Compilers - Chapter 7:

Loop optimisations

Part 3: Loop-invariant code motion

- Lecturer:
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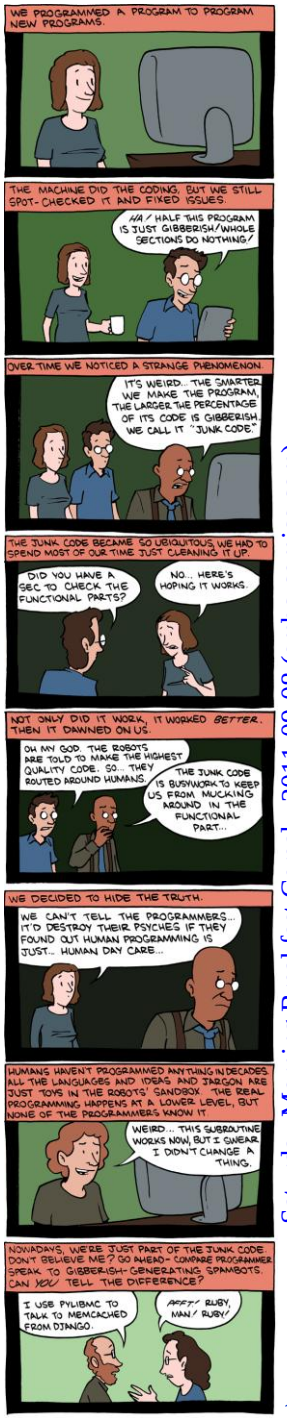

Saturday Morning Breakfast Cereal - 2011-09-08 (smbc-comics.com)

Which instructions can we move out of a loop?

- The next question is exactly which loop-invariant instructions we can move to the pre-header

```
L0:                    A
 t = 0


L1:
 i = i+1
 t = a ⊕ b
 M[i] = t
 if i<N goto L1
L2:
 x = t
```

Which instructions can we move out of a loop?

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 t = a ⊕ b
L_1:
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 ~~t = a ⊕ b~~
 M[i] = t
 if i<N goto L_1
L_2:
 x = t

Which instructions can we move out of a loop?

- It's easy to get it wrong….

```
L0:                    B
 t = 0


L1:
 if i<N goto L2
 i = i+1
 t = a ⊕ b
 M[i] = t
 goto L1
L2:
 x = t
```

Which instructions can we move out of a loop?

- It's easy to get it wrong….

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 t = 0
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 if i<N goto L_2
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 M[i] = t
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Which instructions can we move out of a loop?

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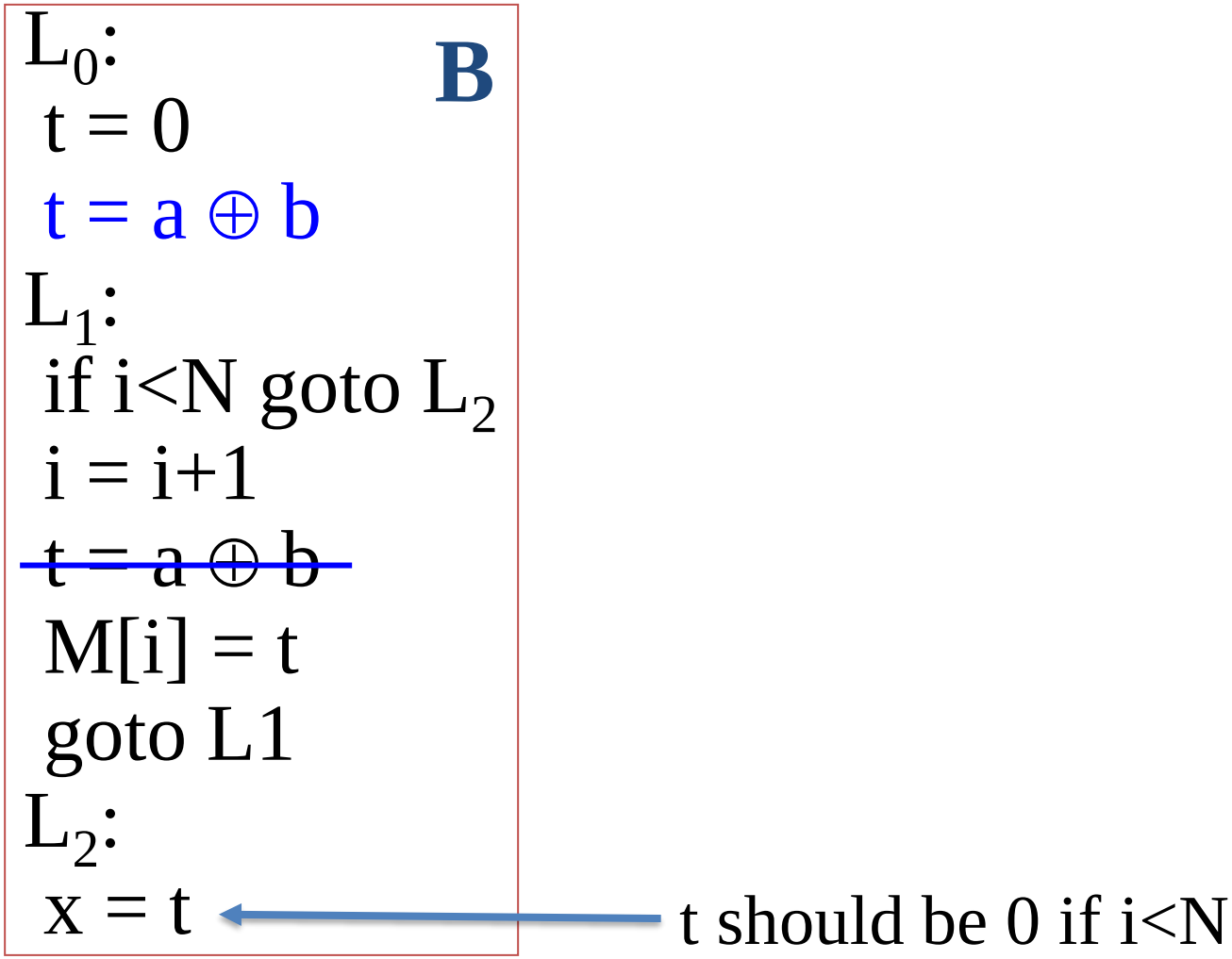

Which instructions can we move out of a loop?

- It's easy to get it wrong….

```
L0:                          C


L1:
 i = i+1
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 M[i] = t
 t = 0
 M[j] = t
 if i<N goto L1
L2:
```

Which instructions can we move out of a loop?

- It's easy to get it wrong….

```
L0:                          C

 t = a ⊕ b
L1:
 i = i+1
 t = a ⊕ b
 M[i] = t
 t = 0
 M[j] = t
 if i<N goto L1
L2:
```

Which instructions can we move out of a loop?

- It's easy to get it wrong….

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L0:                    C
 t = a ⊕ b
L1:
 i = i+1
 t = a ⊕ b
 M[i] = t
 t = 0
 M[j] = t
 if i<N goto L1
L2:
```

- (Just because an *expression* is loop invariant, doesn't mean we can always move the *instruction)*

What about the second iteration?

Which instructions can we move out of a loop?

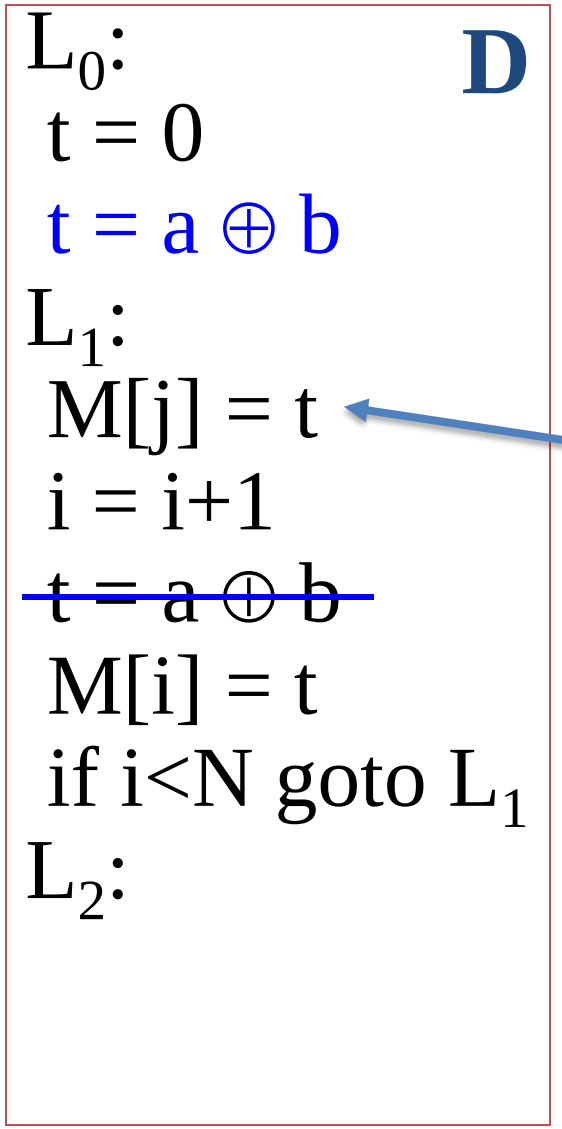
- It's easy to get it wrong….

```
L0:                    D
 t = 0

L1:
 M[j] = t
 i = i+1
 t = a ⊕ b
 M[i] = t
 if i<N goto L1
L2:
 x = t
```

Which instructions can we move out of a loop?

- It's easy to get it wrong….



Which instructions can we move out of a loop?

A

L_0:
t = 0
t = a ⊕ b
L_1:
i = i+1
~~t = a ⊕ b~~
M[i] = t
if i<N goto L_1
L_2:
x = t

Hoist

B

L_0:
t = 0
t = a ⊕ b
L_1:
if i<N goto L_2
i = i+1
~~t = a ⊕ b~~
M[i] = t
goto L1
L_2:
x = t

Don't hoist:
Loop invariant node does not dominate all loop exits

C

L_0:
t = 0
t = a ⊕ b
L_1:
i = i+1
~~t = a ⊕ b~~
M[i] = t
t = 0
M[j] = t
if i<N goto L_1
L_2:

Don't hoist:
More than one definition of t in the loop

D

L_0:
t = 0
t = a ⊕ b
L_1:
M[j] = t
i = i+1
~~t = a ⊕ b~~
M[i] = t
if i<N goto L_1
L_2:

Don't hoist:
t is liveOut from the loop's preheader

Which instructions can we move out of a loop?

- Conditions for hoisting a CFG node

  d: t = a ⊕ b

1	Loop invariant: all reaching defs used by d occur outside loop	Use Reaching Definitions data flow analysis
2	Loop invariant node must dominate all loop exits	Use Dominators analysis
3	There must be just one def of t in loop	Just count them!
4	t must not be liveOut from the loop's preheader	Use Live Variables data flow analysis

This is a bit messy…

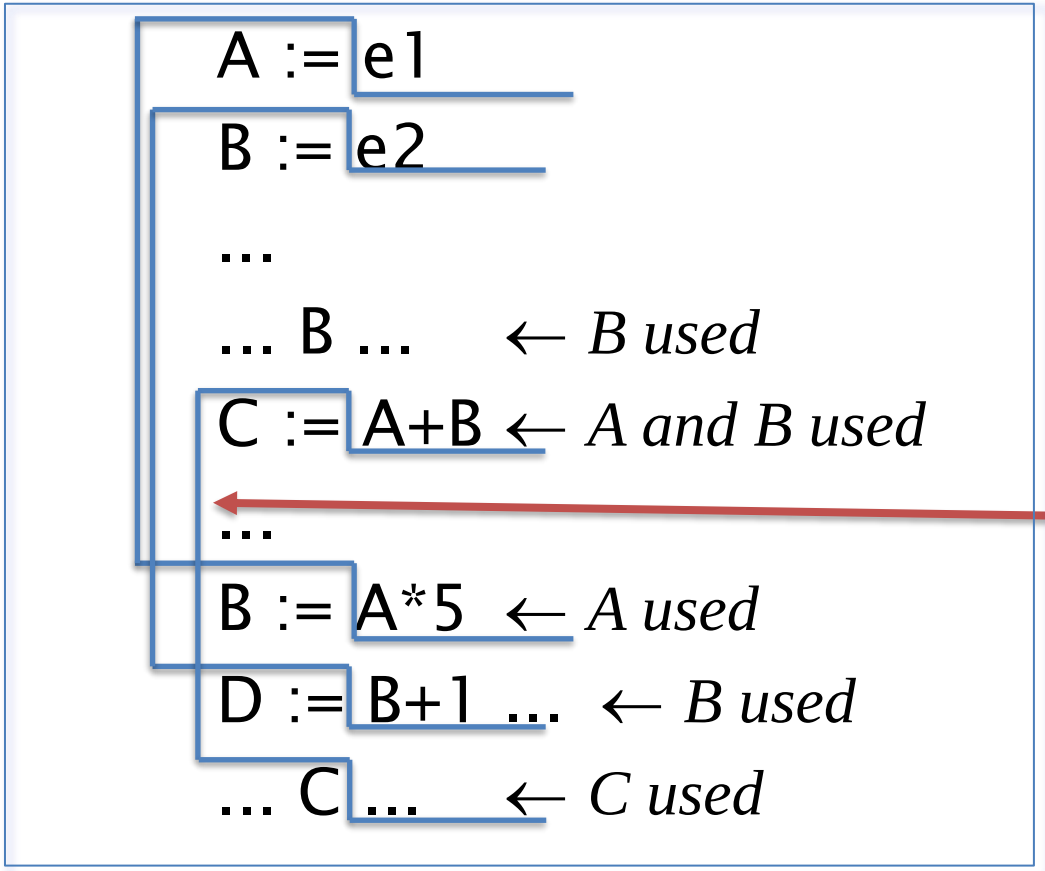
Static Single Assignment form (SSA)

- SSA is a powerful technique for simplifying many optimisation problems
- It is very widely used

- The next few slides illustrate how SSA avoids the side conditions on loop invariant code motion presented earlier
- Converting your code into SSA, and back to code again, takes a bit more work
  - You should know what SSA is, and how it helps – but converting to and from SSA form is beyond the scope of this course
  - but see the textbooks (EaC Section 9.3 pp454, Appel Ch19)

Introducing static single assignment…

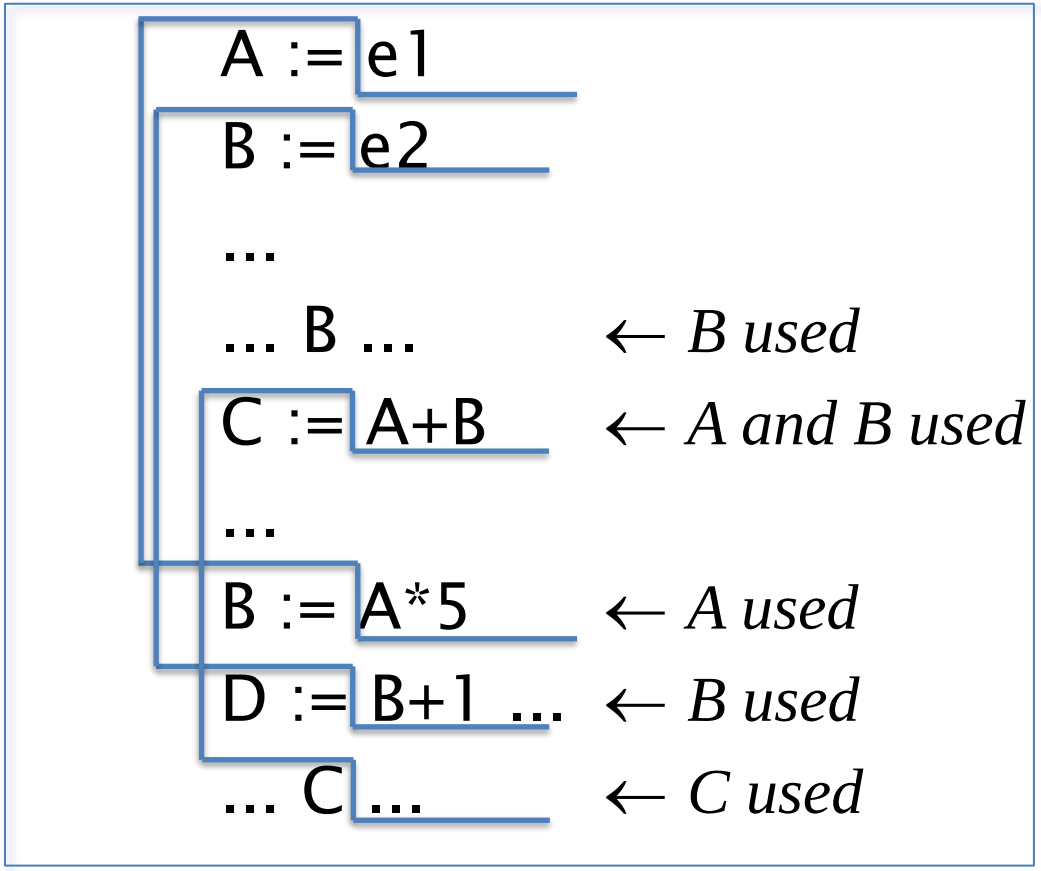
Recall graph colouring for register allocation

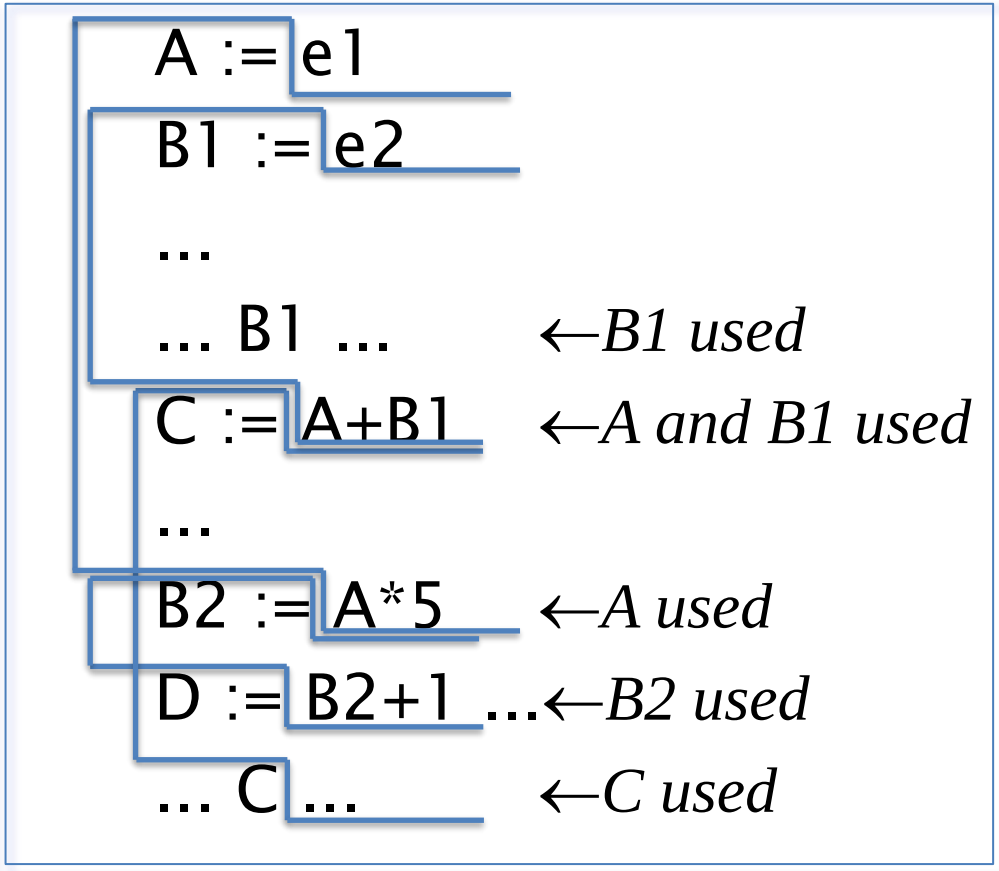


- Note that B is reassigned

Introducing static single assignment…

- Variable B is *reused* – it really has two separate live ranges:

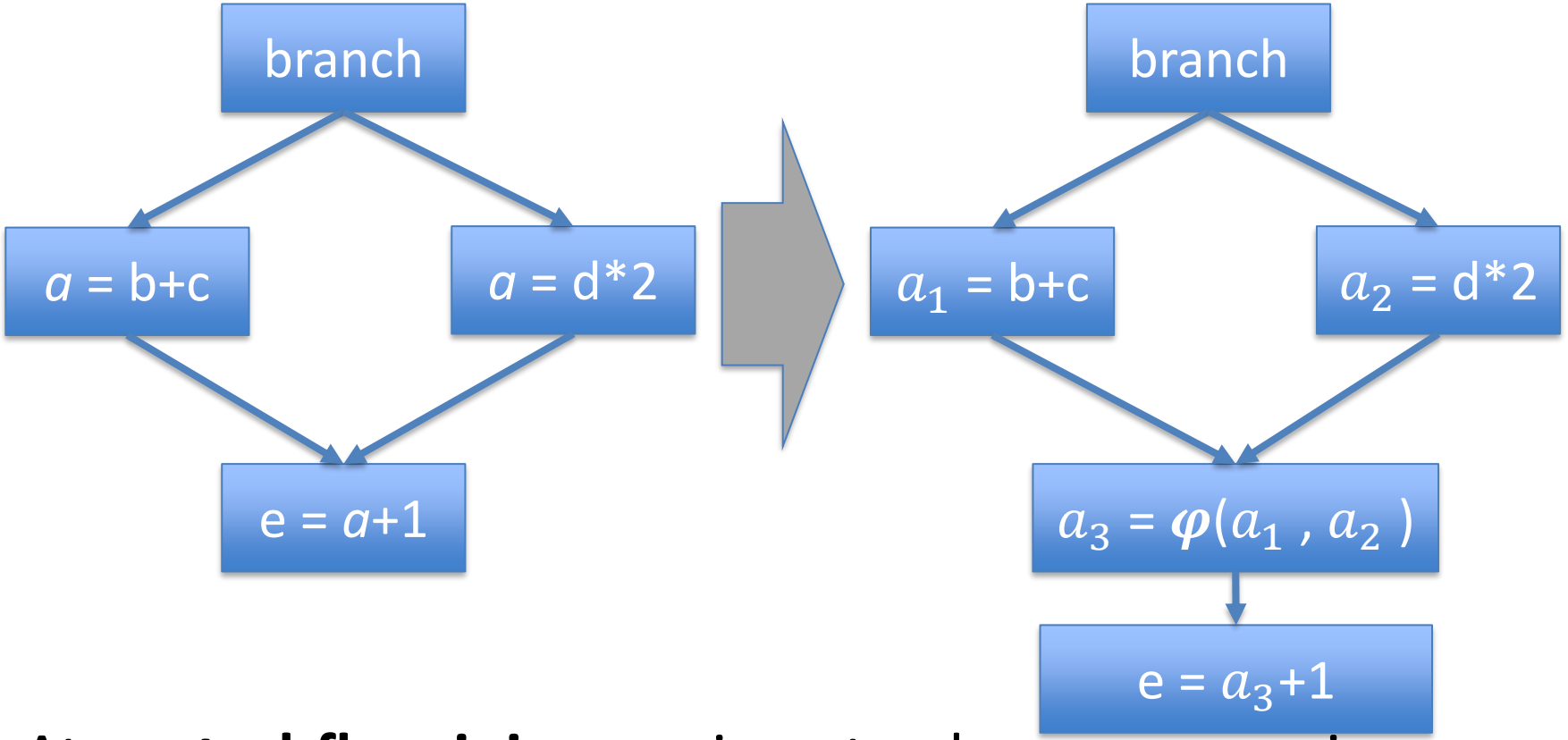




- SSA: introduce a new name each time a variable is assigned
- This helps register allocation by splitting live ranges

Static Single Assignment form (SSA)

- Things are a bit more complicated with branches:



- At **control-flow joins**, we insert a dummy renaming operator $\varphi(a_1, a_2)$ – which magically picks either a_1 or a_2 depending on which path is actually taken
- φ is not really executed – it is eliminated during code generation

Original code

A

L_0:
$t = 0$

L_1:
i = i+1
t = a ⊕ b
M[i] = t
if I<N goto L_1
L_2:
x = t

SSA code

L_0:
$t_1 = 0$

L_1:
i = i+1
t_2 = a ⊕ b
M[i] = t_2
if I<N goto L_1
L_2:
x = t_2

Example A (where hoisting was *valid*)

We should do SSA conversion for variable i as well – this has been omitted for clarity.

Original code

L_0: **C**

L_1:
i = i+1
t = a ⊕ b
M[i] = t
t = 0
M[j] = t
if i<N goto L_1

L_2:

SSA code

L_0: **C**

L_1:
i = i+1
t_1 = a ⊕ b
M[i] = t_1
t_2 = 0
M[j] = t_2
if i<N goto L_1

L_2:

SSA code after hoisting

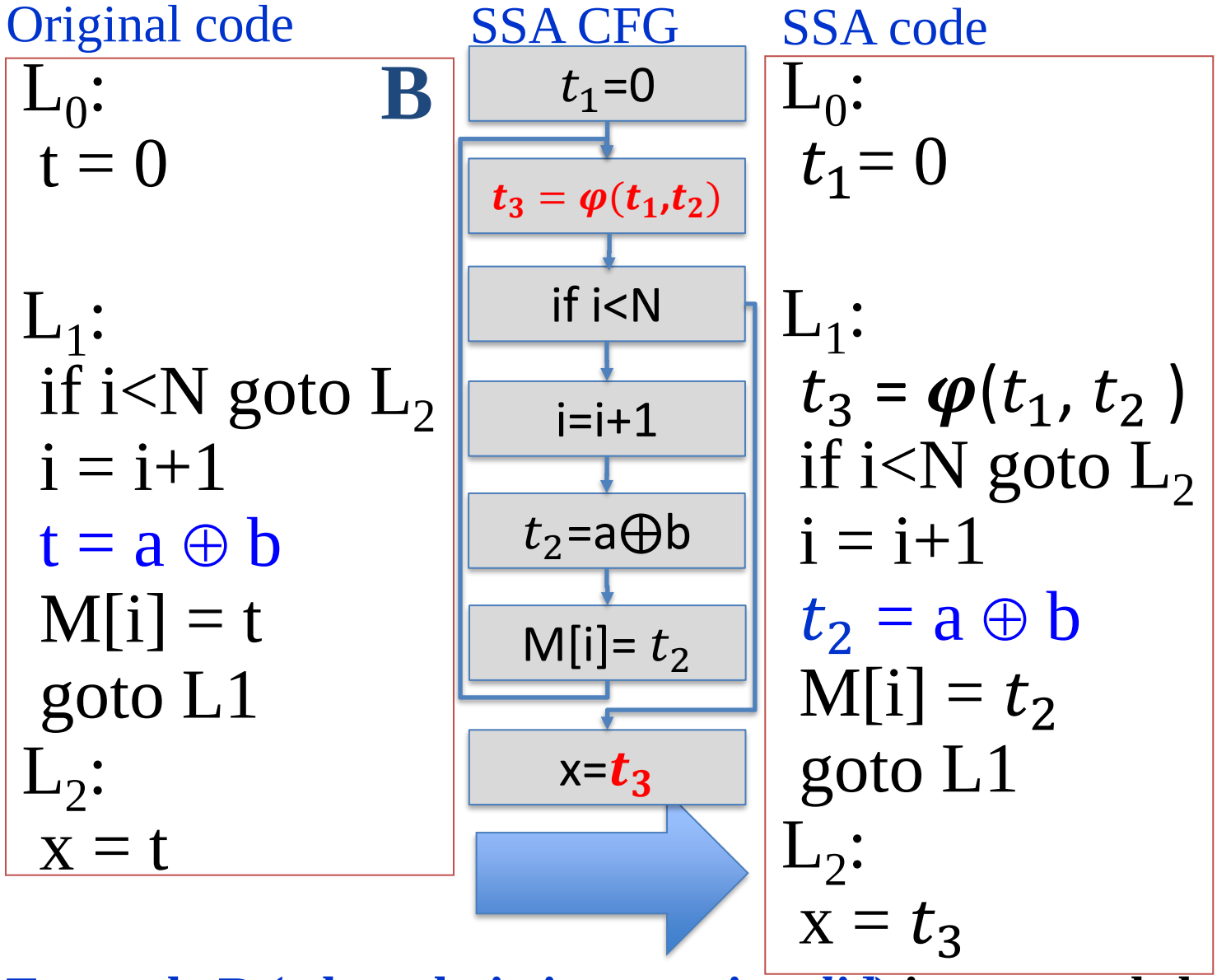
L_0: **C**
t_1 = a ⊕ b

L_1:
i = i+1
~~t_1 = a ⊕ b~~
M[i] = t_1
t_2 = 0
M[j] = t_2
if i<N goto L_1

L_2:

Example C (where hoisting was *invalid*)

Renaming t solves the problem all by itself!



Example B (where hoisting was *invalid*) is more subtle!

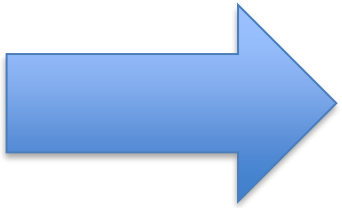
At the control flow join, we have two values for t
"x=t" is reached by two definitions. Adding the "$t_3 = \varphi(t_1 , t_2)$" fixes this.

Original code

L_0:
 t = 0

L_1:
 if i<N goto L_2
 i = i+1
 t = a ⊕ b
 M[i] = t
 goto L1
L_2:
 x = t

B



SSA code

L_0:
 t_1 = 0
 t_2 = a ⊕ b
L_1:
 t_3 = φ(t_1, t_2)
 if i<N goto L_2
 i = i+1
 ~~t_2 = a ⊕ b~~
 M[i] = t_2
 goto L_1
L_2:
 x = t_3

B

Hoist

SSA…

The SSA transformation resolves the problem by introducing separate names

The tricky bit is when we transform the code back out of SSA form again

We really do need more than one variable for t

We need to assign t_3 to the right value on the two different control-flow paths that meet at the phi

We push the assignments to t_3 backwards into the two predecessor paths of the φ

Doing this actually makes the loop-invariant *hoistable* for this example!

SSA code

L_0: **B**
$t_1 = 0$
$t_2 = a \oplus b$
L_1:
$t_3 = \varphi(t_1, t_2)$
if i<N goto L_2
i = i+1
~~$t_2 = a \oplus b$~~
$M[i] = t_2$
goto L_1
L_2:
$x = t_3$

Generated code

L_0: **B**
$t_1 = 0$
$t_2 = a \oplus b$
$t_3 = t_1$
L_1:
if i<N goto L_2
i = i+1
$M[i] = t_2$
$t_3 = t_2$
goto L_1
L_2:
$x = t_3$

Copies introduced when generating code *from* SSA

Original code

D

L_0:
t = 0

L_1:
M[j] = t
i = i+1
t = a ⊕ b
M[i] = t
if i<N goto L_1

L_2:
x = t

SSA code

L_0:
t_1 = 0

L_1:
$t_3 = \varphi(t_1, t_2)$
M[j] = t_3
i = i+1
t_2 = a ⊕ b
M[i] = t_2
if i<N goto L_1

L_2:
x = t_2

SSA code after hoisting

L_0:
t_1 = 0
t_2 = a ⊕ b

L_1:
$t_3 = \varphi(t_1, t_2)$
M[j] = t_3
i = i+1
~~t_2 = a ⊕ b~~
M[i] = t_2
if i<N goto L_1

L_2:
x = t_2

Example D (where hoisting was *invalid*)

Original code | SSA code | SSA code after hoisting | After SSA

D

```
L0:
 t = 0

L1:
 M[j] = t
 i = i+1
 t = a ⊕ b
 M[i] = t
 if i<N goto L1
L2:
 x = t
```

```
L0:
 t1 = 0

L1:
 t3 = φ(t1, t2)
 M[j] = t3
 i = i+1
 t2 = a ⊕ b
 M[i] = t2
 if i<N goto L1
L2:
 x = t2
```

```
L0:
 t1 = 0
 t2 = a ⊕ b
L1:
 t3 = φ(t1, t2)
 M[j] = t3
 i = i+1
 t2 = a ⊕ b
 M[i] = t2
 if i<N goto L1
L2:
 x = t2
```

```
L0:
 t1 = 0
 t2 = a ⊕ b
 t3 = t1
L1:
 t3 = φ(t1, t2)
 M[j] = t3
 i = i+1
 t2 = a ⊕ b
 M[i] = t2
 t3 = t2
 if i<N goto L1
L2:
 x = t2
```

Example D (where hoisting was *invalid*)
SSA makes it valid here too

Which instructions can we move out of a loop?

- Conditions for hoisting a CFG node **after SSA conversion**

d: t = a ⊕ b

1	Loop invariant: all reaching defs used by d occur outside loop	Use Reaching Definitions data flow analysis
2	Loop invariant node must dominate all loop exits	Not a problem any more!
3	There must be just one def of t in loop	Guaranteed by SSA
4	t must not be liveOut from the loop's preheader	Can't happen with SSA (because d must be the only definition of t)

What next…

- Hoisting loop invariants really helps
- But good compilers do lots more…
  - Induction variables:
    - A variable which is incremented by a loop-invariant amount
    - A variable which is a multiple of an induction variable
  - Strength reduction
    - Compute all induction variables by incrementing instead of multiplying
  - Induction variable elimination, rewriting comparisons
  - Array bounds check elimination
    - Range of all induction variables is known on entry to a for loop
  - Common sub-expressions
  - More sophisticated methods – eg partial redundancy elimination
- Now you have seen how to hoist loop-invariants, you can figure the rest out yourself!

Optimisations for high-performance computing

- "Conventional" optimisations *reduce* work done at run-time
- "restructuring" compilers improve performance by finding the *right order* in which to do the computation
- Example: Parallelisation:

Original code:

```
For (i=0;i<N;i++)
  For (j=0;j<M;j++)
   A[i,j] = (A[i,j] + A[i-1,j] + A[i+1,j])* (1/3)
```

Parallel implementation:

```
For (i=0;i<N;i++)
  ParFor (j=0;j<M;j++)
   A[i,j] = (A[i,j] + A[i-1,j] + A[i+1,j])* (1/3)
```

Better parallel implementation?

```
ParFor (j=0;j<M;j++)
  For (i=0;i<N;i++)
   A[i,j] = (A[i,j] + A[i-1,j] + A[i+1,j])* (1/3)
```

Optimisations for high-performance computing

- Another restructuring example:

  Example: matrix transpose:

```
for (i=0;i<N;i++)
  for (j=0;j<M;j++)
    B[i][j] = A[j][i];
```

  Cache-efficient implementation:

```
for (ii=0;ii<N;ii+=IB)
  for (jj=0;jj<M;jj+=JB)
    for (i=ii;i<ii+IB;i++)
      for (j=jj;j<jj+JB;j++)
        B[i][j] = A[j][i];
```

Using Intel i7-7567U, gcc 9.30
gcc -Ofast
N=M=10240
IB=JB=16
Original execution time: 7.2s
Improved execution time: 0.46s

Optimisations for high-level programming languages

- Subtype polymorphism
  - Static resolution of the type of x in x.f( ) enables inlining of method f
- Generics (aka parametric polymorphism)
  - A generic class is parameterised by a type (eg a container by its element type). When is it a good idea to generate specialised code?
- Pattern matching
  - In a language like Haskell, Prolog, Erlang, Elixir, pattern matching on nested data structures is very powerful. Find optimum sequence of tests.
- Dynamic object creation
  - If we allow space to be allocated, but automatically freed, can we sometimes add code to do it instead of relying on garbage collection?
- Lazy evaluation
  - Can an expression be evaluated where it is first referred to, or do we have to build a "closure" representing it?
- Arrays – overloaded arithmetic
  - If we overload arithmetic operators to work on arrays, how to avoid lots of little loops?
- Arrays – slices
  - If we allow a multidimensional array to be sliced, eg A[2:99,2:99], how do we avoid having to manipulate an array descriptor?

Research

- Several Imperial research groups are working on optimising compiler technology, including:
  - Wayne Luk's Custom Computing/Silicon Compilation group
  - Alastair Donaldson's Multicore Programming group
  - Paul Kelly's Software Performance Optimisation group
  - Compiler-related research: Cristian Cadar, Holger Pirk, Nick Wu, Jamie Willis, Hongxiang Fan, Sergio Maffeis, Peter Pietzuch, Herbert Wicklicky, etc
    - And more in EEE including John Wickerson, George Constantinides, Aaron Zhao
    - And more in Maths eg David Ham, Kevin Buzzard
    - And more in Earth Science Engineering eg Gerard Gorman
  - Programming languages: Sophia Drossopoulou, Azalea Raad, Philippa Gardner, and others
- Opportunities: UROP summer placements, individual projects, and PhDs
- Sample projects:
  - Work with computational scientists to make their simulation of tidal turbines/Formula 1/blood flow/glacier flow/weather/medical imaging run fast on 10,000-100,000 cores
  - Design a domain-specific language and compiler to generate high-performance code for 3D robot vision and scene understanding
  - Design a compiler to generate code for computer vision on a camera sensor device with a processor at every pixel
  - Build an automatic program differentiator, that works for parallel programs
  - Check that a program that operates on non-volatile memory always leaves its data in a consistent state, even if it fails at any time

Chapter 7: summary

- Reaching definitions identify instructions that are candidates for loop-invariant code motion
- We can find the headers of the program's natural loops, and insert pre-headers, so that there is a unique place to move loop-invariant instructions to
- It may still not be safe to actual move them! We have to check some subtle side-conditions
- Transforming the program to Static Single Assignment (SSA) form ensures that every use is reached by exactly one definition (which might be a phi). Translating into SSA may require new variables
- SSA makes loop-invariant code motion safe without side-conditions
- Transforming out of SSA may introduce some additional copy instructions
  - SSA also helps with other optimisations, such as register allocation
- Loop-invariant code motion is just one of many optimisations – but it has introduced many of the key ideas and issues
- Nested loops operating on arrays can benefit from loop scheduling optimisations and parallelisation, and there is a rich theory
- Many languages can only be optimised effectively with pointer analysis
- Many high-level language features raise the need for additional optimisations

Textbooks

EaC

- Data flow analysis is covered in Chapter 9
  - Reaching definitions are covered in Section 9.2.4
  - Dominators are covered in Section 9.3.2
- EaC handles loop-invariant code motion somewhat differently from these slides, which are based on Appel's presentation
  - See "Lazy Code Motion" (LCM), page 506
  - LCM resolves the hoisting conditions in a more systematic way than presented here, by combining four different data-flow analyses

Textbooks

- Appel also covers optimisation in depth
  - Chapter 10 introduces DFA through live variable analysis
  - Chapter 17 shows how DFA can be used for many other useful analyses
  - Chapter 18 deals with finding loops, finding induction variables, and implementing loop optimisations (which rely on DFAs)
  - Chapter 19 presents Static Single Assignment, a program representation which provides easy (and space-efficient) access to dependence information such as reaching definitions. This simplifies many loop optimisations
  - Chapter 20 covers instruction scheduling – finding an instruction ordering which makes optimal use of modern CPU architectures
  - Chapter 21 concerns improving cache performance – by prefetching, and by executing loops blockwise
- Another really good source if you're building an optimising compiler is "High-performance compilers for parallel computing", Michael Wolfe (Addison Wesley 1996)

- Fine print:
  - CFG would consist of basic blocks instead of individual instructions
  - For loop optimisations, we would do the DFA on the IR before instruction selection; it's simpler and it avoids complications such a having only two-address instructions
  - See Appel pg388
- Credits: in addition to Appel's book, I found it very useful to study the course notes of Liz White (George Mason University), Laurie Hendren (McGill University) and Chau-Wen Tseng (University of Maryland)

Appendix A: Implementing loop optimisations in Haskell

- The next few slides give a Haskell implementation for some of the ideas presented in this chapter
- This material is provided to provide a concrete illustration of the concepts
- It is the concepts which are important, not the code
- **Do *not* memorise the code** – spend the time reading the textbook instead
- Some of the algorithms used here are rather inefficient – in many cases we just transcribe the mathematical definitions. Efficient algorithms exist – but are considerably more complicated.

Reaching definitions – gen and kill

- Preliminaries: the Gen and Kill sets:

```
nodeGen node | nodeDefs node == []  = []
             | otherwise       = [nodeId node]

nodeKill cfg node = nodeDefSet cfg node \\ [nodeId node]
```

- Suppose t is defined in node. nodeDefSet is set of all the nodeids where t is defined:

```
nodeDefSet (ControlFlowGraph cfg) node
 = case nodeDefs node of
   [t] -> [id | Node id i ds us scs prds <- cfg,
              t `elem` ds]
   []  -> []
   otherwise -> error "nodeDefSet: multiple defs“
```

- Auxiliary functions used in solver overleaf:

```
untilConverges (a:b:rest) | a == b = a
untilConverges (a:b:rest)          = untilConverges (b:rest)

zip2 (rdsin,rdsout) = zip rdsin rdsout
bigU sets = nub (concat sets)
```

Reaching definitions - solver

- Solve the dataflow equations:

```
reachingDefinitionsOf :: CFG -> ( [ (Id,[Id]) ], [ (Id,[Id]) ] )

reachingDefinitionsOf cfg
 = untilConverges (iterate updateRDs initialRDs)
   where
   initialRDs :: ( [ (Id,[Id]) ], [ (Id,[Id]) ] )
   initialRDs = ( [(n,[]) | n<-nodesOf cfg], [(n,[]) | n<-nodesOf cfg] )

   updateRDs :: ( [(Id,[Id])], [(Id,[Id])] ) -> ( [(Id,[Id])], [(Id,[Id])] )
   updateRDs rds = unzip (map (updateRD rds) (zip2 rds))
   updateRD (rdins_sofar,rdouts_sofar) ((id,rdins), (sameid,rdouts))
    = ((id,rdins'), (id,rdouts'))
      where
      rdins' = bigU [retrieve s rdouts_sofar | s <- nodePreds node]
      rdouts' = nodeGen node `union` ((rdInsOf node) \\ nodeKill cfg node )
                where
                rdInsOf node = retrieve (nodeId node) rdins_sofar
      node = idToNode cfg id
```

- We solve the system of simultaneous set equations iteratively
- Initially each node's ReachIn (rdins), and ReachOut (rdouts) set is empty
- The updates successively increase the ReachIn and ReachOut sets until convergence

Use reaching definitions to find loop invariant instructions

- Find the definitions which reach this node which are relevant – that is, which generate the values this node uses:

```
relevantReachingDefinitionsOf :: CFG -> [ (Id,[Id]) ]

relevantReachingDefinitionsOf cfg
 = [(nodeId node, relevantDefs node) | node <- cfgToNodes cfg]
   where
   relevantDefs node
      = [rd | rd <- retrieve (nodeId node) rds_in,
                  nodeDefs (idToNode cfg rd) `intersect` nodeUses node /= []]
   (rds_in, rds_out) = reachingDefinitionsOf cfg
```

Use reaching definitions to find loop invariant instructions

- An instruction is loop invariant if the definitions of all the values it uses are outside the loop:

```
> externallyDependentInstructionsOf cfg loop
>  = [node | node <- [idToNode cfg id | id <- loop],
>          nodeDefs node /= [],
>          relevantDefs node `intersect` loop == [],
>          hoistable node]
>    where
>    relevantDefs node = retrieve (nodeId node)
          (relevantReachingDefinitionsOf cfg)
```

- An instruction is hoistable only if it produces a value (ie not a compare, branch, etc):

```
hoistable (Node id i [] uses succs preds) = False
hoistable (Node id i defs uses succs preds) = True
```

Use reaching definitions to find loop invariant instructions

- Now iteratively add instructions which are l-i because they depend only on l-i instructions. We reverse the result so that when we add them to the pre-header, they are added in dependence-order.

```
> loopInvariantInstructionsOf cfg loop
>  = reverse (untilConverges (iterate updateLIs initialLIs))
>    where
>    initialLIs = externallyDependentInstructionsOf cfg loop
>    updateLIs :: [CFGNode] -> [CFGNode]
>    updateLIs invariantsSoFar
>     = invariantsSoFar `union`
>       [n | n <- map (idToNode cfg) loop,
>            hoistable n,
>            and [hasSingleInvariantDefinition n u | u<-nodeUses n]]
>       where
>       hasSingleInvariantDefinition n u
>        = length defs == 1 && head defs `elem` map nodeId invariantsSoFar
>          where
>          defs = [d | d<-relevantDefs n, u `elem` nodeDefs (idToNode cfg d)]
```

Finding dominators… implementation

```
dominatorsOf :: CFG -> [(Id,[Id])]
dominatorsOf cfg
 = untilConverges (iterate updateDs initialDs)
   where
   initialDs :: [(Id,[Id])]
   initialDs = [ (n, nodesOf cfg) | n <- (nodesOf cfg)]

   updateDs :: [(Id,[Id])] -> [(Id,[Id])]
   updateD ds_sofar (id,d)
    = (id,
        [id] `union` (bigCap [retrieve p ds_sofar | p <- nodePredsOf id])
       )
   updateDs ds = map (updateD ds) ds

   nodePredsOf id = nodePreds (idToNode cfg id)

bigCap [] = []
bigCap sets = foldr1 intersect sets

untilConverges (a:b:rest) | a == b = a
untilConverges (a:b:rest)          = untilConverges (b:rest)
```

- We solve the system of simultaneous set equations iteratively
- Initially each node's Doms set is the set of all the nodes of the CFG
- The updates successively reduce the Doms until convergence

Finding back edges

- A flow graph edge from a node n to a node h that dominates n is called a back edge:

```
backEdges :: CFG -> [(Id,Id)]

backEdges cfg
 = [ (n,h) | n <- nodesOf cfg, h <- nodesOf cfg, n /= h,
             flowedge n h,
             h `dominates` n]
   where
   dominators = dominatorsOf cfg
   a `dominates` b = a `elem` (retrieve b dominators)
   flowedge a b = a `elem` nodePreds (idToNode cfg b)
```

Finding natural loops

- The ***natural loop*** of a backedge (n,h), where h dominates n, is the set of nodes x such that h dominates x and there is a path from x to n not containing h.

```
naturalLoop :: CFG -> (Id,Id) -> (Id, [Id])
--                              backedge   header, nodes

naturalLoop cfg (n,header)
 = (header, real_xs)
   where
   poss_xs = [x | x <- nodesOf cfg, header `dominates` x]
   real_xs = [x | x <- poss_xs, pathExists x n]
   pathExists x n
    = [] /= [path | path <- allpaths, not (header `elem` path)]
      where
      allpaths = findControlFlowPaths cfg x n
   dominators = dominatorsOf cfg
   a `dominates` b = a `elem` (retrieve b dominators)
```

(omit paths via header, and therefore paths via enclosing loops)

(findControlFlowPaths defined next slide)

Finding paths

- I have used a general-purpose path enumeration to find all the paths from one node to another. This is rather wasteful... Some care is needed to avoid following cycles; "mypath" below records the nodes visited so far.

```
findControlFlowPaths :: CFG -> Id -> Id -> [[Id]]

findControlFlowPaths cfg start end = findControlFlowPaths' [] start
  where
  findControlFlowPaths' mypath x
   | x == end              = [[x]]
   | x `elem` mypath       = [[]]
   | otherwise             = map (x:) restOfPath
         where
          extendedpath = x:mypath
          succs = nodeSuccs (idToNode cfg x)
          nonCycleSuccs = succs
         restOfPath = concat (map (findControlFlowPaths' extendedpath) nonCycleSuccs)
```

Building the loop nest tree (a.k.a. the control tree)

- The loop nest tree consists, at each level, of a loop (with its header), and the list of all its subloop trees:

```
data LoopTree = LTree (Id,[Id]) [LoopTree] deriving (Show, Eq)

loopTree :: CFG -> LoopTree

loopTree cfg
 = LTree (0, nodesOf cfg) (makeTrees theloops)
   where
   backedges = backEdges cfg
   theloops = map (naturalLoop cfg) backedges
   makeTrees loops = map makeTree (siblingloops loops)
   makeTree loop
    = LTree loop (makeTrees subloops)
      where
      subloops = [(h,nub l) | (h,l) <- theloops, containedIn (h,l) loop]
```

Building the loop nest tree…

- The children of a given loop are the immediate subloops. A subloop is an immediate subloop if it is not contained in any other loop in the list:

```
siblingloops loops
 = [l1 | l1 <- loops,
       not (any (containedIn l1) [l2 | l2<-loops, l1 /= l2]) ]
```

- To work out whether one loop l1 is strictly contained within another l2, we ask simply whether l1's header is in l2's body:

```
containedIn :: (Id,[Id]) -> (Id,[Id]) -> Bool
containedIn (h1,l1) (h2,l2) = h1 `elem` l2
```

Manipulating the control flow graph…

- To implement hoisting of loop invariants we need a few other functions:
  - Insert a pre-header before each loop header:
    > addPreHeaders :: CFG -> LoopTree -> ([(Id,Id)], CFG)
    > addPreHeaders cfg looptree =
  - Remove a specified list of nodes from a cfg
    > removeNodes :: [CFGNode] -> CFG -> CFG
    > removeNode node cfg = …
  - Insert a specified node n into a cfg after a specified node "target". This only works if the target has only one successor, as is the case with a pre-header.
    > [CFGNode] -> CFG -> Int -> CFG
    > insertNodesAfter nodes cfg target = …
  - Traverse the modified CFG and generate instructions:
    > generateInstructions :: CFG -> [Instruction]
    > generateInstructions cfg = …

Hoisting the loop-invariants…

- Finally, we bring it all together

```
> hoistLoopInvariants cfg looptree
>  = newcfg
>    where
>    newcfg = foldl hoistALoop cfgWithPreheaders loops
>    loops = [(h,l) | (h,l) <- loopsOf looptree, h /= 0]
>    (preheaders, cfgWithPreheaders) = addPreHeaders cfg looptree

>    hoistALoop cfg (header,body)
>     = insertNodesAfter invariants (removeNodes invariants cfg) preheader
>       where
>       invariants = loopInvariantInstructionsOf cfg (header:body)
>       preheader = retrieve header preheaders

> loopsOf (LTree (h,body) subloops)
>  = (h,body) : concat (map loopsOf subloops)
```

(This sketch implementation doesn't check all the hoisting conditions…)

AST

```
(Program
[(Decl "w" Integer),
 (Decl "x" Integer),
 (Decl "y" Integer),
 (Decl "z" Integer)]
[Assign (Var "x") (Const 1),
Assign (Var "w") (Const 100),
Assign (Var "z") (Const 200),
LabelStat "Here",
Assign (Var "x") (Binop Plus
        (Ref (Var "x")) (Const 1)),
Assign (Var "y") (Binop Plus
        (Ref (Var "w")) (Ref (Var "z"))),
IfThenElse (Compare CLT
        (Ref (Var "x")) (Const 10))
  [Goto "Here"] []
] )
```

Example

Original control flow graph:

```
Node 0 (Mov (ImmNum 1) (Reg T1)) [T1] [] [1] []
Node 1 (Mov (ImmNum 100) (Reg T0)) [T0] [] [2] [0]
Node 2 (Mov (ImmNum 200) (Reg T3)) [T3] [] [3] [1]
Node 3 (Mov (Reg T1) (Reg T4)) [T4] [T1] [4] [2,15]
Node 4 (Add (ImmNum 1) (Reg T4)) [T4] [T4] [5] [3]
Node 5 (Mov (Reg T4) (Reg T1)) [T1] [T4] [6] [4]
Node 6 (Mov (Reg T3) (Reg T5)) [T5] [T3] [7] [5]
Node 7 (Mov (Reg T0) (Reg T6)) [T6] [T0] [8] [6]
Node 8 (Add (Reg T5) (Reg T6)) [T6] [T5,T6] [9] [7]
Node 9 (Mov (Reg T6) (Reg T2)) [T2] [T6] [10] [8]
Node 10 (Mov (Reg T1) (Reg T7)) [T7] [T1] [11] [9]
Node 11 (Mov (ImmNum 10) (Reg T8)) [T8] [] [12] [10]
Node 12 (Cmp (Reg T7) (Reg T8)) [] [T7,T8] [13] [11]
Node 13 (Blt "L1") [] [] [14,15] [12]
Node 14 (Bra "L2") [] [] [17] [13]
Node 15 (Bra "LHere") [] [] [3] [13]
Node 16 (Bra "L3") [] [] [17] []
Node 17 Halt [] [] [] [14,16]
```

- Relevant reaching definitions:

relevantReachingDefinitionsOf cfg =
[(0,[]),
(1,[]),
(2,[]),
(3,[0,5]),
(4,[3]),
(5,[4]),
(6,[2]),
(7,[1]),
(8,[7,6]),
(9,[8]),
(10,[5]),
(11,[]),
(12,[11,10]),
(13,[]),
(14,[]),
(15,[]),
(16,[]), (17,[])]

- Loop Tree:

loopTree cfg =
LTree (0,[0,1,2,3,4,5,6,7,8,9,10,
11,12,13,14,15,16,17])
[LTree (3,[4,5,6,7,8,9,10,
11,12,13,15])
[]]

- Loop invariants:

externallyDependentInstructionsOf cfg
[3,4,5,6,7,8,9,10,11,12,13,15] =
[Node 6 (Mov (Reg T3) (Reg T5)) [T5] [T3] [7] [5],
Node 7 (Mov (Reg T0) (Reg T6)) [T6] [T0] [8] [6],
Node 11 (Mov (ImmNum 10) (Reg T8)) [T8] [] [12] [10]]

loopInvariantInstructionsOf (cfgex 15)
[3,4,5,6,7,8,9,10,11,12,13,15] =
[Node 7 (Mov (Reg T0) (Reg T6)) [T6] [T0] [8] [6],
Node 6 (Mov (Reg T3) (Reg T5)) [T5] [T3] [7] [5],
Node 11 (Mov (ImmNum 10) (Reg T8)) [T8] [] [12] [10],
Node 9 (Mov (Reg T6) (Reg T2)) [T2] [T6] [10] [8],
Node 8 (Add (Reg T5) (Reg T6)) [T6] [T5,T6] [9] [7]]

- Code after loop-invariant hoisting:

```
move.l #1, T1
move.l #100, T0
move.l #200, T3
#Preheader for loop with header 3
move.l T0, T6
move.l T3, T5
move.l #10, T8
move.l T6, T2
add.l  T5, T6
```

(continued in next column…)

```
M3:
move.l T1, T4
add.l  #1, T4
move.l T4, T1
#Mov (Reg T3) (Reg T5) moved
#Mov (Reg T0) (Reg T6) moved
#Add (Reg T5) (Reg T6) moved
#Mov (Reg T6) (Reg T2) moved
move.l T1, T7
#Mov (ImmNum 10) (Reg T8) moved
cmp.l  T7, T8
blt    M15
bra    M14

M15:
bra    M3

M14:
bra    M17

M17:
halt
bra    M3
```

Feeding curiosity…

- We have been focused on optimising programs for efficiency. How about optimising (floating-point) programs for *accuracy*? See for example "**Intra-procedural Optimization of the Numerical Accuracy of Programs**", Nasrine Damouche, Matthieu Martel, Alexandre Chapoutot, *Formal Methods for Industrial Critical Systems*, 2015.
- We talk about optimisation – but optimising compilers generally just improve programs, more or less heuristically. What can we say about *optimality*? See for example, "**An algorithm for the optimization of finite element integration loops**", Fabio Luporini, David A. Ham, Paul H. J. Kelly, *ACM TOMS* 2017.
- A major element of compilation ~~not covered in this course~~ is scheduling – both of instructions, and of loops and loop nests. Is finding a new schedule for an existing algorithm "just scheduling", or might it sometimes be a truly new inventive step – in fact a new algorithm? See for example, "**Diamond Tiling: Tiling Techniques to Maximize Parallelism for Stencil Computations**", Uday Bondhugula et al, *IEEE TPDS 2016*.

Feeding curiosity…

- **Generalising LICM**: Loop-invariant code motion deals with expressions which are redundantly computed in iterations of the immediately-enclosing loop. But consider:

```
for i
  for j
    R[i,j] = A[i]*B[i] + C[j]*D[j]
```

We see that "A[i]*B[i]" is loop-invariant. But what about "C[j]*D[j]"?

We can eliminate this redundancy by introducing a new *vector* temporary T:

```
for j
  T[j] = C[j]*D[j]
for i
  p = A[i]*B[i]
  for j
    R[i,j] = p+T[j]
```

Most compilers don't do this (we didn't find any), perhaps because allocating new temporary arrays may lead to unwanted consequences. It does, however, really help some applications.

See Fabio Luporini, Ana Lucia Varbanescu, Florian Rathgeber, Gheorghe-Teodor Bercea, J. Ramanujam, David A. Ham, and Paul H. J. Kelly. **Cross-Loop Optimization of Arithmetic Intensity for Finite Element Local Assembly.** *ACM Trans. Archit. Code Optim. 11, 4, Article 57 (January 2015* https://doi.org/10.1145/2687415

Student question: robust simple LICM without SSA

- I understand why SSA is helpful in compilers in general, but also that the algorithms to generate it are rather complicated. In the case where we just want to do Loop Invariant Code Motion, I think I came up with a simpler scheme which essentially does what the phi nodes do without needing to implement the algorithm required to transform the whole program to SSA:

```
for each loop invariant assignment t = a + b you want to hoist:
  hoist it to the loop preheader with a new name - e.g. t1 = a + b
  replace the assignment in the loop with a move operation t = t1
```

- This seems to be ideal because:
- You don't have to check the conditions for LICM
- You don't have to implement the hard SSA algorithms
- But, I couldn't find any resources in the textbooks or online doing it this way, so I think maybe either:
- I misunderstood and this was exactly what was being suggested in the SSA part - i.e. if we are just doing LICM we don't need to implement full SSA. Though I thought that the SSA method involved putting the t = t1 at the control flow join.
- There is a flaw with this method that I haven't spotted.
- I'd be interested in everyone's opinion. This isn't really exam related; I was just interested. :)

- Excellent observation. I think you are right that this is enough for LICM.
- As you say, what you're proposing essentially leads to the same result as the SSA-based approach presented in the lecture.
- LICM is used in the lecture to motivate SSA, which is used for other more complicated analyses and transformations in modern compilers like LLVM and GCC. What you have pointed out is that LICM is not, by itself, a particularly compelling reason to transform into then out of SSA.
- SSA is however useful for more complicated things. You might enjoy this page, which documents all the analysis and transformation passes in LLVM:

  https://llvm.org/docs/Passes.html#sccp-sparse-conditional-constant-propagation
- In particular, you might look at sccp: Sparse Conditional Constant Propagation [https://llvm.org/docs/Passes.html#id79] (a slightly extended explanation is also here: https://en.wikipedia.org/wiki/Sparse_conditional_constant_propagation). SCCP is based on this paper:

  Constant propagation with conditional branches. Wegman and Zadeck ACM TOPLAS 1991 (https://www.cs.wustl.edu/~cytron/531Pages/f11/Resources/Papers/cprop.pdf)
- The Sparse Simple Constant (SSC) algorithm presented there is linear in the size of the SSA graph; they comment that without SSA, it would be much much slower without SSA, in the worst case slower in proportion to the number of variables in the program. But this is beyond what we had time for in this course.

Student question: robust simple LICM without SSA

For f), **int c = D[N];** is loop invariant, as D, N all come from outside loop. However, it does not dominate all loop exits (condition given in slides for not hoisting). **However, intuitively this doesn't matter** as nothing uses **c** after the loop anyways -> i.e. **c is not liveOut from pre-header.** So do we conform to the "definition" or do we use our intuition when answering this question?

2a Draw the control-flow graph for this Java code fragment:

```
S1:  int a=0;
S2:  int b=0;
S3:  while (a<N) {
S4:    a = a+1;
S5:    if (E[a] == 0) {
S6:       b = b+1;
S7:       int c = D[N];
S8:       E[a] = c+b;
       }
     }
S9:  System.out.println(b);
```

Your control-flow graph should have one node for each of the labelled statements S1-S9.

b Which of the statements labelled S1-S9 above are dominated by statement S4?

c Which of the variables are induction variables?

d List the nodes in each of the natural loops present in this program fragment.

e List the relevant reaching definitions for statements S8 and S9.

f Can S7 be hoisted into the loop's pre-header? Explain.

- According to the lecture slides, S7 can't be hoisted since it does not dominate all loop exits.
- But, as you say, it might not matter, in an example like this where c is not live on exit.
- However, hoisting S7 could actually cause a big problem. Suppose the array D is not allocated, or not big enough: the access to D[N] might not be valid. In fact it might actually be an access violation - if the address of D[N] is not allocated in the process's virtual address space, the access will cause a page fault - hoisting it might actually make the program crash when it shouldn't. [this is what the question was actually looking for]

- Now suppose S7 were something safe, like "c = N/3". Now hoisting it looks safe. But we might wonder whether it's a good idea, as it would put S7 on an always-taken path - whereas the conditional on S5 might very, very rarely activated. We are at risk of making the program slower.

- In some instruction sets, for example Intel's Itanium, there is a special load instruction for situations like this, which loads from memory (eg c=D[N]), but if the address is invalid it doesn't throw a fault - it just marks the target register (c here) as "NaT" ("not a thing"). Then at S8 you would have an instruction to check whether c is NaT - if so, you re-execute the load there so that the fault occurs at the right point.
- (The real reason Itanium programmers might do this is to increase parallelism; for a more detailed explanation and example see https://devblogs.microsoft.com/oldnewthing/20150804-00/?p=91181).