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Firedrake

Loop fusion for finite element assembly in PyOP2

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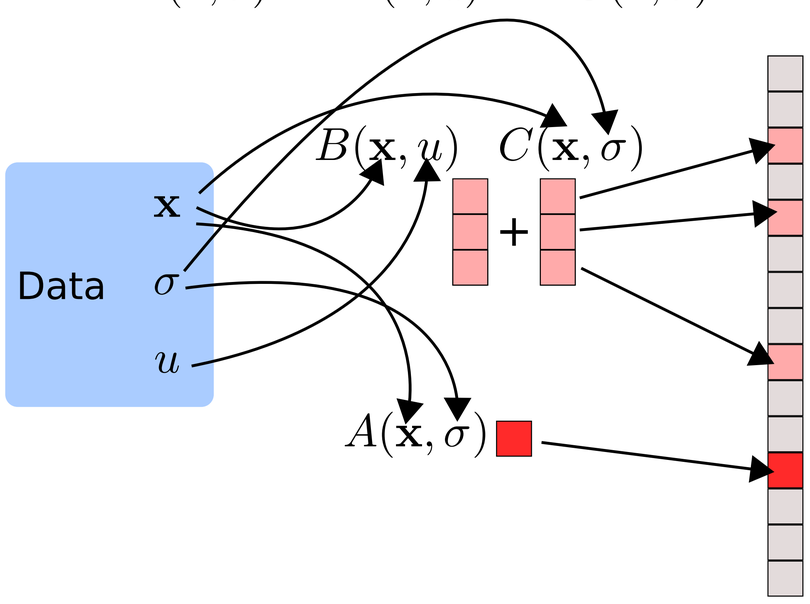
EPSRC Engineering and Physical Sciences Research Council

NERC Science of the Environment

Homogeneous Loop Fusion

$$F = \int \underbrace{\sigma \cdot \tau}_{A(\mathbf{x},\sigma)} + \underbrace{\nabla \cdot \tau u}_{B(\mathbf{x},u)} + \underbrace{\nabla \cdot \sigma v}_{C(\mathbf{x},\sigma)} \, \mathrm{dx}$$




```
BDM = FunctionSpace(mesh, 'BDM', 1)
DG = FunctionSpace(mesh, 'DG', 0)
W = BDM * DG
w = Function(W)
```

```
sigma, u = split(w)
tau, v = TestFunctions(W)
F = (dot(sigma, tau) + div(tau)*u
     + div(sigma)*v)*dx
```

Naive algorithm

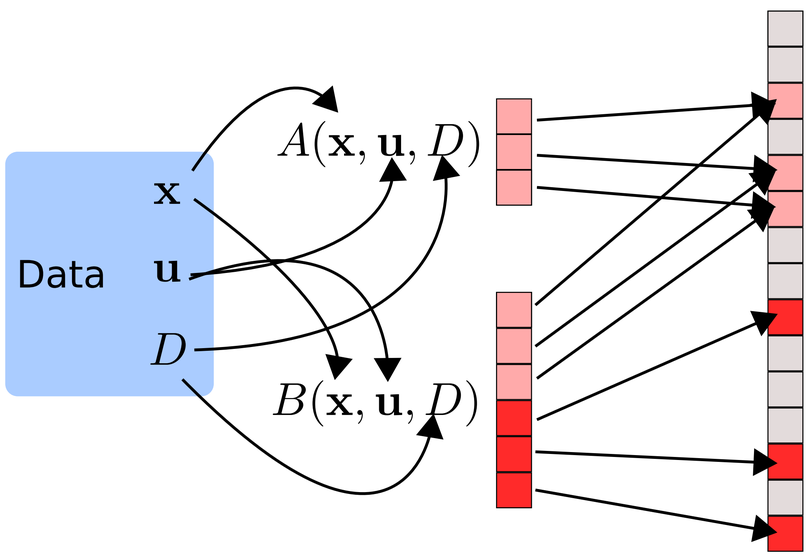
for all c **do**
  R += $A(\mathbf{x}, \sigma)$
end for
for all c **do**
  R += $B(\mathbf{x}, u)$
end for
for all c **do**
  R += $C(\mathbf{x}, \sigma)$
end for

Loop fusion
Reuse of $\mathbf{x}, \sigma$

for all c **do**
  R += $A(\mathbf{x}, \sigma)$
  R += $B(\mathbf{x}, u)$
  R += $C(\mathbf{x}, \sigma)$
end for

Heterogeneous Loop Fusion

$$F = -\int \underbrace{D\mathbf{u} \cdot \nabla\phi}_{A(\mathbf{x},\mathbf{u},D)} \, \mathrm{dx} + \int \underbrace{[\![\phi]\!] \cdot \widetilde{\mathbf{u_n} D}}_{B(\mathbf{x},\mathbf{u},D)} \, \mathrm{dS}$$




```
V = FunctionSpace(mesh, 'DG', 1)
D = Function(V)
phi = TestFunction(V)
```

```
F = (-D*dot(u, grad(phi))*dx
     +dot(jump(phi),
          (un('+')*D('+')
           -un('-')*D('-')))*dS)
```

Naive algorithm

for all c **do**
  R += $A(\mathbf{x}, D)$
end for
for all e **do**
  R += $B(\mathbf{x}, \mathbf{u}, D)$
end for

Loop fusion
Reuse of $\mathbf{x}, \mathbf{u}, D$

for all c **do**
  R += $A(\mathbf{x}, D)$
  for all $e \in \{1, 2, 3\}$ **do**
    if e not seen **then**
      R += $B(\mathbf{x}, \mathbf{u}, D)$
    end if
  end for
end for

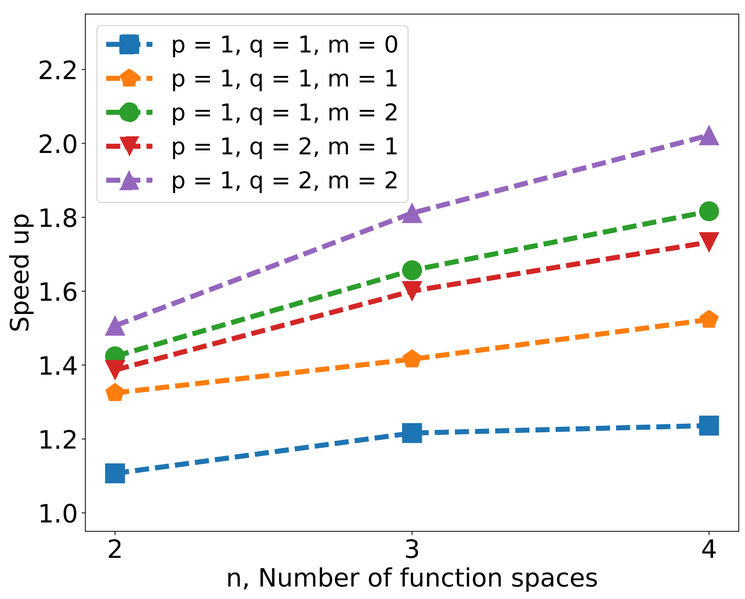
Preliminary results

$$L = \int \sum_{i=1}^{n} \left(\prod_{j=1}^{m} f_j \right) v_i \, \mathrm{dx}$$

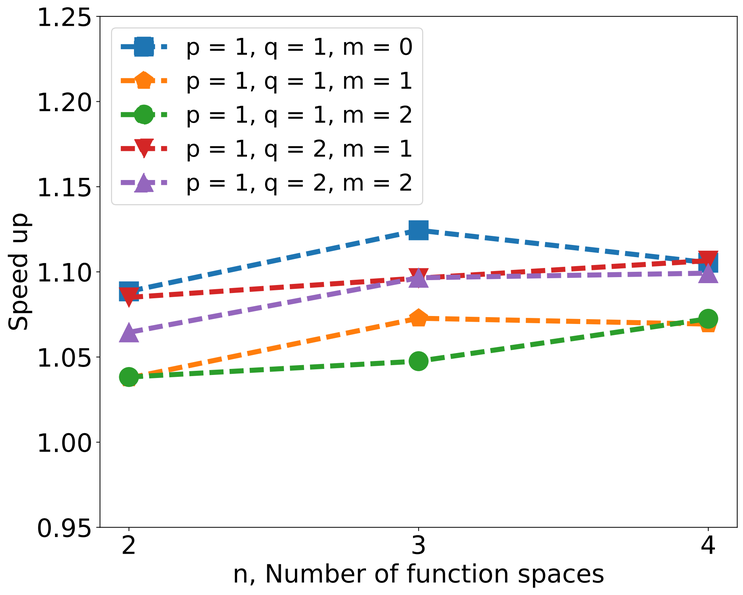
Linear form assembly in mixed function space
Speed up against unfused loop

`n, m, p, q` → More data sharing

```
V = FunctionSpace(mesh, 'CG', p)
W = MixedFunctionSpace([V for _ in range(n)])
# n CG(p) function spaces in mixed space
Q = FunctionSpace(mesh, 'CG', q)
vs = reduce(operator.add, TestFunctions(W))
fs = [Function(Q) for _ in range(m)]
# m CG(q) shared coefficient functions
L= reduce(inner, [vs]+fs) * dx
```



Bad numbering



Reverse Cuthill-McKee

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