (LiveOut(10) - defs(10))$

$LiveOut(10) = \bigcup_{s \in succ(10)} LiveIn(s)$

$LiveIn(11) = uses(11) \cup (LiveOut(11) - defs(11))$

$LiveOut(11) = \bigcup_{s \in succ(11)} LiveIn(s)$

Id		Uses	Defs	Ids of succs
1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #	b		11
11	Blt L1			12,2

- 22 simultaneous equations

LiveIn(1)= uses(1) U (LiveOut(1) – defs(1))

LiveOut(1)=LiveIn(10)

LiveIn(2)= uses(2) U (LiveOut(2) – defs(2))

LiveOut(2)=LiveIn(3)

LiveIn(3)= uses(3) U (LiveOut(3) – defs(3))

LiveOut(3)=LiveIn(4) U LiveIn(8)

LiveIn(4)= uses(4) U (LiveOut(4) – defs(4))

LiveOut(4)=LiveIn(5)

LiveIn(5)= uses(5) U (LiveOut(5) – defs(5))

LiveOut(5)=LiveIn(6)

LiveIn(6)= uses(6) U (LiveOut(6) – defs(6))

LiveOut(6)=LiveIn(7)

LiveIn(7)= uses(7) U (LiveOut(7) – defs(7))

LiveOut(7)=LiveIn(10)

LiveIn(8)= uses(8) U (LiveOut(8) – defs(8))

LiveOut(8)=LiveIn(9)

LiveIn(9)= uses(9) U (LiveOut(9) – defs(9))

LiveOut(9)=LiveIn(10)

LiveIn(10)= uses(10) U (LiveOut(10) – defs(10))

LiveOut(10)=LiveIn(11)

LiveIn(11)= uses(11) U (LiveOut(11) – defs(11))

LiveOut(11)=LiveIn(12) U LiveIn(2)

Id		Uses	Defs	Ids of succs
1	Bra L2			10
2	cmp b a	a,b		3
3	bge L3			4,8
4	mul #7 a	a	a	5
5	mov a b	a	b	6
6	add #1 b	b	b	7
7	bra L4			10
8	mov b a	b	a	9
9	sub #1 a	a	a	10
10	Cmp b #	b		11
11	Blt L1			12,2

- 22 simultaneous equations

Clearer if we substitute in the successors:
succs(11) = {12,2}

Solving the dataflow equations

- We have a system of simultaneous equations for LiveIn(n) and LiveOut(n) for each node n

- How can we solve them?

Solving the dataflow equations

- Idea: Iterate!

```
for each n in CFG {
  LiveIn(n) := {}; LiveOut(n) := {};
}
repeat {
  for each n in CFG {
    LiveIn(n) = uses(n) U (LiveOut(n) – defs(n));
    LiveOut(n) = ⋃ s ∈ succ(n) LiveIn(s);
  }
} until LiveIn and LiveOut stop changing
```

$$\mathrm{LiveOut}(n) = \bigcup_{s \in succ(n)} \mathrm{LiveIn}(s)$$

Iteration… walkthrough

Step 0

Node	uses	defs	succs	in	out
1			10	{ }	{ }
2	a,b		3	{ }	{ }
3			4,8	{ }	{ }
4	a	a	5	{ }	{ }
5	a	b	6	{ }	{ }
6	b	b	7	{ }	{ }
7			10	{ }	{ }
8	b	a	9	{ }	{ }
9	a	a	10	{ }	{ }
10	b		11	{ }	{ }
11			12,2	{ }	{ }

```
for each n in CFG {
  LiveIn(n) := {}; LiveOut(n) := {};
}
repeat {
  for each n in CFG {
    LiveIn(n) = uses(n) U (LiveOut(n) – defs(n));
    LiveOut(n) = U s ∈ succ(n) LiveIn(s);
  }
} until LiveIn and LiveOut stop changing
```

Q: should I process the nodes in order?

- see Appel pg 226 for another example

Iteration… walkthrough

Node	uses	defs	succs	Step 1 in	Step 1 out
1			10	{ }	**{b}**
2	a,b		3	**{a,b}**	{ }
3			4,8	{ }	**{a,b}**
4	a	a	5	**{a}**	**{a}**
5	a	b	6	**{a}**	**{b}**
6	b	b	7	**{b}**	{ }
7			10	{ }	**{b}**
8	b	a	9	**{b}**	**{a}**
9	a	a	10	**{a}**	**{b}**
10	b		11	**{b}**	{ }
11			12,2	{ }	{ }

for each n in CFG {
  LiveIn(n) := {}; LiveOut(n) := {};
}
repeat {
  for each n in CFG {
    LiveIn(n) = uses(n) U (LiveOut(n) – defs(n));
    LiveOut(n) = $\bigcup_{s \in succ(n)}$ LiveIn(s);
  }
} until LiveIn and LiveOut stop changing

Q: should I process the nodes in order?

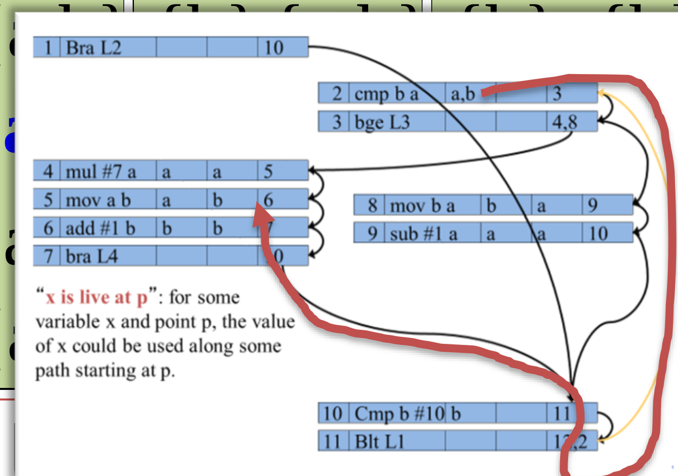
- see Appel pg 226 for another example

Node	uses	defs	succs	Step 1 in	Step 1 out	Step 2 in	Step 2 out	Step 3 in	Step 3 out	Step 4 in	Step 4 out	Step 5 in	Step 5 out	Step 5 in	Step 5 out
1			10	{ }	**{b}**	**{b}**	{b}	{b}	{b}	**{a,b}**	**{a,b}**	{a,b}	{a,b}	{a,b}	{a,b
2	a,b		3	**{a,b}**	{ }	{a,b	**{a,b}**	{a,b	{a,b}	{a,b	{a,b}	{a,b	{a,b}	{a,b	{a,b}
3			4,8	{ }	**{a,b}**	**{a,b}**	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b
4	a	a	5	**{a}**	**{a}**	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}
5	a	b	6	**{a}**	**{b}**	{a}	{b}	**{a}**	{b}	{a}	{b}	{a}	{b}	{a}	**{a,b**
6	b	b	7	**{b}**	{ }	{b}	**{b}**	{b}	{b}	{b}	{b}	{b}	**{a,b}**	**{a,b**	{a,b}
7			10	{ }	**{b}**	**{b}**	{b}	{b}	{b}	{b}	**{a,b}**	**{a,b}**	{a,b}	{a,b}	{a,b
8	b	a	9	**{b}**	**{a}**	{b}	**{a,b}**	{b}	{a,b}	{b}	{a,b}	{b}	{a,b}	{b}	{b}
9	a	a	10	**{a}**	**{b}**	**{a,b}**	{b}	{a,b}	{b}	{a,b}	**{a,b}**	{a,b}	{a,b}	{a,b}	{a,b
10	b		11	**{b}**	{ }	{b}	{ }	{b}	**{a,b}**	**{a,b}**	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}
11			12,2	{ }	{ }	{ }	**{a,b}**	**{a,b}**	{a.b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b

LiveIn(n) = uses(n) U (LiveOut(n) – defs(n));

LiveOut(n) = $\bigcup_{s \in succ(n)}$ LiveIn(s);

Node	uses	defs	succs	Step 1 in	Step 1 out	Step 2 in	Step 2 out	Step 3 in	Step 3 out	Step 4 in	Step 4 out	Step 5 in	Step 5 out	Step 5 in	Step 5 out
1			10	{ }	{b}	{b}	{b}	{b}	{b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}
2	a,b		3	{a,b}	{ }	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}
3			4,8	{ }	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}
4	a	a	5	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}	{a}
5	a	b	6	{a}	{b}	{a}	{b}	{a}	{b}	{a}	{b}	{a}	{b}	{a}	{a,b}
6	b	b	7	{b}	{ }	{b}	{b}	{b}	{b}	{b}	{b}	{b}	{a,b}	{a,b}	{a,b}
7			10	{ }	{b}	{b}	{b}	{b}	{b}	{b}	{a,b}	{a,b}	{a,b}	{a,b}	{a,b}
8	b	a	9	{b}	{a}	{b}	{a,b}	{b}	{a,b}	{b}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	a	a	10	{a}	{b}	{a,b}	{b}	{a,b}	{b}	{a,b}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	b		11	{b}	{ }	{b}	{ }	{b}	{a,b}	{a,b}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11			12,2	{ }	{ }	{ }	{a,b}	{a,b}	{a.b}	{a,b}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



LiveIn(n) = uses(n) U (LiveOut(n) – defs(n));

LiveOut(n) =

Real example: factorial loop

Concrete syntax

```
program
  declare x :
   Integer
  declare a :
   Integer
begin
  a := 1
  for x = 1 to 10
   a := a * x
  end
end
```

Abstract syntax

```
(Program [Decl "a" Integer]
 [(Assign (Var "a") (Const 1)),
  (For "x" (Const 1) (Const 10)
   [(Assign (Var "a")
     (Binop Times (Ref (Var "a")) (Ref (Var "x"))))]
)])
```

Real example: factorial loop

Concrete syntax

```
program
  declare x :
    Integer
  declare a :
    Integer
begin
  a := 1
  for x = 1 to 10
    a := a * x
  end
end
```

Code

```
.data
; Integer variable a has been allocated to T0
.text
move.l #1, T0
move.l #10, T1
move.l #1, T2
bra    L2
L1:
move.l T2, T3
move.l T0, T4
mul.l  T3, T4
move.l T4, T0
add.l  #1, T2
L2:
cmp.l  T1, T2
bgt    L3
bra    L1
L3:
move.l T2, x
```

(updates variable x on exit from loop – a bug! (?))

Real example: factorial loop

Concrete syntax

```
program
  declare x :
   Integer
  declare a :
   Integer
begin
  a := 1
  for x = 1 to 10
    a := a * x
  end
end
```

Code

```
Node 0 (Mov (ImmNum 1) (Reg T0)) [T0] [] [1] []
Node 1 (Mov (ImmNum 10) (Reg T1)) [T1] [] [2] [0]
Node 2 (Mov (ImmNum 1) (Reg T2)) [T2] [] [3] [1]
Node 3 (Bra "L2") [] [] [9] [2]
Node 4 (Mov (Reg T2) (Reg T3)) [T3] [T2] [5] [11]
Node 5 (Mov (Reg T0) (Reg T4)) [T4] [T0] [6] [4]
Node 6 (Mul (Reg T3) (Reg T4)) [T4] [T3,T4] [7] [5]
Node 7 (Mov (Reg T4) (Reg T0)) [T0] [T4] [8] [6]
Node 8 (Add (ImmNum 1) (Reg T2)) [T2] [T2] [9] [7]
Node 9 (Cmp (Reg T1) (Reg T2)) [] [T1,T2] [10] [3,8]
Node 10 (Bgt "L3") [] [] [11,12] [9]
Node 11 (Bra "L1") [] [] [4] [10]
Node 12 (Mov (Reg T2) (Abs "x")) [] [T2] [13] [10]
Node 13 Halt [] [] [] [12]
```

(Node id instrn defs uses succs preds)

Live range analysis for factorial example

	Step 0	Step 1	Step 2	Step 3	Step 4
LiveIns	([(0,[]),	([(0,[]),	([(0,[]),	([(0,[]),	([(0,[]),
	(1,[]),	(1,[]),	(1,[]),	(1,[]),	(1,[]),
	(2,[]),	(2,[]),	(2,[]),	(2,[T1]),	(2,[T1]),
	(3,[]),	(3,[]),	(3,[T1,T2]),	(3,[T1,T2]),	(3,[T1,T2]),
	(4,[]),	(4,[T2]),	(4,[T2,T0]),	(4,[T2,T0]),	(4,[T2,T0]),
	(5,[]),	(5,[T0]),	(5,[T0,T3]),	(5,[T0,T3]),	(5,[T0,T3,T2]),
	(6,[]),	(6,[T3,T4]),	(6,[T3,T4]),	(6,[T3,T4,T2]),	(6,[T3,T4,T2,T1]),
	(7,[]),	(7,[T4]),	(7,[T4,T2]),	(7,[T4,T2,T1]),	(7,[T4,T2,T1]),
	(8,[]),	(8,[T2]),	(8,[T2,T1]),	(8,[T2,T1]),	(8,[T2,T1]),
	(9,[]),	(9,[T1,T2]),	(9,[T1,T2]),	(9,[T1,T2]),	(9,[T1,T2]),
	(10,[]),	(10,[]),	(10,[T2]),	(10,[T2]),	(10,[T2]),
	(11,[]),	(11,[]),	(11,[]),	(11,[T2]),	(11,[T2,T0]),
	(12,[]),	(12,[T2]),	(12,[T2]),	(12,[T2]),	(12,[T2]),
	(13,[])],	(13,[])],	(13,[])],	(13,[])],	(13,[])],
LiveOuts	[(0,[]),	[(0,[]),	[(0,[]),	[(0,[]),	[(0,[]),
	(1,[]),	(1,[]),	(1,[]),	(1,[T1]),	(1,[T1]),
	(2,[]),	(2,[]),	(2,[T1,T2]),	(2,[T1,T2]),	(2,[T1,T2]),
	(3,[]),	(3,[T1,T2]),	(3,[T1,T2]),	(3,[T1,T2]),	(3,[T1,T2]),
	(4,[]),	(4,[T0]),	(4,[T0,T3]),	(4,[T0,T3]),	(4,[T0,T3,T2]),
	(5,[]),	(5,[T3,T4]),	(5,[T3,T4]),	(5,[T3,T4,T2]),	(5,[T3,T4,T2,T1]),
	(6,[]),	(6,[T4]),	(6,[T4,T2]),	(6,[T4,T2,T1]),	(6,[T4,T2,T1]),
	(7,[]),	(7,[T2]),	(7,[T2,T1]),	(7,[T2,T1]),	(7,[T2,T1]),
	(8,[]),	(8,[T1,T2]),	(8,[T1,T2]),	(8,[T1,T2]),	(8,[T1,T2]),
	(9,[]),	(9,[]),	(9,[T2]),	(9,[T2]),	(9,[T2]),
	(10,[]),	(10,[T2]),	(10,[T2]),	(10,[T2]),	(10,[T2,T0]),
	(11,[]),	(11,[]),	(11,[T2]),	(11,[T2,T0]),	(11,[T2,T0]),
	(12,[]),	(12,[]),	(12,[]),	(12,[]),	(12,[]),
	(13,[])])	(13,[])])	(13,[])])	(13,[])])	(13,[])])

Live range analysis for factorial example

	Step 5	Step 6	Step 7	Step 8	Step 9
LiveIns	([(0,[]),	([(0,[]),	([(0,[]),	([(0,[]),	([(0,[]),
	(1,[]),	(1,[]),	(1,[]),	(1,[]),	(1,[T0]),
	(2,[T1]),	(2,[T1]),	(2,[T1]),	(2,[T1,T0]),	(2,[T1,T0]),
	(3,[T1,T2]),	(3,[T1,T2]),	(3,[T1,T2,T0]),	(3,[T1,T2,T0]),	(3,[T1,T2,T0]),
	(4,[T2,T0]),	(4,[T2,T0,T1]),	(4,[T2,T0,T1]),	(4,[T2,T0,T1]),	(4,[T2,T0,T1]),
	(5,[T0,T3,T2,T1]),	(5,[T0,T3,T2,T1]),	(5,[T0,T3,T2,T1]),	(5,[T0,T3,T2,T1]),	(5,[T0,T3,T2,T1]),
	(6,[T3,T4,T2,T1]),	(6,[T3,T4,T2,T1]),	(6,[T3,T4,T2,T1]),	(6,[T3,T4,T2,T1]),	(6,[T3,T4,T2,T1]),
	(7,[T4,T2,T1]),	(7,[T4,T2,T1]),	(7,[T4,T2,T1]),	(7,[T4,T2,T1]),	(7,[T4,T2,T1]),
	(8,[T2,T1]),	(8,[T2,T1]),	(8,[T2,T1,T0]),	(8,[T2,T1,T0]),	(8,[T2,T1,T0]),
	(9,[T1,T2]),	(9,[T1,T2,T0]),	(9,[T1,T2,T0]),	(9,[T1,T2,T0]),	(9,[T1,T2,T0]),
	(10,[T2,T0]),	(10,[T2,T0]),	(10,[T2,T0]),	(10,[T2,T0]),	(10,[T2,T0,T1]),
	(11,[T2,T0]),	(11,[T2,T0]),	(11,[T2,T0]),	(11,[T2,T0,T1]),	(11,[T2,T0,T1]),
	(12,[T2]),	(12,[T2]),	(12,[T2]),	(12,[T2]),	(12,[T2]),
	(13,[])],	(13,[])],	(13,[])],	(13,[])],	(13,[])],
LiveOuts	[(0,[]),	[(0,[]),	[(0,[]),	[(0,[]),	[(0,[T0]),
	(1,[T1]),	(1,[T1]),	(1,[T1]),	(1,[T1,T0]),	(1,[T1,T0]),
	(2,[T1,T2]),	(2,[T1,T2]),	(2,[T1,T2,T0]),	(2,[T1,T2,T0]),	(2,[T1,T2,T0]),
	(3,[T1,T2]),	(3,[T1,T2,T0]),	(3,[T1,T2,T0]),	(3,[T1,T2,T0]),	(3,[T1,T2,T0]),
	(4,[T0,T3,T2,T1]),	(4,[T0,T3,T2,T1]),	(4,[T0,T3,T2,T1]),	(4,[T0,T3,T2,T1]),	(4,[T0,T3,T2,T1]),
	(5,[T3,T4,T2,T1]),	(5,[T3,T4,T2,T1]),	(5,[T3,T4,T2,T1]),	(5,[T3,T4,T2,T1]),	(5,[T3,T4,T2,T1]),
	(6,[T4,T2,T1]),	(6,[T4,T2,T1]),	(6,[T4,T2,T1]),	(6,[T4,T2,T1]),	(6,[T4,T2,T1]),
	(7,[T2,T1]),	(7,[T2,T1]),	(7,[T2,T1,T0]),	(7,[T2,T1,T0]),	(7,[T2,T1,T0]),
	(8,[T1,T2]),	(8,[T1,T2,T0]),	(8,[T1,T2,T0]),	(8,[T1,T2,T0]),	(8,[T1,T2,T0]),
	(9,[T2,T0]),	(9,[T2,T0]),	(9,[T2,T0]),	(9,[T2,T0]),	(9,[T2,T0,T1]),
	(10,[T2,T0]),	(10,[T2,T0]),	(10,[T2,T0]),	(10,[T2,T0,T1]),	(10,[T2,T0,T1]),
	(11,[T2,T0]),	(11,[T2,T0]),	(11,[T2,T0,T1]),	(11,[T2,T0,T1]),	(11,[T2,T0,T1]),
	(12,[]),	(12,[]),	(12,[]),	(12,[]),	(12,[]),
	(13,[])])	(13,[])])	(13,[])])	(13,[])])	(13,[])])

Derive interference graph from live ranges

Recall definition:

- "**x is live at p**": for some variable x and point p, the value of x could be used along some path starting at p.
- Eg: liveOut(7)= [T2,T1,T0]

  "The values of T2, T1 and T0 could be used along some path starting from 7"

- LiveOut:

  [(0,[T0]),
  (1,[T1,T0]),
  (2,[T1,T2,T0]),
  (3,[T1,T2,T0]),
  (4,[T0,T3,T2,T1]),
  (5,[T3,T4,T2,T1]),
  (6,[T4,T2,T1]),
  (7,[T2,T1,T0]),
  (8,[T1,T2,T0]),
  (9,[T2,T0,T1]),
  (10,[T2,T0,T1]),
  (11,[T2,T0,T1]),
  (12,[]),
  (13,[])])

Interference

Find overlapping live ranges

- For each temporary *t*
- For each node *id*
- If *t* is in liveOut(*id*)
- Then interferes(*t*) includes liveOut(*id*)

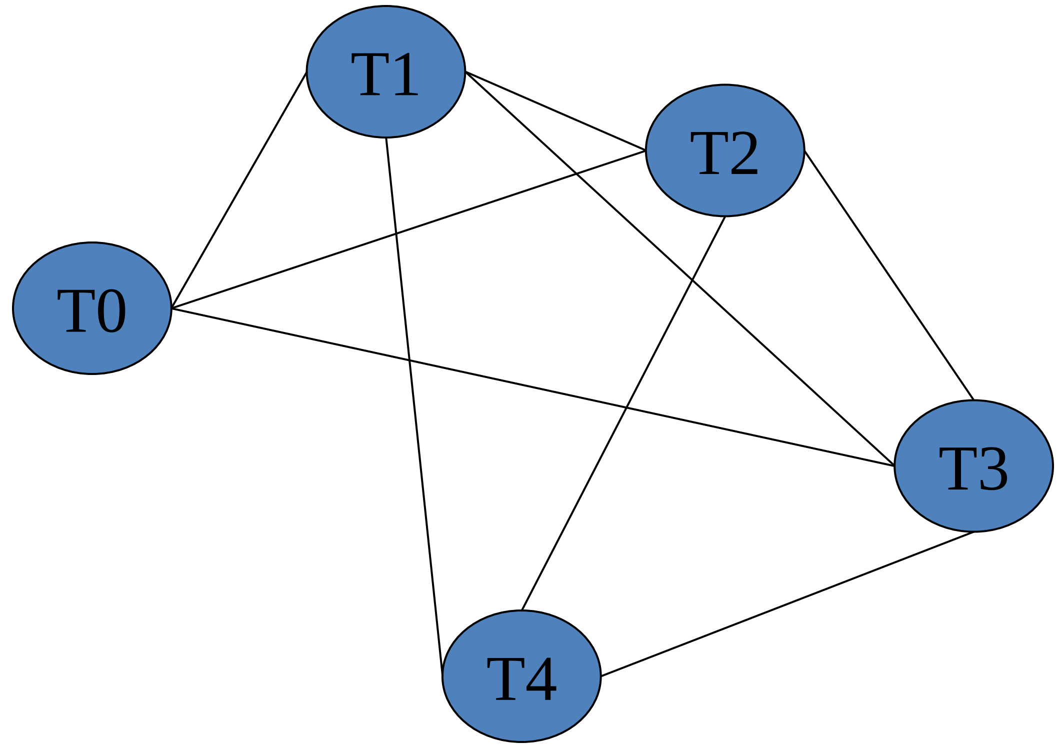
- Interference graph

interferes=

  [(T0,[T0,T1,T2,T3]),
  (T1,[T1,T0,T2,T3,T4]),
  (T2,[T1,T2,T0,T3,T4]),
  (T3,[T0,T3,T2,T1,T4]),
  (T4,[T3,T4,T2,T1])]

Derive interference graph from live ranges

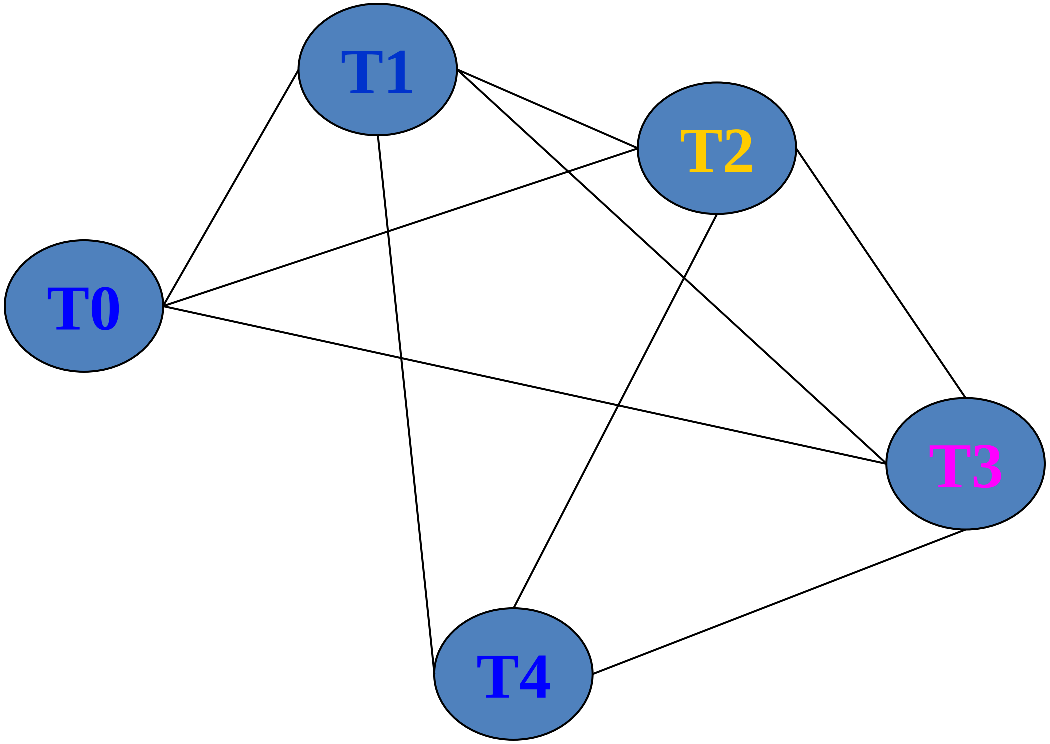
- Interference graph:
  [(T0,[T0,T1,T2,T3]),
  (T1,[T1,T0,T2,T3,T4]),
  (T2,[T1,T2,T0,T3,T4]),
  (T3,[T0,T3,T2,T1,T4]),
  (T4,[T3,T4,T2,T1])]



Use interference graph to assign temporaries

- Interference graph:

  [(T0,[T0,T1,T2,T3]),
  (T1,[T1,T0,T2,T3,T4]),
  (T2,[T1,T2,T0,T3,T4]),
  (T3,[T0,T3,T2,T1,T4]),
  (T4,[T3,T4,T2,T1])]



- Find colouring:

  [(T0,**D0**),(T1,**D1**),(T2,**D2**),(T3,**D3**),(T4,**D0**)]

Applying the colouring:

Before colouring

```
.data
; Integer variable a has been allocated to T0
.text
move.l #1, T0
move.l #10, T1
move.l #1, T2
bra    L2

L1:
move.l T2, T3
move.l T0, T4  (T0 & T4 assigned to D0)
mul.l  T3, T4
move.l T4, T0
add.l  #1, T2

L2:
cmp.l  T1, T2
bgt    L3
bra    L1

L3:
move.l T2, x
```

After colouring

```
.data
; Integer variable a has been allocated to D0
.text
move.l #1, D0
move.l #10, D1
move.l #1, D2
bra    L2

L1:
move.l D2, D3
mul.l  D3, D0
add.l  #1, D2      ((move.l D0 D0 deleted))

L2:
cmp.l  D1, D2
bgt    L3
bra    L1

L3:
move.l D2, x
```

Live variable analysis… summary

- We found we could find live ranges by constructing a *system of dataflow equations* and solving it by iteration
- The algorithm always *terminates*…
- The amount of work per iteration depends on program complexity - #instructions, #temporaries
- The number of iterations needed depends on the order in which the CFG is traversed…
  - See EaC pg445, Appel pg226, pg399
  - Live variable analysis is a ***backwards*** analysis – LiveIn(n) depends on its *successors*
  - Number of iterations depends on program's structural complexity – its "*loop interconnectiveness*"

APPENDIX: Liveness analysis, colouring in Haskell…

- Encode DFA equations:

```
newLiveIn liveIns liveOuts node
 = nodeUses node `union` ( (liveOutsOf node) \\ nodeDefs node )
    where
    liveOutsOf node = retrieve (nodeId node) liveOuts
newLiveOut liveIns liveOuts node
 = bigU [retrieve s liveIns | s <- nodeSuccs node]
   where bigU sets = nub (concat sets)
```

- Do one step: update LiveIn and LiveOut sets for each node:

```
updateLiveness [] (liveIns, liveOuts) = (liveIns, liveOuts)
updateLiveness (node:nodes) (liveIns, liveOuts)
 = updateLiveness nodes (newLiveIns, newLiveOuts)
   where
  newLiveIns  = subst (nodeId node) liveIns  (newLiveIn  liveIns liveOuts node)
  newLiveOuts = subst (nodeId node) liveOuts (newLiveOut newLiveIns liveOuts node)
```

Detailed code is shown in the hope that it will make the concepts clearer; please don't memorize it! Spend the time reading the textbook instead.

Solving DFAs in Haskell… (for completeness!)

- Iterate…

```
iterateUpdates nodes (liveIns, liveOuts)
 = let
    (newLiveIns, newLiveOuts) = updateLiveness nodes (liveIns, liveOuts)
   in
    if newLiveIns == liveIns && newLiveOuts == liveOuts
    then
      (newLiveIns, newLiveOuts)
    else
      iterateUpdates nodes (newLiveIns, newLiveOuts)
```

```
findLiveRanges :: CFG -> ([(Id,[Register])], [(Id,[Register])])

findLiveRanges (ControlFlowGraph cfgnodes)
  = iterateUpdates cfgnodes (initialLiveIns, initialLiveOuts)
    where
    initialLiveIns = initialLiveOuts
    initialLiveOuts = [(id,[]) | id <- map nodeId cfgnodes]
```

(live ranges liveIn & liveOut, each a mapping from node to list of temps)

(an empty list for each node)

- Now build the register interference graph (RIG):

```
buildInterferenceGraph cfg
 = [(t, nub (buildInterferenceList liveOuts t)) | t <- temporaries]        (nub eliminates duplicates)
   where
   (liveIns, liveOuts) = findLiveRanges cfg
   temporaries = findTemporaries cfg                  (findTemporaries lists temps used in code)

buildInterferenceList [] t = []
buildInterferenceList ( (id,livelist) : liveIns) t
  | t `elem` livelist        = livelist ++ buildInterferenceList liveIns t
  | otherwise                = buildInterferenceList liveIns t
```

- If we assign T*i* to D*j*, will we have a conflict?

```
doesntInterfere :: (Register,Register) -> InterferenceGraph -> Bool

doesntInterfere (t,r) ifg
 = actualinterferences == []
   where
   actualinterferences = [ ai | ai <- potentialinterferences, ai == r ]
   potentialinterferences = retrieve t ifg \\ [t]     (retrieve finds the list corresponding to t)
                                                      (remove t itself, which also appears in list)
```

Solving DFAs in Haskell… (for completeness!)

- Colour the graph – find a conflict-free assignment

```
type Colouring = [(Register, Register)]    (temporary, real register)

findColouring cfg ifg
  = let temporaries = findTemporaries cfg
     in findColouring' temporaries ifg


findColouring' :: [Register] -> InterferenceGraph -> Colouring
findColouring' [] ifg  = []
findColouring' (t:ts) ifg
  = let
                                                     (theRealRegisters is [D0,D1..D31])
      possibleMappings = [(t,r) | r <- theRealRegisters]
      validMappings = [(t,r) | (t,r) <- possibleMappings, doesntInterfere (t,r) ifg]
    in
                                                  (updateIFG replaces temps with regs)
      head [ (t,r) : (findColouring' ts (updateIFG ifg (t,r))) | (t,r) <- validMappings ]
```

- If no colouring can be found, this function fails (the list above is empty). If this happens, we will have to "spill" one of the variables to memory and try again.
- This is a quick and dirty but dumb inefficient algorithm; see Appel pg239

Solving DFAs in Haskell… (for completeness!)

- Put it all together…

```
applyColouring :: [Instruction] -> [Instruction]

applyColouring code
  = let
      cfg = buildCFG code
      colouring = findColouring cfg (buildInterferenceGraph cfg)
    in
      map (replaceTemporaries colouring) code
```

(where "replaceTemporaries colouring instruction" updates the instruction to use the specified real registers instead of temporaries)

Feeding curiosity…

- For general programs with unrestricted gotos, the control-flow graph can be any graph, and so can the interference graph. Hence for some fixed $\varepsilon > 0$, we cannot in polynomial time colour within a factor $O(n^{\varepsilon})$ from optimality unless NP=P. It can be approximated with a factor O(*n(log log n)*/$(log\, n)^3$) [Halldorsson 1993]
- But see "**All Structured Programs have Small Tree-Width and Good Register Allocation**", Mikkel Thorup, Journal of Information and Computation, 1998.
- Dataflow analysis can be understood as execution of the program in a special way – replacing the operations with abstract operations on a finite, approximate, abstract machine state. If we design the abstract state representation right, iteration and recursion always converge after a finite number of iterations. See "**Abstract interpretation: a unified lattice model for static analysis of programs by construction or approximation of fixpoints**", Patrick Cousot & Radhia Cousot, POPL 1977.